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RESEARCHES
ON THE
EVOLUTION OF THE STELLAR SYSTEMS

VOLUME I

ON THE UNIVERSALITY OF THE LAW OF GRAVITATION AND ON THE
ORBITS AND GENERAL CHARACTERISTICS OF BINARY STARS

BY

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ASTRONOMER AT THE LOWELL OBSERVATORY IN CHARGE OF A SURVEY OF THE SOUTHERN HEAVENS
FOR THE DISCOVERY AND MEASUREMENT OF NEW DOUBLE STARS AND NEBULAE;
FELLOW OF THE ROYAL ASTRONOMICAL SOCIETY; MITGLIED DER
ASTRONOMISCHEN GESELLSCHAFT, ETC., ETC.

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DEDICATED

TO

DR. BENJAMIN APTHORP GOULD,

THE ARGELANDER OF THE SOUTHERN HEAVENS,

IN TESTIMONY OF A HIGH APPRECIATION OF LIFE-LONG SERVICES
CONSECRATED TO THE ADVANCEMENT OF

AMERICAN SCIENCE.

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*“L’un des phénomènes les plus remarquables du système du monde est celui de tous les mouvements de rotation et de révolution des planètes et des satellites dans le sens de la rotation du soleil et à peu près dans le plan de son équateur. Un phénomène aussi remarquable n’est point l’effet du hasard; il indique une cause générale qui a déterminé tous ces mouvements. . . . * * **

*“Un autre phénomène également remarquable du système solaire est le peu d’excentricité des orbes des planètes et des satellites, tandis que ceux des comètes sont très allongés. . . . * * **

“Quelle est cette cause primitive? J’exposerai sur cela, dans la note qui termine cet ouvrage (Système du Monde) une hypothèse, qui me paraît résulter avec une grande vraisemblance des phénomènes précédents, mais que je présente avec la défiance que doit inspirer tout ce qui n’est point un résultat de l’observation ou de calcul.”

LAPLACE.

INTRODUCTION.

ONE hundred years ago LAPLACE published an outline of the nebular hypothesis, which has since been confirmed and developed by the labors of astronomers. His physical explanation of the evolution of the planets and satellites, under the gradual operation of the laws of nature, was the logical outcome of his profound study of the mechanism of our system, and rested mainly on the common direction of motion and the small eccentricities and mutual inclinations of the orbits. From the concurrence of such remarkable phenomena in a great number of bodies the author of the *Mécanique Céleste* was led to conceive that at a remote epoch in the past, the matter now constituting the planets and satellites was expanded into a vast rotating fiery nebula, which slowly contracted with the radiation of its heat into surrounding space. According to the mechanical principle of the conservation of areas, the contraction accelerated the rotation and thereby increased the oblateness; when the centrifugal force at the equator became equal to the force of gravity the particles ceased to fall towards the centre, and the nebula shed successive rings or zones of vapor from its equatorial periphery. The condensation of the several rings thus abandoned by the contracting mass eventually gave rise to the bodies of the planetary system.

LAPLACE observed that the comets, unlike the planets and satellites, have every degree of inclination and very high eccentricities, and hence he concluded that they were originally foreign to the solar system; accordingly, in the nebular hypothesis, the comets are regarded as small nebulae which have been drawn to the sun in its secular motion among the fixed stars.

The above hypothesis, based on sound dynamical principles and worked out in detail by the philosophic judgement and imaginative genius of LAPLACE, has merited and received the attention of subsequent natural philosophers. Owing to the brief duration of human history compared to the immense ages required for appreciable cosmogonic changes, probably the evolution of the heavenly bodies can never be observed, but must be inferred from a compara-

tive study of existing phenomena; and hence the sublime discovery of the essential process involved in the formation of the planetary system would necessarily mark an epoch in the history of science. The boldness and profound physical insight with which LAPLACE attacked this problem have justly ranked his effort among the greatest achievements of the human intellect. The germ of the general theory of evolution, which has so powerfully influenced the thought of the nineteenth century, may be traced to the recondite speculations of this great geometer.

The strikingly analogous cosmogonic views advanced by KANT in the *Naturgeschichte und Theorie des Himmels* preceded those of LAPLACE by forty-one years, and hence some priority is claimed for the great metaphysician of Königsberg; but since the real vitality of the nebular hypothesis springs from LAPLACE, whose scientific eminence gave it authority commensurate with the development of Physical Astronomy in the eighteenth century, this great cosmogonic speculation is justly dated from the publication of the *Système du Monde* in 1796.

SIR WILLIAM HERSCHEL'S observations on the different types of stars and nebulae led him to consider them of different ages, and to compare the heavenly bodies in such various stages of development to the mixture of growth and decay presented by the trees of an aged forest. The combination of HERSCHEL'S studies on actual phenomena of the heavens with LAPLACE'S dynamical speculations relative to the solar system gave the nebular hypothesis both an observational and a theoretical basis, and hence it soon became an integral part of scientific philosophy. SIR JOHN HERSCHEL'S survey of the entire heavens supplied new and important observations relative to the appearances of the stars and nebulae, and confirmed the general validity of the nebular hypothesis. When, however, LORD ROSSE'S great Reflector resolved certain clusters previously classed as nebulae, the question naturally arose whether with sufficient power all nebulae might not be resolved into discrete stars. Fortunately, the invention of the Spectroscope about 1860, and HUGGINS'S application of it to the heavenly bodies, showed that many of the nebulae are masses of glowing gas gradually condensing into stars, and, so far as possible, realized the postulates laid down by LAPLACE. JOULE'S discovery of the mechanical equivalent of heat and HELMHOLTZ'S application of the resulting laws of thermodynamics to the heat of the sun, established the contraction of the solar nebula, while the subsequent researches of LANE, NEWCOMB, KELVIN and DARWIN have shown the theoretical possibility of most of the development outlined in the *Système du Monde*.

Notwithstanding the general confirmation of the essential parts of LAPLACE'S speculation, some doubt still remains whether the planets and satellites separated as rings or as lumpy masses, and whether rings of anything like regularity could ever condense into single bodies. The most recent investigations of this question indicate that instead of separating as rings or zones which afterwards condensed, the planets and satellites, like the double stars, assumed originally the form of lumpy or globular masses.

In the time of LAPLACE it was supposed that the figures of equilibrium of rotating masses of fluid, whose particles attract one another according to the Newtonian law, are of necessity surfaces of revolution about the axis of rotation, and therefore that a separation could take place only in the form of a ring or zone. But the investigations of JACOBI showed that a homogeneous mass of fluid in the form of an ellipsoid of three unequal axes rotating about its shortest axis could be maintained in equilibrium by the pressure and attraction of its parts; the figure of such a mass is no longer one of revolution, although it is still symmetrical with respect to the axis of rotation. POINCARÉ'S recent investigation of the stability of the equilibrium of the Jacobian ellipsoid showed that when the oblateness has become about two-fifths the equilibrium in this form becomes unstable, and another figure is developed; the body assumes the form of a pear or an hour-glass with two unequal bulbs, and finally breaks up into two comparable, though unequal, masses. Starting from an entirely different point of view, DARWIN made an independent and almost simultaneous investigation of the form assumed by the mass after the Jacobian ellipsoid becomes unstable. Taking two separate masses of fluid revolving as a rigid system in such close proximity that the tidal distortions of figure cause them to coalesce, he determined the resulting figure of equilibrium, and found a dumb-bell form corresponding very closely to the Apoid discovered by POINCARÉ. Though both of these investigations relate to homogeneous masses, and therefore are not strictly applicable to the cases which arise in nature, yet they agree entirely in proving the existence of unsymmetrical forms of equilibrium; and a comparison of these figures with the drawings of double nebulae made by SIR JOHN HERSCHEL leaves no doubt that the process of separation into unequal but comparable masses indicated by these recondite mathematical researches is abundantly illustrated in the evolution of double stars from double nebulae. If this process has played such a prominent part in the genesis of the stellar systems, it is highly probable that the planets and satellites originated in a similar manner, notwithstanding the abnormally rapid increase in density towards the centres of the solar nebula implied by the separation of such inconsiderable masses.

When NEWTON established the law of universal gravitation he also discovered the true *cause* of the tides of the sea, and outlined some of the principal phenomena which follow from the perturbing action of the sun and moon upon the waters which cover the terrestrial spheroid. After the lapse of more than a century LAPLACE attacked this problem from the dynamical point of view, and developed his celebrated analytical theory of oceanic tides, which has been generally adopted in the subsequent researches of astronomers. About two centuries after NEWTON established the cause of the tides, DARWIN was led to consider not only the tides in the mass of fluid spread over the earth's surface, but also those which arise in the body of the globe, owing to its imperfect rigidity. He inquired whether the earth's mass might not be a fluid of great viscosity, and proceeded to develop the theory of *bodily tides*, and to discuss the bearing of these researches on the cosmogonic history of the earth and moon. When the investigation was subsequently extended to other parts of our system, it was found that while LAPLACE's hypothesis as a whole remained unshaken, some appreciable modifications were rendered necessary, especially in the case of the earth and moon, where the relatively large mass-ratio of the component bodies sensibly increased the efficiency of tidal friction. It seemed clear that in the development of the lunar-terrestrial system, the action of tidal friction had been of paramount importance, but that elsewhere the effects had been much less considerable, owing chiefly to the small masses of the attendant bodies.

When we reflect that the planetary system is made up of a great number of very small bodies revolving in almost circular orbits about large central masses, and is therefore different from all other known systems in the heavens, although other systems like it may exist unobserved, it is remarkable that previous investigators have almost invariably approached the problems of Cosmogony from the point of view of the planets and satellites, and that no considerable attempt has been made to inquire into the development of the great number of systems observed among the fixed stars. The short period of time which has elapsed since the explorations of the Telescope have made known the general state of the heavens, with the impossibility of observing any considerable changes, except in the case of double stars, may perhaps account for the natural tendency to focus all effort upon the development of the planets and satellites. But the peculiar character of our system, compared to other known systems in space, renders this procedure incapable of giving us any general law of nature. It is only from a study of the systems of the universe

at large that we may hope to throw light upon the general problems of Cosmogony; among these systems the binary stars are eminently suited for such an investigation.

In the present work we propose to investigate the evolution of the stellar systems. The problem is difficult and the observations are incomplete, and hence in this arduous undertaking we may beg the indulgence of astronomers for such imperfections as the discussion of the subject will necessarily exhibit. The present volume is devoted mainly to the *facts* as made known by the labors of double-star observers since the time of SIR WILLIAM HERSCHEL; the more theoretical inquiry into the Secular Effects of Tidal Friction and the Processes of Cosmogony is reserved for subsequent treatment.

It would seem that the micrometrical measures discussed in this work establish for the first time, on a secure observational basis, the general shape of the real orbits of double stars. It follows from the results here brought to light that the most probable eccentricity among double stars is over 0.45, and since this mean value is deduced from the consideration of forty orbits, which future observations will not alter materially, we see that such high eccentricities are characteristic of the stellar systems. In the solar system the mean eccentricity for the great planets and their satellites does not surpass 0.0389, and hence we see that the *average eccentricity among double stars is about twelve times that found in our own system*. The great number of binary stars and the practical certainty that the properties deduced from forty of the best orbits now available will be confirmed by the stellar systems in general, justifies us in raising this remarkable induction, relative to the eccentricities, to the dignity of a fundamental law of nature. The binary stars are therefore distinguished from the planets and satellites by two striking characteristics:

1. *The orbits are highly eccentric.*
2. *The stars of a system are comparable, and frequently almost equal, in mass.*

The first of these remarkable properties is traced mainly to the condition stated in the second; high eccentricities probably did not belong to these systems originally, but have been *developed* by the secular action of tidal friction, which is a physical cause affecting all cosmical systems.

In developing the theory of gravitation mathematicians have very generally assumed that the attracting masses are rigid solids, and hence it has been easy to overlook the fact that nearly all the bodies of the visible universe are really *fluid*. The stars and nebulae are self-luminous masses of a gaseous, liquid or

semi-solid nature, and hence it is apparent that in such systems enormous bodily tides will necessarily arise from the mutual gravitation of the particles. Tides are cosmic phenomena as universal as gravitation itself; and since tidal friction will operate in every system of fluid bodies which is endowed with a relative motion of its parts, we see that the general agency of bodily tides gives rise to most important secular changes in the figures and motions of the heavenly bodies. The tidal alterations of figure, which modify the attraction on neighboring bodies, will become especially marked in the case of double stars and double nebulae, where two large fluid masses in comparative proximity are subjected to their mutual gravitation; and hence if the bodies of such a system be rotating as well as revolving the secular working of tidal friction becomes an agency of great and indeed of paramount importance. The general theory of all the secular changes which follow from the double tidal action arising in a binary system remains to be developed, but meanwhile the work of DARWIN in connection with the extension which I have given his researches, makes known some of the more important effects.

From our previous investigations it seems exceedingly probable that the great eccentricities now observed among double stars have arisen from the action of tidal friction during immense ages; that the elongation of the real orbits, so unmistakably indicated by the apparent ellipses described by the stars, is the visible trace of a physical cause which has been working for millions of years. It appears that the orbits were originally nearly circular, and that under the working of the tides in the bodies of the stars they have been gradually expanded and rendered more and more eccentric.

Some simple considerations will enable us to see how these general results arise from the secular action of tidal friction. Suppose the two stars of a system to be spheroidal fluid masses of small viscosity, and let us assume, conformably to the motions observed in the solar system and to those which would result from the division of a double nebula, that the two bodies are rotating about axes nearly perpendicular to the plane of orbital motion, and in the same direction as the revolution about the common centre of gravity; also let the angular velocity of rotation considerably surpass that of orbital revolution. Then, as the fluid is viscous, the tides raised in either mass by the attraction of the other will lag, and hence the major axes of the tidal ellipsoids will point in advance of the tide-raising bodies, and the tidal elevations will exercise on them tangential disturbing forces which tend to accelerate the instantaneous velocities and thereby increase the mean distance. The reaction of the revolving bodies upon the tidal protuberances will retard the axial rotations; for the

moment of momentum of the whole system is constant, and the moment of momentum of axial rotation lost by the stars must be just equal to the gain in moment of momentum of orbital motion. Thus the rotations of the stars are diminished, while the mean distance is correspondingly increased.

But the tangential disturbing force is found to vary inversely as the seventh power of the distance, and hence when the orbit is eccentric the accelerating force at periastron is very much greater than at apastron. The result is that at periastron the disturbing force increases the apastron distance by an abnormally large amount, while at apastron it increases the periastron distance by a very small amount. Thus while the ellipse is being gradually expanded, the apastron is driven away so rapidly compared to the slight recession of the periastron that the orbit grows more and more eccentric. When the axial rotations are sufficiently reduced by the transfer of axial to orbital moment of momentum this change of the system will finally cease; under conditions different from those mentioned above the eccentricity and major axis may decrease, and various other changes take place.

The causes here briefly sketched appear to be sufficient to account for the development of double stars, and the tidal theory might therefore be regarded as satisfactory; yet if the explanation be deemed incomplete it is easy to adduce considerations which exclude other conceivable hypotheses. Let us imagine the x -axis to represent the region of eccentricity, and divide this line into convenient parts, making the intervals, say, 0.1; then we may erect ordinates denoting the number of orbits falling in a given region, and thus illustrate the distribution of orbits as regards the eccentricity. The irregular line which results from connecting the points determined by a finite number of orbits would become a smooth curve if the number were indefinitely increased. In case of the double stars we obtain what is essentially a probability curve with the maximum near 0.45; the slope on either side appears to be somewhat gradual, but the curve vanishes at zero and unity.

If we make a similar representation for the orbits of comets, we shall find a very high maximum at the eccentricity *unity*; in this case both slopes are extraordinarily steep, though perhaps the curve descends with less rapidity on the side towards the origin, on account of the considerable number of periodic comets which have been gradually accumulated by the perturbing action of the planets. The corresponding curve for the planets and satellites has a high maximum near 0.0389; and while both slopes are steep, that on the side from the origin is the more gradual by virtue of the somewhat unusual eccentricities of *Hyperion*, the Moon and *Mercury*.

If we inquire into the physical meaning of these illustrations, it is easy to see that the distribution of the cometary orbits about the parabolic eccentricity indicates, as LAPLACE first pointed out, that the comets have been drawn to our system from the regions of the fixed stars. The curve for the planets and satellites proves merely that the eccentricities were originally small, and that, under the minimized effects of tidal friction resulting from such inconsiderable masses, they have never been much increased. The curve for the orbits of double stars is of such a nature that we cannot, as in the case of comets, assign to these systems a fortuitous origin; for in this event the eccentricities would surpass, equal or approximate unity, and the periods of revolution, if finite, would be of immense duration; nor could any cause be assigned for the reduction of the eccentricity and period if it be admitted that anything which might properly be called a system could arise from the approach of separate stars. On the other hand the stellar orbits have no close analogy with those of the planets and satellites, for they are densest in the region of mean elliptic eccentricity, and thus almost equally removed from the two extremes presented in the solar system. They were therefore of this mean form originally, or have been made so by a cause which has left a distinct impress upon the nature of the systems. The secular alteration in the figure of equilibrium of a greatly expanded mass like a double nebula would of necessity be very gradual, and hence it follows that the mass cut off under the increased centrifugal force incident to slowly accelerated rotation would begin to revolve in an orbit of comparatively small eccentricity. Indeed, were the initial eccentricity considerable the two nebulae would come into grazing collision at periastron, and in consequence of the resistance encountered the system would rapidly degenerate into a single mass. When at length the bodies are separated, each mass will contract and gain correspondingly in velocity of axial rotation, and tidal friction will begin expanding and elongating the orbit; nothing but this secular process would be adequate to develop the mean eccentricities observed in the immensity of space. If then tidal friction be sufficient to account for the elongation of the real orbits of double stars, we shall be justified in concluding that it is the true cause of the phenomenon. Accordingly, it does not seem probable that the conclusions reached in the *Inaugural Dissertation* which I presented to the Faculty of the University of Berlin will be materially altered, but some of the many problems connected with the general theory of tides still need additional elucidation. If we shall be able to explain the origin and development of double stars, the abundance of such systems will raise a presumption that the agencies and processes involved are more or less general throughout the universe, and no inconsiderable light

will be thrown upon the laws of Cosmogony. By extending our researches to the various classes of nebulae and clusters, additional knowledge will be gained, and in the course of time it will be possible to approach the general problem of cosmical evolution.

For more than two centuries Celestial Mechanics has been occupied with the confirmation of the Newtonian law, and with the development of theories for the precise determination of the figures and motions of the heavenly bodies. In the writings of NEWTON and LAPLACE the attracting masses are essentially solid spheroids covered by a fluid in equilibrium. The theories of the orbital motions and perturbations of the planets, and of the figures and rotations of these bodies about their centres of gravity, are treated mainly from the point of view of rigid dynamics, and little account is taken of the fact that so far as known the heavenly bodies are masses of viscous fluid. The work of DARWIN on the precession of a viscous spheroid and on the secular effects of bodily tidal friction marks an epoch in the history of Celestial Mechanics, which will eventually become a science of the equilibrium and motion of fluids, and must take account of not only the attractions due to undisturbed figures, but also the forces arising from tidal deformation, with the resulting secular changes in the motions of the heavenly bodies.

Physical Astronomy has been devoted heretofore to first approximations under the law of universal gravitation, in particular, to the development of methods for tracing the exact paths of the heavenly bodies through past and future centuries; the theories thus developed are applicable to all periods of recorded history and are justly considered the most imposing monuments yet reared by the human intellect. But the ultimate aim of Astronomy is not only to explain and to predict phenomena which the course of time will make known to observers, but also to determine the secular effects of cumulative causes, and, by approaching the primitive condition of the universe, to discover the origin and to trace the evolutionary history of the stars. As the slow processes of cosmical development are forever withheld from the direct vision of the astronomer, and can be discovered only by the investigation of the continued effects of laws and causes now at work in the heavens, the solution of this sublime problem will be an achievement not unworthy of the human mind.

HAWLEY HOUSE,
5326 Washington Ave., Chicago,
May 6, 1896.

CHAPTER I.

ON THE DEVELOPMENT OF DOUBLE-STAR ASTRONOMY, AND ON THE MATHEMATICAL THEORIES OF THE MOTIONS OF BINARY STARS.

§ 1. *Historical Sketch of Double-Star Astronomy from Herschel to Burnham.*

THE suggestive relation of certain prominent stars, in contrast with the irregular manner in which the multitude are strewn over the surface of the celestial sphere, presented to the minds of the ancients the appearance of arrangement or classification; the more or less obvious constellations thus invented for bright and widely-separated objects were of various sizes, and frequently of an arbitrary character. The condensation of the stars into natural groups, such as the *Pleiades*, *Coma Berenices*, and the clouds in the Milky Way, must have attracted early attention, but no one attempted a philosophical inquiry into the cause of such arrangement until MITCHELL took up the question in 1767, and showed from the theory of probability that a real physical connection was strongly indicated. Further considerations of a similar character led him to predict in advance of observation that compound stars would be found revolving about their common centres of gravity. LAMBERT had surmised the existence of possible stellar systems in 1761, and GIORDANO BRUNO, CASSINI, and MAUPERTUIS had advanced even earlier conjectures of the same kind. The argument for physical connection of closely associated stars, based on the theory of probability, has since been greatly extended by WILLIAM STRUVE, and a practical verification of theory is furnished by the evidence of orbital motion in about 500 out of the 5000 interesting double stars catalogued by modern observers.

The designation *double-star* ($\delta\iota\pi\lambda\omicron\upsilon\varsigma$) was first employed by PTOLEMY in describing the appearance of ν *Sagittarii*. The first object of the kind ever discovered with the Telescope was probably ζ *Ursae Majoris*, which appeared double to RICCIOLI about the middle of the seventeenth century. The quadruple system θ *Orionis* was detected by HUYGHENS in 1656, and the wide pair γ *Arietis* by HOOKE some eight years later. While observing a comet at Pondicherry, India, in December, 1689, FATHER RICHAUD separated the com-

ponents of α *Centauri*, and thus secured the first record of a star, which has proved to be binary. The duplicity of γ *Virginis* was accidentally discovered by BRADLEY and POUND in 1718, and subsequently re-discovered by CASSINI and MESSIER, while observing occultations, with a view of finding evidence of an atmosphere surrounding the moon.

α *Geminorum* was resolved in 1719, 61 *Cygni* in 1753, and β *Cygni* in 1755; but although these sporadic discoveries had been made, no systematic search for double stars was attempted until 1777, when CHRISTIAN MAYER, of Mannheim, began to collect a list of these remarkable objects. Having reached the conclusion that faint stars near larger ones are essentially revolving planets, he searched the heavens attentively with an eight-feet mural circle, by BIRD, and discovered in all some seventy-two pairs, including γ *Andromedae*, ζ *Cancrī*, α *Herculis*, ϵ *Lyrae* and β *Cygni*. Unfortunately, the wide objects within the reach of such a telescope seldom have any appreciable relative motion, and hence the stars discovered by MAYER give very little evidence of the physical connection which he expected.

The real history of double-star discovery and measurement, dates from the explorations begun by SIR WILLIAM HERSCHEL in 1779. This indefatigable observer sought to grapple with the unsolved problem of stellar parallax, which had engaged the attention of astronomers since the time of COPERNICUS. Rejecting the methods recommended by GALILEO, FLAMSTEED and BRADLEY, he proposed one of his own, depending on the measurement of position-angles of two stars of unequal magnitudes from opposite sides of the earth's orbit. HERSCHEL supposed the double stars to be mere groups of perspective, and hence he hoped to detect the relative parallax due to the orbital motion of the earth. He resolved to examine every star in the heavens with the utmost attention under a very high power; the superiority of his telescope gave him an advantage over previous observers; and moreover, his improved optical appliances were supplemented by great energy and boundless enthusiasm. During the interval from 1779 to 1784 he made an extensive catalogue of double stars, some of which he hoped would ultimately prove to be suitable for measurement of parallax. In 1782 he communicated to the Royal Society a catalogue of 269 double stars, 227 of which were new, and followed it three years later by a second catalogue containing 434 such objects. For the next fifteen years the attention of the great observer was devoted to, among other things, the measurement of these pairs, with a view of finding those best adapted to parallax determination. Slight changes were observed from the first, but in most cases the shifting of the relative positions of the objects was attributed

either to the proper motions of the stars, or to the secular motion of the sun in space. The motions were so slow that it took the observations of many years to prove conclusively that certain double stars are moving in regular orbits. This unexpected and astonishing result was finally announced by HERSCHEL in 1802, and demonstrated during the following year by his elaborate memoirs on binary stars. These investigations supplied the first satisfactory evidence that some of the double stars constitute genuine stellar systems maintained by the action of universal gravitation. HERSCHEL's celebrated papers dealt with the motions of such objects as ξ *Ursae Majoris*, 70 *Ophiuchi*, γ *Virginis*, α *Geminorum*, η *Coronae Borealis*, ξ *Boötis*, η *Cassiopeae*, ζ *Herculis*, μ^2 *Boötis*; and in some cases assigned rough estimates of the periods of revolution. The interest in an announcement which opened up fields of inquiry of the widest scope, was fully commensurate with the inherent importance of the discovery; and yet, notwithstanding the splendor of the achievement, double stars were little observed during the first twenty years of this century.

SIR JOHN HERSCHEL began some preliminary work on double stars in 1816, and was soon joined by SIR JAMES SOUTH. During the next ten years these two observers published several series of observations made either conjointly or separately; and when SIR JOHN HERSCHEL made his survey of the Southern Hemisphere, over 2000 pairs were discovered and roughly measured. The conscientious records which he has left us in the *Results* of his observations at the Cape of Good Hope, as well as the catalogues since published, and his elegant researches on the orbits of double stars, ensure to him a distinguished place among those astronomers who have labored to advance our knowledge of binary systems.

The systematic survey of the part of the heavens between the north pole and fifteen degrees south declination, executed by WILLIAM STRUVE between the years 1824 and 1836, will long remain the most important contribution to double-star Astronomy ever made by one man. The instrument used was the Dorpat 9.9-inch refractor by FRAUNHOFER; the results furnished the material of the *Mensurae Micrometricae* which includes careful observations of 3112 double and multiple stars, besides records of his previous work with smaller instruments. The labors of WILLIAM STRUVE abolished HERSCHEL's cumbersome method of referring position-angles to the quadrants, and reduced double-star Astronomy to a scientific basis by reckoning the angle continuously from 0° to 360° . Out of this extensive work grew other reforms, such as the superior classification and arrangement of the results, and in this way STRUVE laid the foundations of the subsequent development of the science.

Among the other observers who contributed to this branch of Astronomy prior to 1850, we may mention especially MÄDLER, BESSEL, and DAWES. The measures of DAWES take high rank for quality and serve as an example of what may be done by private observers with limited appliances. Other deceased observers especially deserving of mention for important contributions to the records of double-star Astronomy are SECCHI, KAISER, KNOTT, ENGLEMAN, JEDRZEJEWICZ, and, above all, BARON DEMBOWSKI.

Though the last-mentioned observer worked privately and with a small instrument, his measures are more extensive and perhaps more accurate than those of any other observer either living or dead. Covering the period from 1854 to 1878, the work included measures of all the pairs in the *Mensurae Micrometricae* accessible to his 7-inch glass, besides numerous observations of pairs more recently discovered by himself, OTTO STRUVE, BURNHAM and ALVAN CLARK. The twenty thousand precise measures executed by this great astronomer were collected after his death, edited by OTTO STRUVE and SCHIAPARELLI, and published in two large quarto volumes by the *Accademia dei Lincei* of Rome.

Beginning prior to 1840 and extending over the next fifty years, the double-star work of the illustrious OTTO STRUVE furnishes by far the longest and most homogeneous set of observations yet made by any astronomer. Besides records of the numerous stars discovered by himself and by his father, OTTO STRUVE's work includes reliable data for the most important stars discovered by other previous and contemporary observers. Many of his own stars are close and have proved to be comparatively rapid, and hence will soon yield satisfactory orbits.

Among living observers the names of OTTO STRUVE, HALL, DUNÉR, SCHIAPARELLI, TARRANT, BIGOURDAN, MAW, GLASENAPP, TEBBUTT, STONE, COMSTOCK, KNORRE, SEABROKE, DOBERCK, PERROTIN, HOUGH, and BURNHAM will be familiar to the reader. Each has contributed important material for the study of the stellar systems, but the work of STRUVE, HALL, SCHIAPARELLI, and BURNHAM is especially important to the computer, as covering a long series of years and thus supplying homogeneous material for the determination of the orbits of revolving binaries.

Prior to 1870 it had been generally held by such authorities as DAWES that the subject of double stars was practically exhausted by the discoveries of the HERSCHELS and the systematic surveys of the STRUVES. As the latter had swept over all the brighter stars in the northern heavens, including about 140,000 objects, we may refer with a certain pleasure to the epoch-making discoveries since made by BURNHAM, who has detected nearly 1300 important pairs which had escaped all previous observers. BURNHAM's stars are either very close or

the companion is very faint, and their high importance lies in their rapid orbital motion. This characteristic of BURNHAM'S stars has already enabled us to deduce a number of most interesting orbits. It is probable that during the next half century his stars will yield more good orbits than all the other stars previously discovered put together. When we remember that the aim of the observer is to determine the paths of the stars with a view of throwing light upon the character of the stellar systems, it is clear that the measurement of these close objects, which will yield a large number of orbits within a reasonable time, is the most pressing duty of the observer of the future. Many distinguished observers have devoted their attention to the sidereal studies begun by the HERSCHELS and developed by the STRUVES, but none have labored more devotedly or achieved more splendid discoveries than the illustrious BURNHAM.

§ 2. *Laplace's Demonstration of the Law of Gravitation
in the Planetary System.*

SUPPOSE we denote by X and Y the forces which act on a planet, resolved along the coördinate axes, and directed towards the origin at the centre of the sun; let the plane of the orbit be taken as the plane of xy . Then we have, as the equations of motion,

$$\frac{d^2x}{dt^2} + X = 0 \quad , \quad \frac{d^2y}{dt^2} + Y = 0. \quad (1)$$

If we multiply the first equation by $-y$, and the second by x , and add the results, we find

$$\frac{d(xy - yx)}{dt^2} + xY - yX = 0. \quad (2)$$

But $\frac{xy - yx}{dt}$ is the double areal velocity, and by KEPLER'S law the areas described by the radius-vector of the planet are proportional to the time. Therefore we have

$$xY - yX = 0, \quad (3)$$

or the forces X and Y are related as the coördinates x and y ; which indicates that the attractive force is directed to the origin of coördinates. Therefore we conclude that the force which retains the planets in their orbits is directed to the centre of the sun.

We may now investigate the law of this force at different distances. On multiplying the first of (1) by dx , and the second by dy , adding and integrating, we have

$$\frac{dx^2 + dy^2}{dt^2} + 2\int(Xdx + Ydy) = 0. \quad (4)$$

If we denote the double areal velocity by c , we shall have

$$dt = \frac{x dy - y dx}{c},$$

and hence the last equation gives

$$\frac{c^2(dx^2 + dy^2)}{(x dy - y dx)^2} + 2 \int (X dx + Y dy) = 0. \quad (5)$$

In polar coordinates,

$$x = r \cos v \quad ; \quad y = r \sin v \quad ; \quad r = \sqrt{x^2 + y^2},$$

and we find

$$dx^2 + dy^2 = r^2 dv^2 + dr^2 \quad ; \quad x dy - y dx = r^2 dv.$$

If we now denote by F the central force which acts on the planet, we shall have

$$X = F \cos v \quad ; \quad Y = F \sin v \quad ; \quad F = \sqrt{X^2 + Y^2}.$$

Hence we get

$$X dx + Y dy = F \cos v (\cos v dr - r \sin v dv) + F \sin v (\sin v dr + r \cos v dv) = F dr.$$

Therefore

$$\frac{c^2(r^2 dv^2 + dr^2)}{r^4 dr^2} + 2 \int F dr = 0, \quad (6)$$

and we find

$$dv = \frac{c dr}{r \sqrt{-c^2 - 2r^2 \int F dr}}. \quad (7)$$

If the force F were a known function of r , we might find v by the process of quadrature. But since the force is unknown, although the species of curve it causes the planets to describe is known, we may differentiate equation (6), and obtain

$$F = \frac{c^2}{r^3} - \frac{c^2}{2} \frac{d \left\{ \frac{dr^2}{r^4 dv^2} \right\}}{dr}. \quad (8)$$

KEPLER found from observation that the planets and comets respectively move in ellipses and parabolas, which are conic sections. The polar equation of a conic may be written

$$\frac{1}{r} = \frac{1 + e \cos(v - \omega)}{a(1 - e^2)}, \quad (9)$$

whence we find

$$\frac{dr}{r^2 dv} = \frac{e \sin(v - \omega)}{a(1 - e^2)},$$

$$\text{or} \quad \frac{dr^2}{r^4 dv^2} = \frac{e^2 - e^2 \cos^2(v - \omega)}{a^2(1 - e^2)^2}. \quad (10)$$

If we reduce the second member by (9),

$$e \cos(v - \omega) = -1 + \frac{a(1 - e^2)}{r},$$

we shall easily find

$$\frac{dr^2}{r^4 dv^2} = \frac{2}{ar(1 - e^2)} - \frac{1}{r^2} - \frac{1}{a^2(1 - e^2)^2};$$

and hence we get

$$\frac{d \left\{ \frac{dr^2}{r^4 dv^2} \right\}}{dr} = -\frac{2}{ar^2(1 - e^2)} + \frac{2}{r^3}. \quad (11)$$

Thus equation (8) becomes

$$F = \frac{e^2}{a(1 - e^2)} \cdot \frac{1}{r^2}. \quad (12)$$

Therefore we conclude that the force which causes the planets and comets to move in conic sections about the sun varies inversely as the square of the distance from the sun's centre. Such is the demonstration by which LAPLACE was led to the law of universal gravitation; it rests solely on phenomena, and is independent of any hypothesis. The original demonstration by NEWTON was based on geometrical methods, and is given in the *Principia*, Lib. I., Sec. III., Prop. XI.

The laws of KEPLER made use of in these demonstrations are taken as fundamental facts discovered from observation; but planetary observations in the time of KEPLER were not sufficiently exact to ensure entire rigor in these laws, and besides no account was taken of the mutual gravitation of the planets. Hence it will be seen that the accuracy of the laws of KEPLER, even in the time of NEWTON, could be maintained only within given limits.

It is never possible to realize the conditions of undisturbed motion assumed by KEPLER, and hence the problem presented to astronomers can be solved only by successive approximations. Assuming that the facts embodied in KEPLER's laws are strictly true, NEWTON's reasoning shows that the law of gravitation is mathematically exact; if on the other hand we assume the accuracy of the law of NEWTON, we are led directly to the laws of KEPLER as phenomena which would arise under the operation of gravitation. The laws of KEPLER are sensibly correct, and on the admissible supposition that they are entirely rigorous,* astronomers have applied the law of gravitation to the disturbed motions of the planets, with a view of explaining observed inequalities, and of discovering from theory other perturbations which have been subse-

*The third law is here supposed to be corrected for the planetary masses neglected by KEPLER.

quently verified by observation. This development of the planetary theories has occupied the attention of astronomers for over two centuries, and in every case where doubt has arisen the accuracy of the Newtonian law has been verified.

The range of possible inaccuracy has been gradually narrowed, until at present the data of Astronomy show that if the law of nature departs at all from that given by NEWTON, the deviation must be extremely slight. Indeed, the law of gravitation, taken in connection with its simplicity, is so thoroughly established as to authorize the belief that it is rigorously the law of nature. Its brilliant confirmation and extension since the time of NEWTON, especially by LAPLACE, leaves but few, and generally insignificant, motions yet unexplained; and since we know that the slightest deviation from the law of inverse squares would become very perceptible in the motions of the perihelia of the orbits of the planets and the periplaneta of the orbits of the satellites, and no such outstanding phenomena have been disclosed by observation, except in the case of the perihelion of the orbit of *Mercury*, which may be explained in a different manner, it is hardly possible to doubt that the few anomalous phenomena yet remaining will finally be explained in perfect accord with the law of NEWTON.

The strongest proof of the rigor of this law is to be found in the fact that it accounts for both the regular and the irregular motions of the heavenly bodies, and in the hands of LAPLACE and his successors has become a means of discovery as real as observation itself.

A law which explains satisfactorily the figures, the secular variations, and the delicate long-period inequalities of the planets, and above all the numerous perturbations to which the moon is subjected, certainly has a strong claim to be regarded as a fundamental law of nature, and is incontestibly the sublimest discovery yet achieved in any science.

§ 3. *Investigation of the Law of Attraction in the Stellar Systems.*

The labors of NEWTON and LAPLACE on the mechanism of the solar system established the law of gravitation with all the rigor which modern observations could demand; but neither of these two great geometers attempted to apply this law to other systems existing in space. The close of the career of LAPLACE, just a century after that of NEWTON, marks an epoch in the verification of the Newtonian law, since in this year SAVARY devised the first method for determining the orbits of double stars; he justly based his theory on the principle

of gravitation which the author of the *Mécanique Céleste* had recently tested with such thoroughness for the regions about the sun traversed by the planets and comets. The method developed by SAVARY has been improved and rendered more practical by the labors of subsequent geometers, and consequently at the present time there is no considerable body of phenomena which appear to be irreconcilable with the law of NEWTON. Indeed, when proper allowance is made for the large but inevitable errors of our micrometrical measures, all modern observations of binary stars may be explained either by the theory of two bodies revolving under the law of gravitation, or by the action of unseen bodies perturbing the regular elliptical motion. This accordance of observation with theory, while it increases enormously the probability of the Newtonian law, does not furnish an independent criterion; and therefore it is desirable to ascertain the most general form of the expressions which will cause a particle to describe a conic, so that we may determine whether any other law can explain the phenomena. In the case of double stars, micrometrical measures enable us to study only the apparent orbits, which are projections of the real orbits upon the plane tangent to the celestial sphere. The apparent orbits are ellipses, and therefore we may conclude that the real orbits are also conics of the same species. When the orbit is projected the centre of the real ellipse will fall upon the centre of the apparent ellipse, but the positions of the projected foci are not determinate unless the position of the real ellipse is known. Astronomers are accustomed to assume that Newtonian gravitation is the attractive force, and as this requires that the principal star shall be in the focus of the real ellipse, it then becomes easy to deduce the corresponding node, inclination and other elements. It is observed that the principal star is not in the centre of the ellipse, and therefore we infer that the force does not vary directly as the distance. But since the areas swept over by the radius vector are proportional to the times, we may conclude that the force is central; and since the apparent motion of 42 *Comæ Berenices* is rectilinear, it is clear that the orbit is a plane curve, or conic section. As other forces besides gravitation could cause a particle to describe a conic, BERTRAND proposed the following problem to the Paris Academy of Sciences: "*Knowing that a material particle under the action of a central force always describes a conic, it is required to find the expression of this force.*"*

Before presenting the solutions developed by DARBOUX and HALPHEN, we shall give an exposition of the geometrical method by which NEWTON treated the same problem.

In the Scholium to Proposition XVII, Liber I, of the *Principia*, NEWTON

* *Comptes Rendus*, April 9, 1887.

derived the general expression for the force which will cause a particle to describe a conic section, the centre of force occupying any internal point. The demonstration given by NEWTON depends upon several preceding propositions; a more direct but similar solution of the same problem has been published by PROFESSOR GLAISHER in the *Monthly Notices*, Vol. XXXIX.

This investigation is as follows: Let C be the centre of the ellipse, P any point occupied by the particle, Q the point occupied by the particle at the next instant, PZ the tangent at P , PG the diameter through P , CD the semi-conjugate diameter to PG , O the centre of attraction, QS a right line parallel

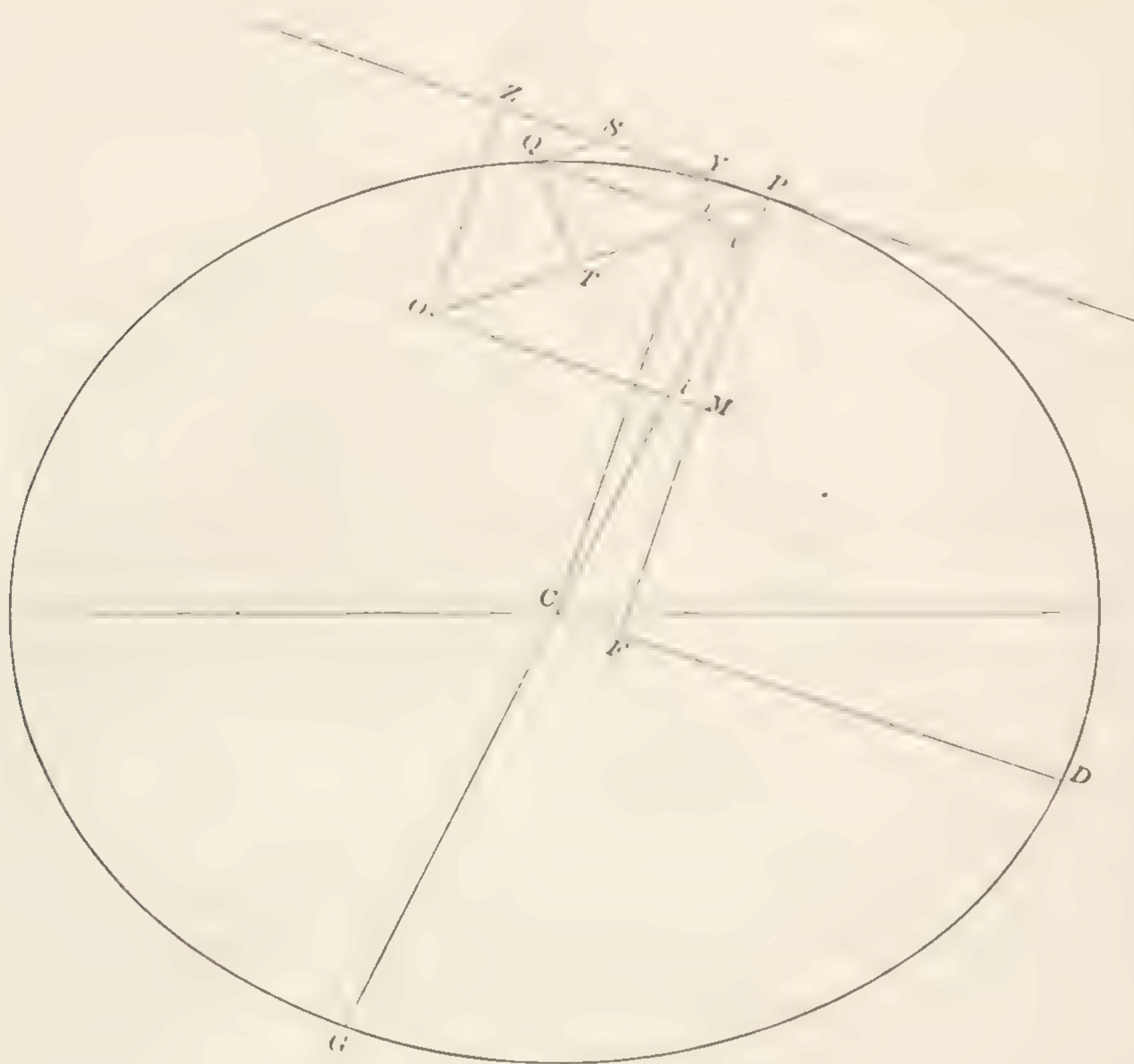


Fig. 1.

to OP , OZ and CY perpendiculars on the tangent from O and C , PF the perpendicular on CD from P , QT the perpendicular from Q on OP , Qr and OM perpendiculars on PF from Q and O , x the intersection of Qr with OP , i the intersection of OM with CP ; and R the required force tending to O .

Then we shall have

$$R = \frac{2h^2}{OP^2} \cdot \frac{QS}{QT^2}, \quad (1)$$

where h denotes the areal velocity.

By the similar triangles QTx and PMO ,

$$\frac{QT}{Qx} = \frac{PM}{OP}. \quad (2)$$

By conic sections,

$$\frac{\overline{Qv^2}}{Pv \cdot Gv} = \frac{CD^2}{CP^2}. \quad (3)$$

And from the figure,

$$\frac{Pv}{QS} = \frac{Pv}{Px} = \frac{Pi}{OP} = \frac{CP}{PF} \cdot \frac{PM}{OP}. \quad (4)$$

Therefore by (3) and (4) we find

$$\frac{\overline{Qv^2}}{QS \cdot Gv} = \frac{\overline{CD^2}}{CP \cdot PF} \cdot \frac{PM}{OP}. \quad (5)$$

In the limit $Qx = Qv$, and hence (2), (3) and (5) give

$$\frac{\overline{QT^2}}{QS} = \frac{2\overline{CD^2}}{PF} \cdot \left(\frac{PM}{OP}\right)^3. \quad (6)$$

Substituting in (1), we obtain

$$R = \frac{h^2}{OP^2} \cdot \frac{PF}{CD^2} \left(\frac{OP}{PM}\right)^3 = \frac{h^2}{a^2b^2} \left(\frac{PF}{PM}\right)^3 \cdot OP = \frac{h^2}{a^2b^2} \left(\frac{CY}{OZ}\right)^3 OP, \quad (7)$$

which is the required law of force.

§ 4. *Analytical Solution of Bertrand's Problem Based on that Developed by Darboux; Solution of Halphen.*

The equations of acceleration are,

$$\frac{d^2x}{dt^2} = -R \frac{x}{r} = -R \cos \theta \quad ; \quad \frac{d^2y}{dt^2} = -R \frac{y}{r} = -R \sin \theta, \quad (1)$$

where R is the attractive force, at unit distance. Multiplying the first by $-y$ and the second by x , and adding, we get

$$x \frac{d^2y}{dt^2} - y \frac{d^2x}{dt^2} = 0. \quad (2)$$

On integrating we obtain

$$x \frac{dy}{dt} - y \frac{dx}{dt} = h. \quad (3)$$

In polar coördinates this equation becomes

$$r^2 \frac{d\theta}{dt} = h = \text{double areal velocity.} \quad (4)$$

Let us now put $u = \frac{1}{r}$, and then

$$r = \frac{\cos \theta}{u}; \quad \frac{dx}{dt} = - \frac{u \sin \theta + \cos \theta \frac{du}{d\theta}}{u^2} \frac{d\theta}{dt}. \quad (5)$$

By equation (4) this becomes

$$\frac{dx}{dt} = -h \left(u \sin \theta + \cos \theta \frac{du}{d\theta} \right), \quad (6)$$

$$\text{and } \frac{d^2x}{dt^2} = -h^2 u^2 \left(u \cos \theta + \cos \theta \frac{d^2u}{d\theta^2} \right). \quad (7)$$

From (7) and (1) we get

$$R = \frac{h^2}{r^2} \left(u + \frac{d^2u}{d\theta^2} \right), \quad (8)$$

where the centre of force is at the origin.

This equation is perfectly general for the determination of R when the equation of the path is known. To get the central force, R , which will cause a particle to describe any given path, we find the value of $\left(u + \frac{d^2u}{d\theta^2} \right)$ for that path, and multiply it by $\frac{h^2}{r^2}$. Therefore, to find the law of R , when the path is a conic section, we have the general equation,

$$ax^2 + 2bxy + cy^2 + 2dx + 2fy = g. \quad (9)$$

Putting $r = \frac{1}{u}$, and transforming to polar coördinates, we have

$$\frac{a \cos^2 \theta}{u^2} + \frac{2b \sin \theta \cos \theta}{u^2} + \frac{c \sin^2 \theta}{u^2} + \frac{2d \cos \theta}{u} + \frac{2f \sin \theta}{u} = g,$$

from which we obtain

$$u = \frac{f \sin \theta + d \cos \theta}{g} + \frac{1}{g} \sqrt{(f^2 + cg) \sin^2 \theta + 2(fd + bg) \sin \theta \cos \theta + (d^2 + ag) \cos^2 \theta}. \quad (10)$$

This equation reduces to the form

$$u = A \sin \theta + B \cos \theta + \sqrt{C \sin 2\theta + D \cos 2\theta + H}, \quad (11)$$

where

$$A = \frac{f}{g}; \quad B = \frac{d}{g}; \quad C = \frac{fd + bg}{g^2}; \quad D = \frac{d^2 + ag - f^2 - cg}{2g^2}; \quad H = \frac{d^2 + ag + f^2 + cg}{2g^2}.$$

From (11) we derive

$$\frac{d^2 u}{d\theta^2} = \frac{-A \sin \theta - B \cos \theta - C^2 - D^2 - (C \sin 2\theta + D \cos 2\theta)^2 - 2H(C \sin 2\theta + D \cos 2\theta)}{(C \sin 2\theta + D \cos 2\theta + H)^{3/2}}. \quad (12)$$

Therefore by (8) we get

$$R = \frac{h^2}{r^2} \frac{(H^2 - C^2 - D^2)}{(C \sin 2\theta + D \cos 2\theta + H)^{3/2}}. \quad (13)$$

This is the general expression for R whatever be the constants a, b, c, d, f and g .

Since by (11) we have

$$u - A \sin \theta - B \cos \theta = \sqrt{C \sin 2\theta + D \cos 2\theta + H},$$

we may write (13)

$$R = \frac{h^2}{r^2} \frac{(H^2 - C^2 - D^2)}{\left(\frac{1}{r} - A \sin \theta - B \cos \theta\right)^3}, \quad (14)$$

which is another general expression for R .

When the conic is an ellipse with the origin at the centre, equation (9) takes the form $ax^2 + cy^2 = ac$, and from (13) or (14) we find after reduction

$$R = \frac{h^2 r}{ac}. \quad (15)$$

The force varies directly as r , which is the well-known law.

When the centre of force is on the x -axis between the centre and one of foci at a distance m from the centre, equation (9) becomes

$$ax^2 + 2amx + cy^2 = a(c - m^2),$$

and we find from (13)

$$R = \frac{h^2}{r^2} \frac{(ac)^{1/2}}{[(a - c + m^2) \cos^2 \theta + c - m^2]^{3/2}}. \quad (16)$$

Since $a - c + m^2$ is always negative, the force at unit distance is a maximum in the direction of the apsides and is a minimum when $\theta = \frac{\pi}{2}$. We have from (14), in this case,

$$R = \frac{h^2 c^2 r}{a(c - m^2 - mx)^3}. \quad (17)$$

This expression can readily be transformed into (16).

When the origin is at one of the foci (13) or (14) gives

$$R = \frac{h^2}{r^2} \frac{c^{1/2}}{a}, \quad (18)$$

which is the Newtonian law.

This is also deducible from (16) by putting $m^2 = c - a$.

When the centre of force is on the x -axis between one of the foci and the nearest apse, at a distance n from the centre, we obtain from (13)

$$R = \frac{h^2}{r^2} \frac{(ac)^{1/2}}{[(a-c+n^2)\cos^2\theta + c - n^2]^{3/2}} \quad (19)$$

Since $a - c + n^2$ is always positive, the force at unit distance is a maximum when $\theta = \frac{\pi}{2}$, and a minimum at the apsides. From (14) it is easy to obtain

$$R = h^2 \frac{c^2}{a} \frac{r}{(c - n^2 - nx)^3}, \quad (20)$$

which may be transformed into (19).

When the centre of force is on the minor axis at a distance k from the centre, equation (13) gives

$$R = \frac{h^2}{r^2} \frac{(ac)^{1/2}}{[(a-c-k^2)\cos^2\theta + c]^{3/2}}. \quad (21)$$

Since $a - c - k^2$ is always negative the force at unit distance is a maximum when $\theta = 0$, and a minimum when $\theta = \frac{\pi}{2}$. In this case we obtain from (14)

$$R = \frac{a^2}{c} \frac{r}{(a - k^2 - ky)^3}. \quad (22)$$

When the centre of force is within the ellipse, at a distance p from the y -axis, and q from the x -axis, we get from (13)

$$R = \frac{h^2}{r^2} \frac{(ac)^{1/2}}{[2pq \sin\theta \cos\theta + (a-c-q^2+p^2)\cos^2\theta + c - p^2]^{3/2}}, \quad (23)$$

which becomes (19) when $q = 0$, and (21) when $p = 0$. We also obtain from (14)

$$R = \frac{h^2 a^2 c^2 r}{(ac - ap^2 - cq^2 - cpy - apx)^3}, \quad (24)$$

which becomes (20) when $q = 0$, and (22) when $p = 0$.

The foregoing values of R are real and positive, and represent all the laws consistent with the observed motions of binary stars.

It may be interesting to note that when the centre of force is at one of the apsides or at one end of the minor axis, our general formulæ (13) and (14) give indeterminate results. In this case we take the equation of the ellipse with the origin at the end of one of the axes, and calculate R by (8). When the centre of force is at the apse, we obtain after reduction

$$R = \frac{h^2}{r^2} \frac{\sqrt{c}}{a \cos^3 \theta}. \quad (25)$$

When the centre of force is at the end of the minor axis, we find

$$R = \frac{h^2}{r^2} \frac{\sqrt{a}}{c \sin^3 \theta}. \quad (26)$$

In both of these cases the origin is taken in the positive direction from the centre of the ellipse; if the other ends of the axes be chosen the signs of (25) and (26) will be reversed.

When $c = a$ in (25) or (26) the conic becomes a circle, and the expression reduces to the well-known law

$$R = \frac{8h^2 a^{5/2}}{r^5}. \quad (27)$$

The expression for the force at external points may be derived in a manner entirely similar to that for points within.

*Solution of Halphen.**

Let m be the mass of the central body, and R an unknown function of x and y . Then we have the equations

$$m \frac{d^2 x}{dt^2} = -R \frac{x}{r}, \quad m \frac{d^2 y}{dt^2} = -R \frac{y}{r}. \quad (28)$$

R is to be determined by the condition that the orbit of the particle is a conic section. Let

$$\frac{dx}{dt} = x', \quad \frac{dy}{dt} = y'; \quad R = -mur, \quad (29)$$

where u is an unknown function of x and y .

* TISSERAND'S *Mécanique Céleste*, Tome I, Cap. I, where the original solution has been somewhat modified.

From (28) and (29) we obtain

$$\frac{dx'}{dt} = ux \quad ; \quad \frac{dy'}{dt} = uy. \quad (30)$$

By this equation we have

$$\frac{d}{dt} F(x, y, x', y') = x' \frac{\partial F}{\partial x} + y' \frac{\partial F}{\partial y} + u \left(x \frac{\partial F}{\partial x'} + y \frac{\partial F}{\partial y'} \right). \quad (31)$$

We now proceed to find the differential equation which is common to all conics. The general equation of a conic has the form,

$$Ax^2 + 2Bxy + Cy^2 + 2Fx + 2Gy + H = 0, \quad (32)$$

in which there are five arbitrary constants. Taking x as the independent variable and differentiating five times in succession we have, in LAGRANGE'S notation,

$$\left. \begin{aligned} Cyy' &+ B(xy' + y) + Ax + Gy' + F = 0 \\ C(yy'' + y'^2) &+ B(xy'' + 2y') + A + Gy'' = 0 \\ C(yy''' + 3y'y'') &+ B(xy''' + 3y'') + Gy''' = 0 \\ C(yy^{iv} + 4y'y''' + 3y''^2) &+ B(xy^{iv} + 4y''') + Gy^{iv} = 0 \\ C(yy^v + 5y'y^{iv} + 10y''y''') &+ B(xy^v + 5y^{iv}) + Gy^v = 0 \end{aligned} \right\} \quad (33)$$

We now have to eliminate the five constants in (32) and (33). We notice that the last three equations of (33) are homogeneous, containing only the three constants C , B and G , and we can eliminate them by equating to zero the determinant

$$\Delta = \begin{vmatrix} yy''' + 3y'y'' & xy''' + 3y'' & y''' \\ yy^{iv} + 4y'y''' + 3y''^2 & xy^{iv} + 4y''' & y^{iv} \\ yy^v + 5y'y^{iv} + 10y''y''' & xy^v + 5y^{iv} & y^v \end{vmatrix} \quad (34)$$

By elementary principles of Determinants equation (34) reduces to

$$\Delta = \begin{vmatrix} 0 & 3y'' & y''' \\ 3y'' & 4y''' & y^{iv} \\ 10y''' & 5y^{iv} & y^v \end{vmatrix} \quad (35)$$

Expanding (35) and returning to the differential notation, we have

$$9 \left(\frac{d^2y}{dx^2} \right)^2 \frac{d^5y}{dx^5} - 45 \frac{d^2y}{dx^2} \frac{d^3y}{dx^3} \frac{d^4y}{dx^4} + 40 \left(\frac{d^3y}{dx^3} \right)^2 = 0. \quad (36)$$

This is the general differential equation of a conic section. We now calculate $\frac{d^2y}{dx^2} \cdots \frac{d^5y}{dx^5}$ from the relations expressed in (29), (30) and (31).

We have

$$\frac{dy}{dx} = \frac{y'}{x'},$$

therefore

$$x' \frac{d^2 y}{dx^2} = \frac{x' u y - y' u x}{x'^2},$$

or

$$x'^3 \frac{d^2 y}{dx^2} = (x' y - y' x) u. \quad (37)$$

Since the force is central, by the law of areas, $(x' y - y' x)$ is constant. Therefore we derive

$$\left. \begin{aligned} x'^5 \frac{d^3 y}{dx^3} &= (x' y - y' x) \left(x' \frac{du}{dt} - 3u^2 x' \right) \\ x'^7 \frac{d^4 y}{dx^4} &= (x' y - y' x) \left(x'^2 \frac{d^2 u}{dt^2} - 10u x x' \frac{du}{dt} - 3u^2 x'^2 + 15u^3 x^2 \right) \\ x'^9 \frac{d^5 y}{dx^5} &= (x' y - y' x) \left[x'^3 \frac{d^3 u}{dt^3} - 15u x x'^2 \frac{d^2 u}{dt^2} - 10x x'^2 \left(\frac{du}{dt} \right)^2 \right. \\ &\quad \left. + \frac{du}{dt} (105u^2 x^2 x' - 16u x'^3) + 45u^3 x x'^2 - 105u^4 x^3 \right]. \end{aligned} \right\} \quad (38)$$

Substituting these values in (36) and reducing, we obtain

$$9u^2 \frac{d^3 u}{dt^3} - 45 \frac{du}{dt} \frac{d^2 u}{dt^2} + 40 \left(\frac{du}{dt} \right)^3 = 9u^3 \frac{du}{dt}. \quad (39)$$

Putting $u = w^{-3/2}$, in which w is a function of x and y , (39) reduces to

$$\frac{d^3 w}{dt^3} = w^{-3/2} \frac{dw}{dt}. \quad (40)$$

When we remember that

$$\frac{dx'}{dt} = x w^{-3/2} \quad \text{and} \quad \frac{dy'}{dt} = y w^{-3/2},$$

and that w is a function of x and y , we get

$$\left. \begin{aligned} \frac{dw}{dt} &= x' \frac{\partial w}{\partial x} + y' \frac{\partial w}{\partial y} \\ \frac{d^3 w}{dt^3} &= x'^3 \frac{\partial^3 w}{\partial x^3} + 3x'^2 y' \frac{\partial^3 w}{\partial x^2 \partial y} + 3x' y'^2 \frac{\partial^3 w}{\partial x \partial y^2} + y'^3 \frac{\partial^3 w}{\partial y^3} \\ &\quad + w^{-3/2} \left(x' \frac{\partial w}{\partial x} + y' \frac{\partial w}{\partial y} \right) + 3w^{-3/2} (x' y + y' x) \frac{\partial^2 w}{\partial x \partial y} \\ &\quad + 3w^{-3/2} \left(x x' \frac{\partial^2 w}{\partial x^2} + y y' \frac{\partial^2 w}{\partial y^2} \right) \\ &\quad - \frac{3}{2} w^{-5/2} \left(x \frac{\partial w}{\partial x} + y \frac{\partial w}{\partial y} \right) \left(x' \frac{\partial w}{\partial x} + y' \frac{\partial w}{\partial y} \right). \end{aligned} \right\} \quad (41)$$

Substituting these values in (40), we obtain

$$\left. \begin{aligned} 0 &= x'^3 \frac{\partial^3 w}{\partial x^3} + 3x'^2 y' \frac{\partial^3 w}{\partial x^2 \partial y} + 3x' y'^2 \frac{\partial^3 w}{\partial x \partial y^2} + y'^3 \frac{\partial^3 w}{\partial y^3} \\ &+ \frac{3}{2} x' w^{-5/2} \left[2w \left(x \frac{\partial^2 w}{\partial x^2} + y \frac{\partial^2 w}{\partial x \partial y} \right) - \frac{\partial w}{\partial x} \left(x \frac{\partial w}{\partial x} + y \frac{\partial w}{\partial y} \right) \right] \\ &+ \frac{3}{2} y' w^{-5/2} \left[2w \left(y \frac{\partial^2 w}{\partial y^2} + x \frac{\partial^2 w}{\partial x \partial y} \right) - \frac{\partial w}{\partial y} \left(x \frac{\partial w}{\partial x} + y \frac{\partial w}{\partial y} \right) \right] \end{aligned} \right\} \quad (42)$$

This equation holds true whatever be the value of t , and hence when $t = 0$, in which case x, y, x', y' may be any four quantities mutually independent of one another. Then (42) gives the following equations

$$\frac{\partial^3 w}{\partial x^3} = 0; \quad \frac{\partial^3 w}{\partial x^2 \partial y} = 0; \quad \frac{\partial^3 w}{\partial x \partial y^2} = 0; \quad \frac{\partial^3 w}{\partial y^3} = 0. \quad (43)$$

$$\left. \begin{aligned} 2w \left(x \frac{\partial^2 w}{\partial x^2} + y \frac{\partial^2 w}{\partial x \partial y} \right) - \frac{\partial w}{\partial x} \left(x \frac{\partial w}{\partial x} + y \frac{\partial w}{\partial y} \right) &= 0, \\ 2w \left(y \frac{\partial^2 w}{\partial y^2} + x \frac{\partial^2 w}{\partial x \partial y} \right) - \frac{\partial w}{\partial y} \left(x \frac{\partial w}{\partial x} + y \frac{\partial w}{\partial y} \right) &= 0. \end{aligned} \right\} \quad (44)$$

We obtain from (43), when we denote the arbitrary constants by a, b, c, f, g, h ,

$$w = ax^2 + 2bxy + cy^2 + 2fx + 2gy + h. \quad (45)$$

Forming the differentials and substituting in (44), we obtain

$$\left. \begin{aligned} (bf - ag)xy + (cf - bg)y^2 + (f^2 - ah)x + (fg - bh)y &= 0, \\ (bg - cf)xy + (ag - bf)x^2 + (fg - bh)x + (g^2 - ch)y &= 0. \end{aligned} \right\} \quad (46)$$

Since these equations hold true for all values of x and y , we find

$$ag - bf = 0, \quad bg - cf = 0. \quad (47)$$

$$f^2 - ah = 0, \quad g^2 - ch = 0, \quad fg - bh = 0. \quad (48)$$

From (48) we have

$$fh(ag - bf) = 0, \quad gh(bg - cf) = 0. \quad (49)$$

Then, if none of the quantities f, g, h vanishes, (47) follows from (48), and it is sufficient to verify the latter.

We may write (45) in the form

$$w = \frac{1}{h} [(fx + gy + h)^2 - (f^2 - ah)x^2 - (g^2 - ch)y^2 - 2(fg - bh)xy], \quad (50)$$

which, in consequence of (48), becomes

$$w = \frac{(fx + gy + h)^2}{h}. \quad (51)$$

Therefore, since $u = w^{-3/2}$, we have by (29)

$$R_1 = mh^{3/2} \frac{r}{(fx + gy + h)^3}, \quad (52)$$

which is an expression for the force sought. When $h = 0$, (48) leads to $f = 0$ and $g = 0$. In this case we have

$$w = ax^2 + 2bxy + cy^2, \quad (53)$$

from which we get

$$R_2 = m \frac{r}{(ax^2 + 2bxy + cy^2)^{3/2}}. \quad (54)$$

This is another expression for the force, whatever be the constants a , b and c .

When $f = 0$, (47) and (48) give $ag = bg = ah = bh = 0$, $g^2 = ch$, from which $a = b$.

In this case we get from (50)

$$w = \frac{(gy + h)^2}{h}, \quad (55)$$

which gives the same result as (52), when $f = 0$.

Thus there are two laws of force, and only two, which answer the question; but the forces R_1 and R_2 contain both the radius vector r , and the polar angle

$$\theta = \tan^{-1} \frac{y}{x}.$$

If the forces depend upon r alone, as is natural to suppose, we should have in R_1 , $f = g = 0$; and in R_2 , $a = c$ and $b = 0$. Then we find

$$R_1 = mr \quad ; \quad R_2 = \frac{m}{r^2}. \quad (56)$$

The first of these laws is excluded by observation; the second is the law of Newtonian gravitation.

§ 5. *Theory of the Determination, by Means of a Single Spectroscopic Observation, of the Absolute Dimensions, Parallaxes and Masses of Stellar Systems whose Orbits are Known from Micrometrical Measurement.**

Recent researches on the orbits of double stars have led me to develop the suggestion, first thrown out by FOX TALBOT in 1871† and since somewhat varied by others,‡ for determining the absolute dimensions, parallaxes and masses of stellar systems by spectroscopic observation of the relative motion of the companion in the line of sight. A simple and general theory of this motion may be derived from the application of the hodograph of the ellipse, and hence we shall now investigate the nature of this curve.

Let x, y be the coördinates of a point in the ellipse; x', y' those of the corresponding point in the hodograph; then we shall have

$$x' = \frac{dx}{dt} \quad , \quad y' = \frac{dy}{dt} . \quad (1)$$

Suppose M to be attracting the mass in the focus of the ellipse; and let r and θ be the polar coördinates of the particle moving in the orbit, and we have

$$\frac{d^2x}{dt^2} = -\frac{Mx}{r^3} = -\frac{M}{r^2} \cos \theta \quad , \quad \frac{d^2y}{dt^2} = -\frac{My}{r^3} = -\frac{M}{r^2} \sin \theta . \quad (2)$$

By the principle of the conservation of areas resulting from central forces, we have the equation

$$r^2 \frac{d\theta}{dt} = \text{double areal velocity} = C ,$$

or

$$r^2 = C \frac{dt}{d\theta} ;$$

and hence

$$\frac{d^2x}{dt^2} = -\frac{M}{C} \cos \theta \frac{d\theta}{dt} \quad , \quad \frac{d^2y}{dt^2} = -\frac{M}{C} \sin \theta \frac{d\theta}{dt} . \quad (3)$$

If we integrate we obtain

$$\frac{dx}{dt} + a = -\frac{M}{C} \sin \theta \quad , \quad \frac{dy}{dt} + b = +\frac{M}{C} \cos \theta , \quad (4)$$

* *Astronomische Nachrichten*, No. 3314.

† Report of British Association, 1871, Part II. p. 34; CLERKE'S "System of the Stars," p. 201, and "History of Astronomy during the 19th Century," third edition, p. 467.

‡ RAMBAUT, *M. N.*, March, 1890; WILSING, *A. N.*, 3198; also a paper on the determination of orbits from spectroscopic observation of the velocity-components in the line of sight, by LEHMAN-FILHÉS, *A. N.*, 3242.

where a and b are the arbitrary constants of integration. But since

$$\sin \theta = \frac{y}{r}, \quad \cos \theta = \frac{x}{r},$$

we find

$$\frac{dx}{dt} + a = -\frac{M}{C} \frac{y}{r}, \quad \frac{dy}{dt} + b = +\frac{M}{C} \frac{x}{r}. \quad (5)$$

By means of equation (1) we have

$$x' + a = -\frac{M}{C} \frac{y}{r}, \quad y' + b = +\frac{M}{C} \frac{x}{r},$$

and on squaring and adding we obtain

$$(x' + a)^2 + (y' + b)^2 = \frac{M^2}{C^2}, \quad (6)$$

which shows that the hodograph of the ellipse is a circle of radius $\frac{M}{C}$.

The following geometrical proof will render the application somewhat more intelligible.

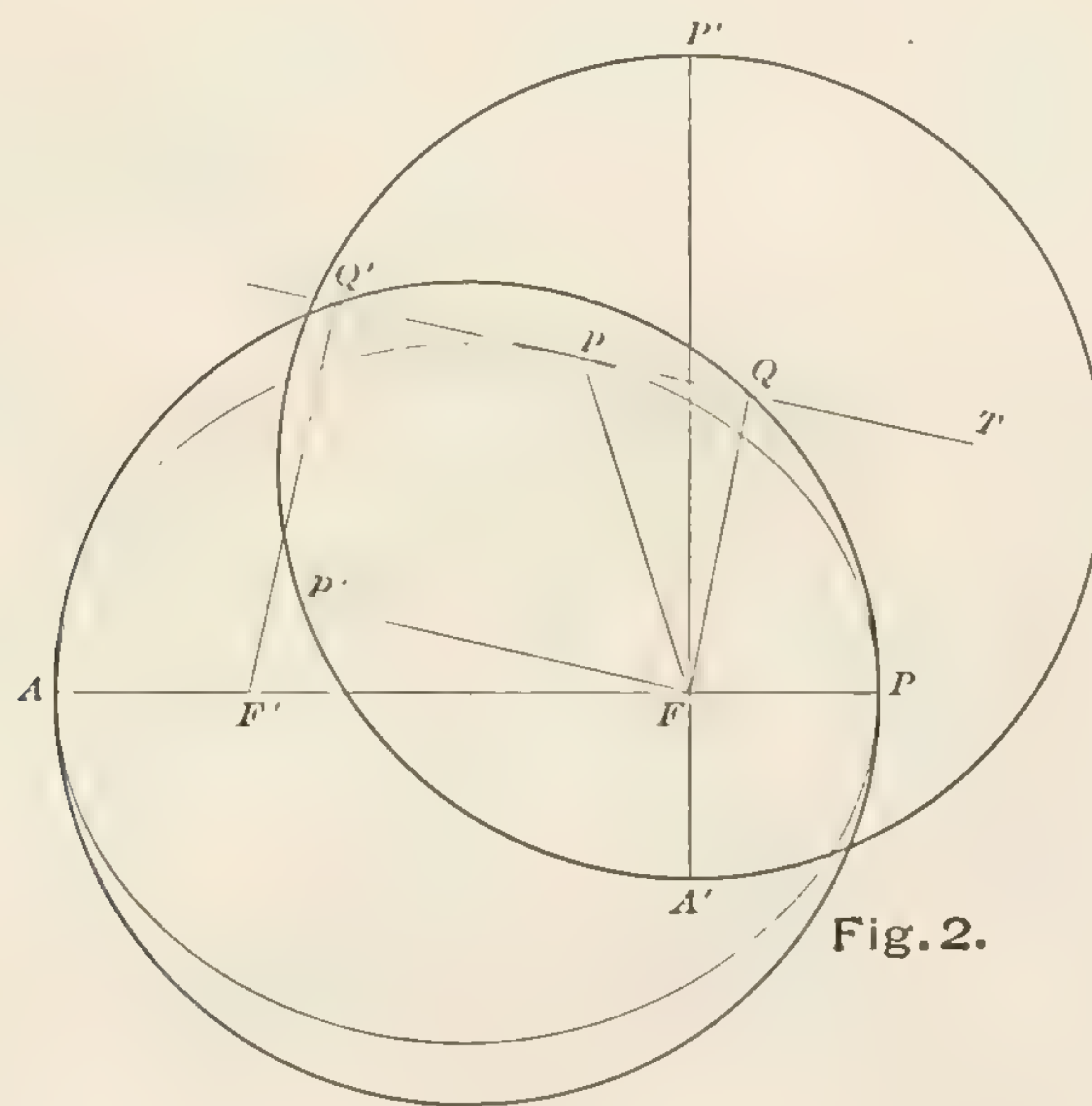


Fig. 2.

In the figure let PpA be the ellipse described by the particle p ; PA being the major axis, and F and F' the two foci. Let pT be the tangent to the ellipse at p , and let the perpendicular from the focus upon the tangent be denoted by FQ . Then by definition the radius vector of the point in the hodograph is parallel to the tangent pT and proportional to the velocity at

the point p . It is well known from the law of the conservation of areas that this velocity is always inversely as the perpendicular $F'Q$, or directly proportional to the length of $F'Q'$. But the locus of Q or Q' is known to be the auxiliary circle described upon the major axis as a diameter. Therefore we see that the hodograph is of the same form as the locus of Q' , but since the point p' in the hodograph is on a radius vector parallel to pT' , its situation relative to the focus F' will always be 90° in advance of Q .

The shape and situation of the hodograph relative to the ellipse is shown in the figure. Thus, when p is in periastron the point of the hodograph is in the direction perpendicular to the major axis, and at a distance proportional to $F'Q$, which is then equal to $F'P$; and similarly for other points of the orbit. For the sake of clearness we have made the hodograph in the figure of the same size as the auxiliary circle of the ellipse, but if the radius vector in the hodograph is to represent the velocity in the ellipse the scale of the hodograph ought in reality to be greatly reduced.

If the orbit of a double star is given we may at once construct the form of the hodograph, the position relative to the ellipse being the same as in the preceding figure. Moreover if the velocity of the companion about the central star is known in absolute units for any point of the orbit, we may determine the velocity for any other point by means of the hodograph. For the magnitude of the velocity will be the length of the radius vector of the hodograph which is parallel to the tangent of the orbit at the point in question, and can easily be computed or measured graphically directly from the diagram.

When the elements of a binary are known, we may determine the component of the velocity in the line of sight as follows: Suppose ρ to be the radius vector of the point in the hodograph, and ω to be the angle made by the radius vector ρ with the ascending node, and therefore identical with the angle made by the tangent to the orbit with the line of nodes; and let i be the inclination of the plane of the orbit to the plane tangent to the celestial sphere. Then we evidently have, as the component towards the earth,

$$\kappa = \rho \sin \omega \sin i. \quad (7)$$

The angle i is an element of the star's orbit and is known; the angle ω can be computed from the theory of the ellipse, or can be determined directly from the diagram; and when ρ is known in absolute units the component in the line of sight is perfectly determined.

We shall now show how to compute ω and ρ for any given orbit. The

radius vector of the star r and the true anomaly v must be computed by the usual process, and then we find the radius vector with respect to the other focus

$$r' = 2a - r;$$

and we have the angle γ by means of the equation

$$\sin \gamma = \frac{r \sin v}{r'} . \quad (8)$$

The angle ψ between the radii vectores drawn to the two foci is evidently equal to $v - \gamma$, and hence

$$\frac{\psi}{2} = \frac{v - \gamma}{2} . \quad (9)$$

It is also easy to see that q , the angle made by the tangent with the latus rectum of the ellipse, is given by

$$q = v - \frac{1}{2}\psi . \quad (10)$$

When the value of q is determined, it is clear that

$$\omega = \lambda + 90^\circ + q = v + \lambda + 90^\circ - \frac{1}{2}\psi , \quad (11)$$

so that we easily find the angle of the radius vector ρ from the ascending node.

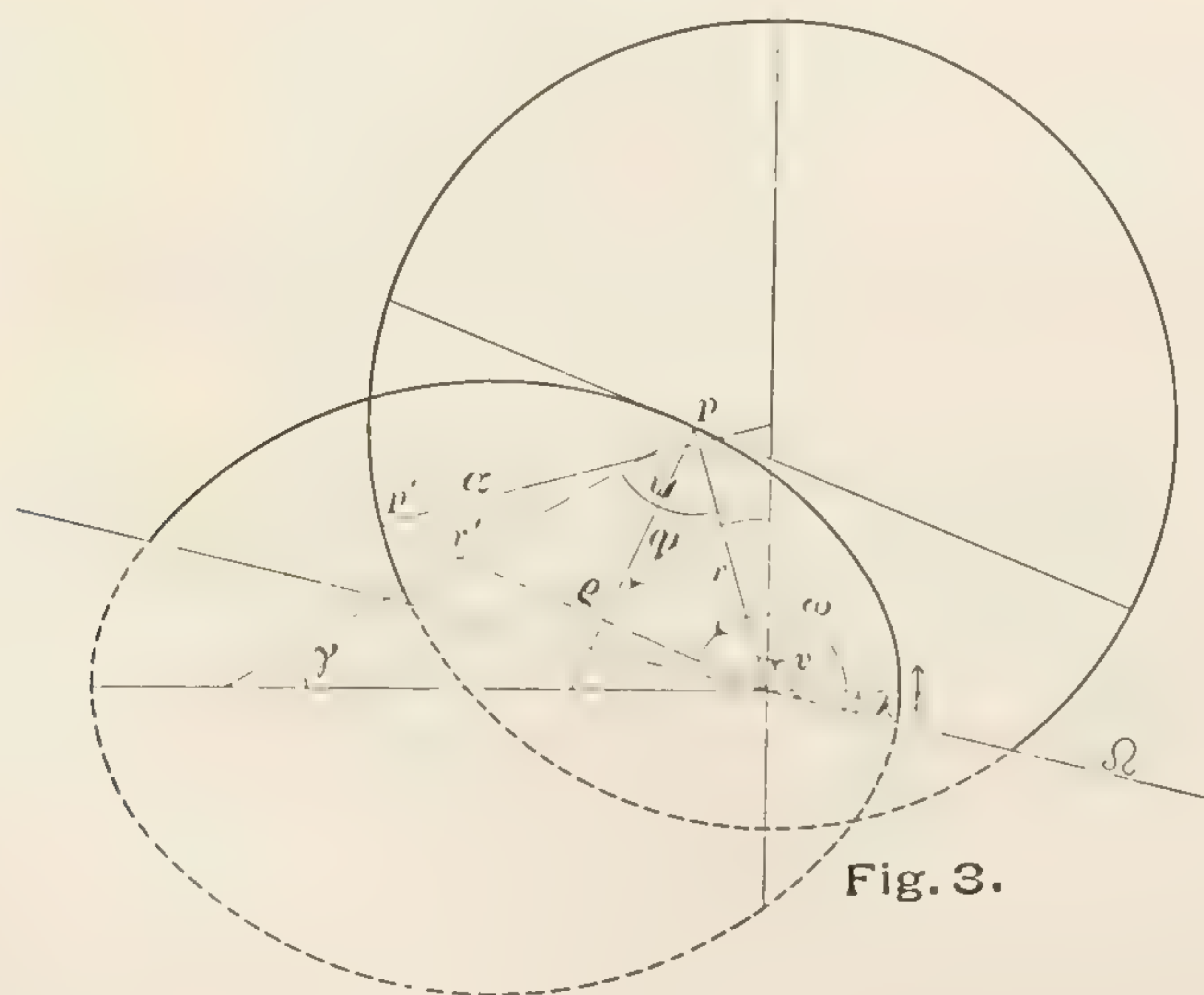


Fig. 3.

We may compute the length of this radius vector in the hodograph in the following manner. Let the radius of the circle be denoted by a , its value being

supposed known in absolute units; the linear eccentricity will be ae , and we shall have

$$\alpha^2 = \rho^2 + a^2e^2 - 2\rho ae \cos q;$$

on solving for ρ we find

$$\rho = \alpha [e \cos q + \sqrt{1 - e^2 \sin^2 q}]. \quad (12)$$

Thus when α , the radius of the hodograph, is known in absolute units, we are enabled by means of (11) and (12) to predict the motion in the line of sight for any instant whatever.

Now suppose we determine the relative motion of the companion in the line of sight by means of a modern Spectrograph such as that at Potsdam; this will give us results freed from the effect of the proper motion of the system in space, as well as the secular motion of the sun and the orbital motion of the earth. Then by equation (7) we have

$$\rho = \frac{\kappa}{\sin \omega \sin i}, \quad (13)$$

in which κ is furnished by spectroscopic measurement, and ω and i are found from the orbit deduced from micrometrical measures.

A single observation therefore gives us the absolute velocity in the orbit, and this fixes the scale of the hodograph. For since we have

$$\rho = \alpha [e \cos q + \sqrt{1 - e^2 \sin^2 q}],$$

and e and q are known, we may determine the radius of the hodograph by

$$\alpha = \frac{\rho}{[e \cos q + \sqrt{1 - e^2 \sin^2 q}]}. \quad (14)$$

Having determined κ by observation, we get the absolute value of ρ by (13) and of α by (14), and we may then predict the value of κ in absolute units for any time whatever. In practice it will be desirable to measure the motion in the line of sight when the function κ is a maximum, in order that an error in κ may have a minimum effect upon the radius of the hodograph.

When α is thus determined in absolute units, the problem arises to find the absolute dimensions of the system, the masses of the stars, and their distance from the earth. Suppose we choose two epochs separated by a convenient interval of time, say a year or a fractional part of a year, when the companion is near apastron, and the velocity changes slowly. We shall denote the radii vectores by r_1 and r_2 , and the interval of time by $t_2 - t_1$. The length of the included elliptic arc can be expressed rigorously only by means of an elliptic

integral, but as the evaluation of this integral would be inconvenient in practice and for a short arc unnecessarily exact, we shall determine the length of the arc by mechanical quadrature. Thus we have

$$\text{arc} = \int_{t_1}^{t_2} \rho \, dt = \bar{\rho} (t_2 - t_1),$$

where $\bar{\rho}$ is the average velocity of the interval, easily deduced from the hodograph. If the interval is short compared to the time of revolution, so that the arc may be put equal to its sine, we shall have approximately

$$\frac{r_1 + r_2}{2} \sin(v_2 - v_1) = \bar{\rho} (t_2 - t_1),$$

or

$$r_1 + r_2 = \frac{2\bar{\rho} (t_2 - t_1)}{\sin(v_2 - v_1)}.$$

Now v_2 and v_1 are known true anomalies, and r_1 and r_2 are given in units of the major axis by the polar equation

$$\frac{r}{a} = \frac{(1 - e^2)}{1 + e \cos v}.$$

Hence, with r_1 and r_2 thus expressed numerically, we find

$$a = \frac{2\bar{\rho} (t_2 - t_1)}{(r_1 + r_2) \sin(v_2 - v_1)}. \quad (15)$$

Here the interval $t_2 - t_1$ must be expressed in the same units as $\bar{\rho}$, preferably in kilometres per second. The length of the major semi-axis of the orbit is thus found in kilometres, and the absolute dimensions of the system are determined.

The parallax of the system is equal to the major semi-axis of the orbit in seconds of arc divided by the major semi-axis in astronomical units; or the distance of the system from the earth is equal to the major semi-axis in astronomical units divided by the sine of the angle subtended by the major semi-axis in seconds of arc; thus

$$D = \frac{a}{\sin a''}. \quad (16)$$

If $M_1 + M_2$ denote the combined mass of the system, $M + m$ the combined mass of the sun and earth, a the major semi-axis of the orbit of the companion, and P the period of revolution, R the distance of the earth from the sun, and

T the length of the sidereal year, we have, by the well known extension of KEPLER'S law:

$$M_1 + M_2 = \frac{a^3}{R^3} \cdot \frac{T^2}{P^2} (M+m). \quad (17)$$

If as usual we put $M+m=1$, $R=1$, and $T=1$, and express a and P in these units, we find

$$M_1 + M_2 = \frac{a^3}{P^2}, \quad (18)$$

where the mass of the system will be expressed in units of the combined mass of the sun and earth. The mass of the system is thus determined absolutely.

In conclusion it seems proper to add that this investigation was stimulated by an elegant proof of MR. F. R. MOULTON, that the aberrational orbit of a fixed star is the hodograph of the ellipse in which the earth moves, and therefore a circle. The idea brought out in MR. MOULTON'S proof caused me to revert to the motion of binaries in the line of sight, and hence no small part of the credit is due to him for the interesting application of SIR W. R. HAMILTON'S hodograph given above.

§ 6. *Rigorous Method for Testing the Universality of the Law of Gravitation.**

It remains to consider how we may use the foregoing results to test the law of NEWTON. It is evident that the law of gravitation can be tested by comparing the observed with the theoretical motion of the companion in the line of sight. We may choose a system whose orbit is accurately known and whose stars are suitable for exact spectroscopic measurement of the component κ ; we then determine from one or more observations at a suitable epoch the absolute dimensions of the orbit, as explained in the preceding theory, and predict the motion in the line of sight for other parts of the orbit, perhaps for a whole revolution. If we then determine by spectroscopic measurement the value of the component κ independent of any theory, and find that the theoretical results are confirmed by actual observations, we may consider the result a direct observational proof that the force which retains the companion in its orbit is Newtonian gravitation.

For we know from micrometrical measures that the areas described by the radius vector of the companion are proportional to the time, and therefore that

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the force is central; and the observations of 42 *Comae Berenices*, whose motion happens to be in the plane of vision, indicate that the orbit is a plane curve. The motion being in a plane and the force being central, we must be able to show that the principal star is in the focus of the real ellipse. This can be done if we can show by spectroscopic observations that the inclination and node resulting from the theory of gravitation account perfectly for the motion in the line of sight.

We therefore assume the law of gravitation in deriving the elements of the orbit and in predicting the motion in the line of sight, as heretofore explained; spectroscopic observation will enable us to test the results of theory experimentally. If the theoretical results are confirmed by observation throughout a revolution — thus showing that the node and inclination are identical with those resulting from the theory of gravitation — we may regard the observations as giving a direct and incontestible proof of the validity of the law of NEWTON in the stellar systems.

If we desire to ascertain whether any other inclination and node — in other words, any other law of force — could give rise at every point of the orbit to a relative motion in the line of sight identical with that resulting from the law of gravitation, we may proceed as follows: Suppose that some other inclination and node and orbital velocity be possible; they will differ by unknown quantities from those values resulting from the theory of gravitation, and we shall have the relation

$$\rho \sin i \sin \omega = \rho' \sin (i + \gamma) \sin (\omega + \delta) .$$

By expanding and reducing we find

$$\rho = \rho' \{ \cos \gamma \cos \delta + \cos \gamma \cot \omega \sin \delta + \sin \gamma \cot i \cos \delta + \sin \gamma \sin \delta \cot i \cot \omega \} .$$

But we observe that ω is a variable angle depending on the position of the body in the orbit; and since $\omega = 0$, or $\omega = \pi$ would render the cotangent infinite, and ρ is known to be finite for every point, (the two bodies never come into contact but are always separated by a certain distance), it follows that those terms depending on $\cot \omega$ must vanish, or $\delta = 0$, and the line of nodes becomes the same as that resulting from the theory of gravitation. Our expression thus takes the form

$$\rho = \rho' (\cos \gamma + \sin \gamma \cot i) = \rho' . K ,$$

where K is a constant.

Therefore, if the inclination differs by γ from the value given by the theory of gravitation, it will follow that the velocity at every point of the real orbit

must be multiplied by a constant factor. But since no alteration of the inclination can change the radius vector at the line of nodes, it follows that at these points the orbital velocities would necessarily be the same however the inclination might vary. And since we have seen that the line of nodes is identical with that given by the theory of gravitation, we conclude that the velocities in the orbits could not differ throughout by a constant ratio. Hence it is evident that $\cos \gamma + \sin \gamma \cot i = 1$, or $\gamma = 0$, and the inclination is identical with that resulting from the theory of gravitation. It follows therefore that no other conceivable law of attraction could produce the same relative motion in the line of sight as the law of inverse squares. Consequently if observation shall give for every point a relative motion in the line of sight which accords with theory, we may confidently conclude that Newtonian gravitation is the force which retains the stars in their orbits.

§ 7. *On the Theoretical Possibility of Determining the Distances of Star-Clusters and of the Milky Way, and of Investigating the Structure of the Heavens by Actual Measurement.**

The practical problem of measuring the parallaxes of the fixed stars is one of the greatest of modern Astronomy, and has been solved heretofore very imperfectly. The quantity to be deduced is so very small that accidental and systematic errors often wholly obscure the element desired, and render the probable errors of most of our parallaxes painfully large compared to the minute quantities sought. Moreover, the method of relative parallax, which is the only one in general use, aside from its theoretical inaccuracy, is encumbered with many practical difficulties, the chief of which is in finding suitable comparison stars; and hence not a few astronomers have practically abandoned hope of determining the distances of the fixed stars with any considerable degree of precision. None have felt these difficulties more keenly than those astronomers who have attempted investigations requiring exact knowledge of the masses and dimensions of the stellar systems. At the present time the only parallaxes of binaries which lay claim to any considerable precision are those of α Centauri ($0''.75$), α Canis Majoris ($0''.38$), 70 Ophiuchi ($0''.162$), and η Cassiopeae ($0''.154$). To this list we might perhaps add a few spectroscopic binaries whose parallaxes have been investigated, but even then the number of systems

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would remain very small, and altogether insufficient to support any sound generalization respecting the masses and dimensions of binary stars as a class.

If we consider single, instead of double stars, it will be evident that while a much larger number have been measured for parallax, and in a good many cases reliable values have been derived, yet in the majority of instances the divergence of results obtained by different observers, may fairly be taken to indicate that our knowledge of stellar parallax is still very limited; and owing to the small dimensions of the earth's orbit, very little hope has been entertained of material improvement in time to come.

The method which we have developed in section 5 is full of promise for the case of binary stars. This method is theoretically applicable to any pair where the components have an angular separation of $0''.1$, and a single application of the spectrograph at a suitable epoch gives us the absolute dimensions, mass and parallax of the system.

As $0''.1$ is about the present limit of exact micrometrical or heliometrical measurement, and as this angle would correspond to the parallax of a fixed star at the distance of 36 light-years (eight times the distance of *α Centauri*) we see that all smaller parallaxes determined by methods heretofore in use must necessarily remain very uncertain. On the other hand the spectroscopic method will apply satisfactorily to much more distant systems — to pairs which have an angular separation of $0''.1$, and where an observer by the ordinary method would find that our sun had a parallax of this amount. This is equivalent to using the major semi-axis of the stellar orbit for a base line instead of the mean distance of the earth from the sun; and thus the parallaxes deduced by the spectroscopic method might be as much smaller than $0''.1$ as the major axis of the stellar orbit is larger than that of the earth, provided of course that the combined mass of the stars is great enough to give a relative motion of the companion in the line of sight which can be measured with the desired precision.

Thus, by the usual method the parallax of *α Centauri* would be just measurable at the distance of 36 light-years, and would amount to $0''.1$; and as the major semi-axis of the orbit would there subtend an angle of $2''.2$, the spectroscopic method could be applied at 22 times that distance, or when the system is removed from us by about 800 light-years. Of course we can never hope to measure the distance of a system so remote by the ordinary method, since at the distance of 800 light-years the parallax would amount to only $0''.0045$. If the mass and dimensions of the system be larger than those of *α Centauri*, the spectroscopic method would enable us to measure a parallax correspondingly

smaller. While at present little is known of the magnitude of binary systems, it seems probable that in some cases at least the masses and dimensions will much surpass those of α *Centauri*. It is therefore probable that it will occasionally be possible to determine the distances of systems removed from us by several thousand light-years.

The present state of Astronomy does not permit us to make a confident assertion with regard to the distances of the clusters or of the Milky Way, but it seems exceedingly probable that both are very remote. In each of these species of stellar aggregation there exists a considerable but unknown number of binary stars which can be detected with our present optical means. Thus, BURNHAM has searched for double stars in several of the great northern clusters, such as *Praesepe*, the *Pleiades* and the great clusters in *Perseus*, *Hercules*, &c. (Publications of Lick Obs., vol. II. pp. 211-216), and discovered a number of pairs which promise to be physically connected. He observes that interesting stars are apparently more frequent in wide clusters like the *Pleiades*, *Praesepe*, and the great cluster in *Perseus*, than in the more compact clusters like that in *Hercules*. Yet he has discovered an important pair in this dense globular cluster, and SIR JOHN HERSCHEL has likewise detected double stars of special interest in several of the great clusters of the southern hemisphere. It is not to be doubted that many more such objects will be detected when the clusters generally are critically examined under the powers of our great modern refractors.

When the orbits of these binaries have been found by exact micrometrical measurement, the spectroscopic method will eventually afford the means for determining their immense distances, not by probable assumptions but by exact computation. It is evident therefore that if we are ever to determine the distances of clusters from the earth — and no sound ideas of the nature of these masses of stars can be formed until such determination is made — we must first search the clusters critically for binary stars, and determine their orbits by micrometrical measurement. If, when the orbit is known, it shall appear that the binary has the same proper motion as the adjacent stars of the group, there will be a strong presumption that the system forms a part of the cluster. If the pair be also of about the same magnitude as its neighbors, and of the same color and spectral type, we may conclude with practical certainty that the binary is intimately connected with the mass of stars in which it is projected.

Determination of the parallax of the binary will therefore give the distance of the cluster from the earth, and supply all desired information as to the dimensions of the cluster, the brilliancy of its stars, their mutual distances, &c. If in like manner any group of stars in the Milky Way could be carefully

searched for binary systems, and some intimate connection of a pair with neighboring stars shown to exist, a determination of its orbit and an application of the spectroscopic method would lead to a knowledge of the distance of that part of the Milky Way. By extending the same process to all parts of the Galaxy it will be possible in the course of time to ascertain the nature of that immense aggregation of stars, and throw light upon the construction of the heavens. While the spectroscopic method applies only to binary stars, it is evident that their great abundance and universal distribution in space will some day give a means for determining with precision and certainty the actual structure of the sidereal universe.

We must not expect that the immense possibilities here outlined will be practically realized at once, or even in the near future, yet giant refractors like the 40-inch Yerkes Telescope will give such power for separating close double stars and for supplying a great amount of light for the spectroscopic study of faint objects, that an application of these ideas may not be found impossible in the course of the coming century. If there be spectroscopic or photographic difficulties, the progress of spectroscopic Astronomy during the last thirty years justifies the belief that such obstacles will not continue to be insurmountable. The great philosophic interest attaching to the foregoing method for investigating the structure of the visible universe by exact spectroscopic measurement, instead of by the doubtful processes of gauging employed by HERSCHEL and STRUVE, appears to be a sufficient justification for considering what is at present only a theoretical possibility. The history of Astronomy shows that it is not always the theories that can be realized in a decade or even in a century which in the long run exercise the most important influence on the development of science.

§ 8. *Historical Sketch of the Different Methods for Determining Orbits of Double Stars.*

It is assumed that the law of gravitation governs the motions of double stars, and therefore that the orbits are ellipses with the principal stars in the foci. From the nature of conic sections the centre of the real ellipse will be projected into the centre of the apparent ellipse. But in general the foci of the real ellipse will not fall upon the foci of the apparent ellipse. If, however, a line be drawn from the centre of the apparent ellipse to the principal star and prolonged in either direction until it intersects the curve, the result will define the projection of the real major axis. The diameter of the

ellipse conjugate to this line will be the projection of the minor axis. Thus: It is easy to fix the positions of the real major and minor axes as seen in the apparent orbit. Since all parts of the major axis are shortened in the same ratio, the eccentricity of the real orbit may be deduced from the apparent orbit, by dividing the distance from the centre to the principal star by the major semi-axis as seen in projection. The end of this axis which is nearest the principal star will be the periastron; that farthest away, the apastron; the dates corresponding to the passage of the companion through these points will give the epochs of periastron and apastron passage respectively. It is evident that only one diameter of the real ellipse will suffer no shortening, owing to projection, and this is the diameter parallel to the line of nodes. If from points on the apparent ellipse perpendiculars be drawn to this diameter, and then increased in the ratio of $\cos i$ to 1, we shall get points of the real orbit whose projections give points on the apparent orbit.

The observations of a double star are expressed in polar coördinates, ρ and θ , which give the angular separation of the components in seconds of the arc of a great circle, and the position-angle of the companion with respect to the meridian. The companion is thus referred to the principal star regarded as fixed, and hence the observations give the means of finding only the relative orbit of one star about the other. The absolute orbit of either star about the centre of gravity of the system has a form similar to that of the relative orbit, but the linear dimensions are reduced in the ratio of M_2 or M_1 to $M_1 + M_2$, where M_1 and M_2 are the masses of the stars. The absolute orbits of the stars have the same shape, but are reversed in relative position. The centre of gravity of a pair of stars can be determined only by the criterion that the centre of gravity of a system moves uniformly in a right line; and as most of the systems have too little motion to define this point with any considerable degree of precision, owing to the imperfect state of our absolute positions as determined by the meridian circle, it is in general impossible to define the absolute orbits or relative masses of the stars. With few exceptions, therefore, astronomers have contented themselves heretofore with determining the relative orbit of one body about the other.

The first method for determining the orbit of a double star was proposed by SAVARY in 1827 (*Connaissance des Temps*, 1830). This method is closely analogous to those used for planets and comets, in so far as it rests on the treatment of four complete observations for the definition of the seven elements. The problem is solved by elaborate geometrical constructions, such as characterize work in pure mathematics rather than the practical processes which must

be invoked by the working computer. SAVARY's principal equation is based on the difference between the sector and triangle, the area derived from the time being equated with an expression involving the products of the semi-axes and eccentric angles of the apparent ellipse. The method is thus ill adapted to the determination of an orbit from such positions as are furnished by the measures of double stars.

ENCKE recast the method of SAVARY, from the point of view of a practical computer, and deduced formulae similar to those used by astronomers in their work on planets and comets. Rejecting the equations depending on conjugate diameters, so much employed by the French geometer, he based his formulae on recognized astronomical processes and developed tables to facilitate their application. As SAVARY had applied his method to ξ *Ursae Majoris*, ENCKE was led to illustrate his computations on the equally well-known system of 70 *Ophiuchi* (*Berliner Astronomisches Jahrbuch*, 1832).

SIR JOHN HERSCHEL took up the problem about 1830, and sought to improve the processes by a graphical method which enabled him to make use of all the observational material, and to eliminate the grosser errors of the individual observations. He was convinced that in order to obtain orbits of a satisfactory character, it would be necessary to correct the angles by an interpolating curve, one axis representing the time, the other the angle, and that the distances must be rejected altogether, except for the determination of the major axis. He proceeds by successive approximations to deduce normal places for the angles, and by gradual improvement of his graphical results renders them consistent with an ellipse, and finally obtains a satisfactory apparent orbit. The elements are then deduced by formulae not very different from those employed by SAVARY. The method is illustrated by applications to γ *Virginis*, α *Geminorum*, σ *Coronae Borealis*, ξ *Ursae Majoris*, and 70 *Ophiuchi* (*Memoirs*, Royal Astronomical Society, Vol. V).

While the process of interpolation invented by HERSCHEL has been extensively employed, and in some cases is very useful, I am satisfied that in general it is better to plot the observations directly and to make a trial ellipse the interpolating curve. This enables us to use both angles and distances and secures all the advantage of judgement which HERSCHEL considered so essential. It often happens that the length of the radius vector changes with extreme rapidity, and as the areas are constant this will imply very great and unequal changes in the angular motion; when the angular velocity of the radius vector is so variable in different parts of the apparent ellipse the course of the interpolating curve becomes altogether uncertain. Under these conditions it is much

better to use the observations directly. It is also recognized that modern measures of distance should be allowed an equal or nearly equal weight in the determination of orbits.

After SAVARY, ENCKE and HERSCHTEL had given such an impetus to the study of sidereal systems, the work was carried forward by MÄDLER and VILLARCEAU, both of whom published a number of orbits with some minor improvements in the processes of computation.

KLINKERFUES took up the subject about 1856, and in the course of work on several orbits developed very elegant formulae and more practical methods than any which had been used before. His analytical method is marked by rigor and generality, but in the present state of double-star Astronomy is not so practicable as the graphical method treated in section 10.

THIELE, some years later, devised an elegant graphical method which has many good points, and is much admired by those who are inclined to determine all the elements geometrically. It will be found in the *Astronomische Nachrichten*, Band LII.*

Among the more recent investigations those of PROFESSOR KOWALSKY are remarkable for their extreme elegance and great generality. This method, depending on the general equation of a conic, is all that can be desired from a mathematical point of view, and as simplified by GLASENAPP has been extensively used by several computers. The original exposition of the method will be found in the *Proceedings of the Imperial University of Kasan* for 1873; the valuable modification introduced by GLASENAPP is given in the *Monthly Notices*, Vol. XLIX, p. 278.

Other recent investigations which are worthy of special notice include those of SEELIGER (*Inaugural Dissertation* of SCHORR, Munich, 1889), and of ZWIERS (*Astronomische Nachrichten*, No. 3336).

It is singular that nearly all the methods given above have been developed from the point of view of analysis rather than of practical Astronomy. BURNHAM has recently rendered double-star Astronomy a conspicuous service by reviving the method of representing observations first employed by WILLIAM STRUVE (*Mensurae Micrometricae*, last plate). This consists in plotting the points as determined by the micrometer, and in finding from the places thus laid down the apparent ellipse which best satisfies the observations. We have used a modification of this method throughout the present work, and have discussed it in connection with the graphical method of KLINKERFUES, which supplies the process for deriving the elements from the apparent orbit.

* It is also explained by PROFESSOR HALL in *The Astronomical Journal*, No. 324.

§ 9. *Kowalsky's Method.*

We shall now give an exposition of the elegant method of KOWALSKY, which seems likely to be the one that will ultimately be adopted by astronomers. The general equation of the ellipse with the origin at any point, here taken at the principal star, is

$$F = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0; \quad (1)$$

which may be reduced to the form

$$Ax^2 + 2Hxy + By^2 + 2Gx + 2Fy + 1 = 0. \quad (2)$$

This equation contains five unknown constants, and hence five values of x and y will enable us to determine the constants of the ellipse. Each observation gives one equation by means of the relations

$$x_0 = \rho_0 \cos \theta_0, \quad y_0 = \rho_0 \sin \theta_0,$$

where the x -axis is directed to the north-point. And hence five observations at different epochs will give a determination of the apparent orbit. In practice it is found that a larger number of observations is desirable, and if the observations are sufficiently good, the best results will generally be obtained by a least-square adjustment of the residuals.

When the apparent ellipse is determined, the problem arises to express the elements of the real orbit in terms of the constants which fix the apparent orbit.

It is evident that projection does not alter the diameter coinciding with the line of nodes, and this enables us to pass from the apparent to the real orbit. The real orbit is evidently the curve determined by the intersection of the orbit-plane with the elliptical cylinder whose right section is the apparent orbit. In the apparent orbit the axis of x is directed to the north-point, but in passing to the real orbit we shall direct the new axis of x to the ascending node, while the new axis of y will be taken in the plane of the real orbit, and the origin retained at the principal star. Calling the new system of coördinates x', y', z' , it is evident that we shall have

$$\left. \begin{aligned} x &= x' \cos \Omega - y' \sin \Omega \cos i + z' \sin \Omega \sin i \\ y &= x' \sin \Omega + y' \cos \Omega \cos i - z' \cos \Omega \sin i \\ z &= \quad \quad + y' \sin i + z' \cos i. \end{aligned} \right\} \quad (3)$$

If we put $z' = 0$, we shall have the coördinates of a point in the plane of the real orbit. Thus our expressions are simplified, and become equations for turning the axis of x through the angle Ω , and that of y through the angle i . If we put

$$x = x' \cos \Omega - y' \sin \Omega \cos i, \quad y = x' \sin \Omega + y' \cos \Omega \cos i,$$

in (2), we shall obtain the equation of the intersection of the plane $x'y'$ with the elliptical cylinder, which is the equation of the real ellipse. Thus we have, on omitting the accents,

$$\left. \begin{aligned} & A(x \cos \Omega - y \sin \Omega \cos i)^2 \\ & + 2H(x \cos \Omega - y \sin \Omega \cos i)(x \sin \Omega + y \cos \Omega \cos i) \\ & + B(x \sin \Omega + y \cos \Omega \cos i)^2 + 2G(x \cos \Omega - y \sin \Omega \cos i) \\ & + 2F(x \sin \Omega + y \cos \Omega \cos i) + 1 = 0. \end{aligned} \right\} \quad (4)$$

The equation of the real ellipse referred to its centre is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1. \quad (5)$$

If we shift the origin to the focus, we must increase x by ae , and the equation becomes

$$\frac{(x+ae)^2}{a^2} + \frac{y^2}{b^2} - 1 = 0, \quad (6)$$

when referred to the principal star.

Now suppose λ to be the angle from the node to the periastron, measured in the plane of the real orbit; then if we turn the axis of x back to the line of nodes, the new coördinates are

$$x \cos \lambda + y \sin \lambda \quad ; \quad -x \sin \lambda + y \cos \lambda.$$

By means of these values of x and y , equation (6) becomes

$$\frac{(x \cos \lambda + y \sin \lambda + ae)^2}{a^2} + \frac{(-x \sin \lambda + y \cos \lambda)^2}{b^2} - 1 = 0; \quad (7)$$

the origin is taken at the focus and the axis of x is directed to the node.

Now this equation is necessarily identical with (4), which also represents the true ellipse referred to the same axes. Hence, when multiplied by a constant factor ϵ the coefficients of the variables must equal the corresponding ones in the equation deduced from the apparent orbit, so that (7) and (4) give

$$\epsilon \left(\frac{\cos^2 \lambda}{a^2} + \frac{\sin^2 \lambda}{b^2} \right) = A \cos^2 \Omega + B \sin^2 \Omega + H \sin 2\Omega. \quad (8)$$

$$\epsilon \left(\frac{\sin^2 \lambda}{a^2} + \frac{\cos^2 \lambda}{b^2} \right) = (A \sin^2 \Omega + B \cos^2 \Omega - H \sin 2\Omega) \cos^2 i. \quad (9)$$

$$\epsilon \left(\frac{1}{a^2} - \frac{1}{b^2} \right) \sin 2\lambda = (-A \sin 2\Omega + B \sin 2\Omega + 2H \cos 2\Omega) \cos i. \quad (10)$$

$$\epsilon \frac{e \cos \lambda}{a} = G \cos \Omega + F \sin \Omega. \quad (11)$$

$$\epsilon \frac{e \sin \lambda}{a} = (-G \sin \Omega + F \cos \Omega) \cos i. \quad (12)$$

$$\epsilon (e^2 - 1) = +1. \quad (13)$$

This last equation gives

$$\epsilon = -\frac{1}{1-e^2} \quad \text{and} \quad \frac{\epsilon e}{a} = -\frac{e}{p}.$$

Also, since

$$\frac{\epsilon^2 p^2}{a^2} = \frac{e^2}{p^2} = \epsilon \left(-\frac{1}{1-e^2} \right) \frac{e^2}{a^2} = \epsilon \left(\frac{1}{1-e^2} \right) \frac{b^2 - a^2}{a^4} = \epsilon \frac{b^2 - a^2}{b^2 a^2} = \epsilon \left(\frac{1}{a^2} - \frac{1}{b^2} \right),$$

we have

$$\frac{e^2}{p^2} = \epsilon \left(\frac{1}{a^2} - \frac{1}{b^2} \right).$$

Now (11) and (12) give

$$e \sin \lambda = -p (F \cos \Omega - G \sin \Omega) \cos i \quad ; \quad e \cos \lambda = -p (F \sin \Omega + G \cos \Omega).$$

Multiplying (11) by (12) and reducing, we find

$$\frac{e^2}{p^2} \sin 2\lambda = (F^2 \sin 2\Omega - G^2 \sin 2\Omega + 2FG \cos 2\Omega) \cos i.$$

From (10) we have

$$\frac{e^2}{p^2} \sin 2\lambda = (-A \sin 2\Omega + B \sin 2\Omega + 2H \cos 2\Omega) \cos i,$$

and hence

$$(F^2 - G^2 + A - B) \sin 2\Omega + 2(FG - H) \cos 2\Omega = 0. \quad (14)$$

If we subtract (9) from (8), we get

$$\frac{e^2}{p^2} \cos 2\lambda = \epsilon \left(\frac{\cos^2 \lambda - \sin^2 \lambda}{a^2} - \frac{\cos^2 \lambda - \sin^2 \lambda}{b^2} \right);$$

and the difference of the squares of $e \cos \lambda$ and $e \sin \lambda$ gives another value of $\frac{e^2}{p^2} \cos 2\lambda$. Equating these two values of $\frac{e^2}{p^2} \cos 2\lambda$, and solving for $\cos^2 i$, we find

$$\cos^2 i = \frac{(F^2 - B) \sin^2 \Omega + (G^2 - A) \cos^2 \Omega + (FG - H) \sin 2\Omega}{(F^2 - B) \cos^2 \Omega + (G^2 - A) \sin^2 \Omega - (FG - H) \sin 2\Omega}. \quad (15)$$

The forms of the numerator and denominator show that if we put $\cos^2 i = \frac{P}{Q}$, and hence $\tan^2 i = \frac{Q-P}{P} = \frac{Q+P}{P} - 2$, we shall get

$$\tan^2 i = \frac{F^2 + G^2 - (A+B)}{P} - 2.$$

The first member of (9) gives

$$e \left(\frac{\sin^2 \lambda}{a^2} + \frac{\cos^2 \lambda}{b^2} \right) = \frac{e^2}{\rho^2} \sin^2 \lambda - \frac{1}{\rho^2},$$

and therefore we obtain

$$\frac{e^2}{\rho^2} \sin^2 \lambda - \frac{1}{\rho^2} = (A \sin^2 \Omega + B \cos^2 \Omega - H \sin 2\Omega) \cos^2 i.$$

By squaring (12) we find

$$\frac{e^2}{\rho^2} \sin^2 \lambda = (F^2 \cos^2 \Omega + G^2 \sin^2 \Omega - FG \sin 2\Omega) \cos^2 i.$$

Therefore we have

$$\frac{1}{\rho^2} = [(F^2 - B) \cos^2 \Omega + (G^2 - A) \sin^2 \Omega - (FG - H) \sin 2\Omega] \cos^2 i. \quad (16)$$

Comparing this with (15), we find $\frac{1}{\rho^2} = P$; and hence

$$\frac{2}{\rho^2} + \frac{\tan^2 i}{\rho^2} = F^2 + G^2 - (A+B). \quad (17)$$

Now since

$$\frac{1}{\rho^2} = P = (F^2 - B) \sin^2 \Omega + (G^2 - A) \cos^2 \Omega + (FG - H) \sin 2\Omega,$$

we easily find

$$\frac{2}{\rho^2} = F^2 + G^2 - (A+B) - (F^2 - B) \cos 2\Omega + (G^2 - A) \cos 2\Omega + 2(FG - H) \sin 2\Omega. \quad (18)$$

Hence (17) gives

$$\frac{\tan^2 i}{\rho^2} = (F^2 - G^2 + A - B) \cos 2\Omega - 2(FG - H) \sin 2\Omega. \quad (19)$$

If we multiply this equation by $\sin 2\Omega$, and (14) by $\cos 2\Omega$, and subtract the last result from the first, we get

$$\frac{\tan^2 i}{\rho^2} \sin 2\Omega = -2(FG - H).$$

If we use $\cos 2\Omega$ and $\sin 2\Omega$, and add the products, we have

$$\frac{\tan^2 i}{p^2} \cos 2\Omega = F^2 - G^2 + A - B.$$

Therefore we finally obtain the following set of equations:

$$\left. \begin{aligned} \frac{\tan^2 i}{p^2} \sin 2\Omega &= -2(FG - H), \\ \frac{\tan^2 i}{p^2} \cos 2\Omega &= F^2 - G^2 + A - B, \\ \frac{2}{p^2} + \frac{\tan^2 i}{p^2} &= F^2 + G^2 - (A + B), \\ e \sin \lambda &= -p(F \cos \Omega - G \sin \Omega) \cos i, \\ e \cos \lambda &= -p(F \sin \Omega + G \cos \Omega), \\ a &= \frac{p}{1 - e^2}. \end{aligned} \right\} \quad (20)$$

These formulae enable us to find $\Omega, i, p, \lambda, e, a$; we may then find v at any epoch by the formula

$$\tan(v + \lambda) = \frac{\tan(\theta - \Omega)}{\cos i}, \text{ and } E \text{ by } \tan \frac{1}{2} E = \sqrt{\frac{1-e}{1+e}} \tan \frac{1}{2} v.$$

We find M by KEPLER'S equation

$$M = E - e'' \sin E.$$

And since $M_2 - M_1 = n(t_2 - t_1)$, we see that

$$n = \frac{M_2 - M_1}{t_2 - t_1},$$

and

$$P = \frac{360^\circ (t_2 - t_1)}{M_2 - M_1}, \quad T = \frac{nt - M}{n}. \quad (21)$$

PROFESSOR GLASENAPP has proposed a simple method for cases in which good drawings of the apparent orbits have been made, but it is not desired to adjust the results by the method of Least Squares, owing to the uncertainty of the data furnished by observation. In the present state of double-star Astronomy this method is very practicable, and can be advantageously employed in the determination of orbits.

In the equation (2)

$$Ax^2 + 2Hxy + By^2 + 2Gx + 2Fy + 1 = 0,$$

we put $y = 0$, and then find the roots of

$$Ax^2 + 2Gx + 1 = 0. \quad (22)$$

This may be written

$$x^2 + \frac{2G}{A}x + \frac{1}{A} = 0, \quad \text{or} \quad (x-x_1)(x-x_2) = x^2 - (x_1+x_2)x + x_1x_2 = 0,$$

where x_1 and x_2 are the roots of the equation, or the abscissae of the points of the orbit on the x -axis.

Hence, by the theory of equations, we have

$$A = \frac{1}{x_1x_2}.$$

Also

$$\frac{2G}{A} = -(x_1+x_2), \quad \text{or} \quad G = -\frac{A(x_1+x_2)}{2} = -\frac{(x_1+x_2)}{2x_1x_2}.$$

In like manner, putting $x = 0$, we find

$$By^2 + 2Fy + 1 = 0, \quad \text{or} \quad B = \frac{1}{y_1y_2}; \quad F = -\frac{y_1+y_2}{2y_1y_2}.$$

Hence when the coördinates of the intersections of the orbit with the axes of x and y are known directly from the apparent orbit, we have the four constants A, B, F, G .

And the other constant is given by

$$H = -\frac{Ax^2 + By^2 + 2Gx + 2Fy + 1}{2xy}.$$

In finding H we must take a point (x, y) such that the product $x.y$ has a large value. It may be desirable to take the mean of several values of H .

When all the constants A, B, F, G, H , have been derived, we find the elements by equations (20) and (21).

§ 10. *Graphical Method of Klinkerfues.*

Suppose α and β to denote the lengths of the real major and minor semi-axes when projected on the plane tangent to the celestial sphere, and A and B to be their position-angles. Then we readily find

$$\left. \begin{aligned} \alpha^2 \cos^2(A-\Omega) + \alpha^2 \sin^2(A-\Omega) \sec^2 i &= a^2, \\ \beta^2 \cos^2(B-\Omega) + \beta^2 \sin^2(B-\Omega) \sec^2 i &= b^2. \end{aligned} \right\} \quad (1)$$

But it is evident that the sum of these equations is the square of the chord between the vertices of the major and minor axes; and the square of the same chord is given by

$$\{a \cos(A-\Omega) - \beta \cos(B-\Omega)\}^2 + \{a \sin(A-\Omega) - \beta \sin(B-\Omega)\}^2 \sec^2 i = a^2 + b^2.$$

Therefore we have

$$\cos(A-\Omega) \cos(B-\Omega) + \sin(A-\Omega) \sin(B-\Omega) \sec^2 i = 0; \quad (2)$$

and hence

$$\cos^2 i = \tan(A-\Omega) \tan(\Omega-B). \quad (3)$$

This equation determines the inclination when the node is known, as the angles A and B are taken directly from the apparent orbit.

If we divide the second of equations (1) by the first, we get

$$\frac{b^2 a^2}{a^2 \beta^2} = \frac{\cos^2(B-\Omega) + \sin^2(B-\Omega) \sec^2 i}{\cos^2(A-\Omega) + \sin^2(A-\Omega) \sec^2 i},$$

and on substituting for $\sec^2 i$ its value, we find

$$\frac{b^2 a^2}{a^2 \beta^2} = -\frac{\sin 2(B-\Omega)}{\sin 2(A-\Omega)}. \quad (4)$$

In this equation a and β are given directly by the apparent orbit, and as e is known, we have also the ratio $\frac{b^2}{a^2} = 1 - e^2$. Therefore the only unknown quantity is 2Ω , which we may determine in the following manner. Since the left member of (4) is the square of a real quantity, the right member must be essentially positive, and we may put

$$\tan \zeta = \frac{ba}{a\beta} = \sqrt{\frac{\sin 2(B-\Omega)}{\sin 2(A-\Omega)}}, \quad (5)$$

and since

$$\sec 2\zeta = \frac{\sin 2(A-\Omega) + \sin 2(B-\Omega)}{\sin 2(A-\Omega) - \sin 2(B-\Omega)} = \tan(A+B-2\Omega) \cot(A-B),$$

we get

$$\tan(A+B-2\Omega) = \sec 2\zeta \tan(A-B). \quad (6)$$

The angle ζ is known from its tangent, and hence we easily find Ω .

In (3) it is to be observed that $\cos^2 i$ is necessarily positive and smaller than unity, and hence we have to choose between two values of Ω differing by 180° . As it is thus impossible to distinguish between the ascending and descending node, we may arbitrarily take the ascending node between 0° and 180° , and find i by means of (3)

$$\cos^2 i = \tan(A-\Omega) \tan(\Omega-B)$$

The angular distance from the node to the periastron is denoted by $\pi - \Omega = \lambda$, and is given by the equation

$$\tan(A - \Omega) = \cos i \tan \lambda,$$

or by using (3) we obtain*

$$\tan^2 \lambda = \frac{\tan(A - \Omega)}{\tan(\Omega - B)}. \quad (7)$$

If u denote the argument of the latitude, we have

$$u = v + \lambda = v + \pi - \Omega, \quad \text{and} \quad \tan u = \sec i \tan(\theta - \Omega),$$

where θ is the observed position-angle at the given epoch. The latitude l is given by $\sin l = \sin i \sin u$.

From the apparent radius vector ρ , we may find the corresponding true radius vector by

$$r = \rho \sec l.$$

The major semi-axis is then found by the polar equation

$$a = \frac{r(1 + e \cos v)}{1 - e^2}. \quad (8)$$

If we take the apastron as the point in question, l will be given by

$$\sin l = \sin i \sin \lambda;$$

and since ρ is taken directly from the diagram of the apparent orbit, we easily find r . Then, since $v = 180^\circ$, we have

$$a = \frac{\rho \sec l}{1 + e}. \quad (9)$$

To find the time of revolution we take two observations which are widely separated in time, and find the intervening change in the mean anomaly; or we may find from the diagram the part of the area swept over during this interval compared to the whole area of the apparent ellipse. If θ_1 and θ_2 be the two angles of position, and u_1 and u_2 the corresponding arguments of the latitude, we shall have

$$\begin{aligned} \tan u_1 &= \sec i \tan(\theta_1 - \Omega), \\ \tan u_2 &= \sec i \tan(\theta_2 - \Omega), \end{aligned}$$

and then

$$v_1 = u_1 - \lambda \quad ; \quad v_2 = u_2 - \lambda;$$

whence the mean anomalies are easily found. Instead of computing the change of the mean anomaly, it is generally preferable to measure up the area swept

* $A - \Omega$ and λ must lie in the same or in opposite quadrants. Throughout this work λ is taken in the direction of the motion.

over by the radius vector during the interval, and determine the period by the law of areas.

Suppose that t_1 and t_2 be the dates of two widely-separated observations; then the double area swept over by the radius vector will be

$$\int_{t_1}^{t_2} \rho^2 \frac{d\theta}{dt} dt.$$

Putting a', b' for the major and minor semi-axes of the apparent ellipse, it is evident that the time of revolution will be given by

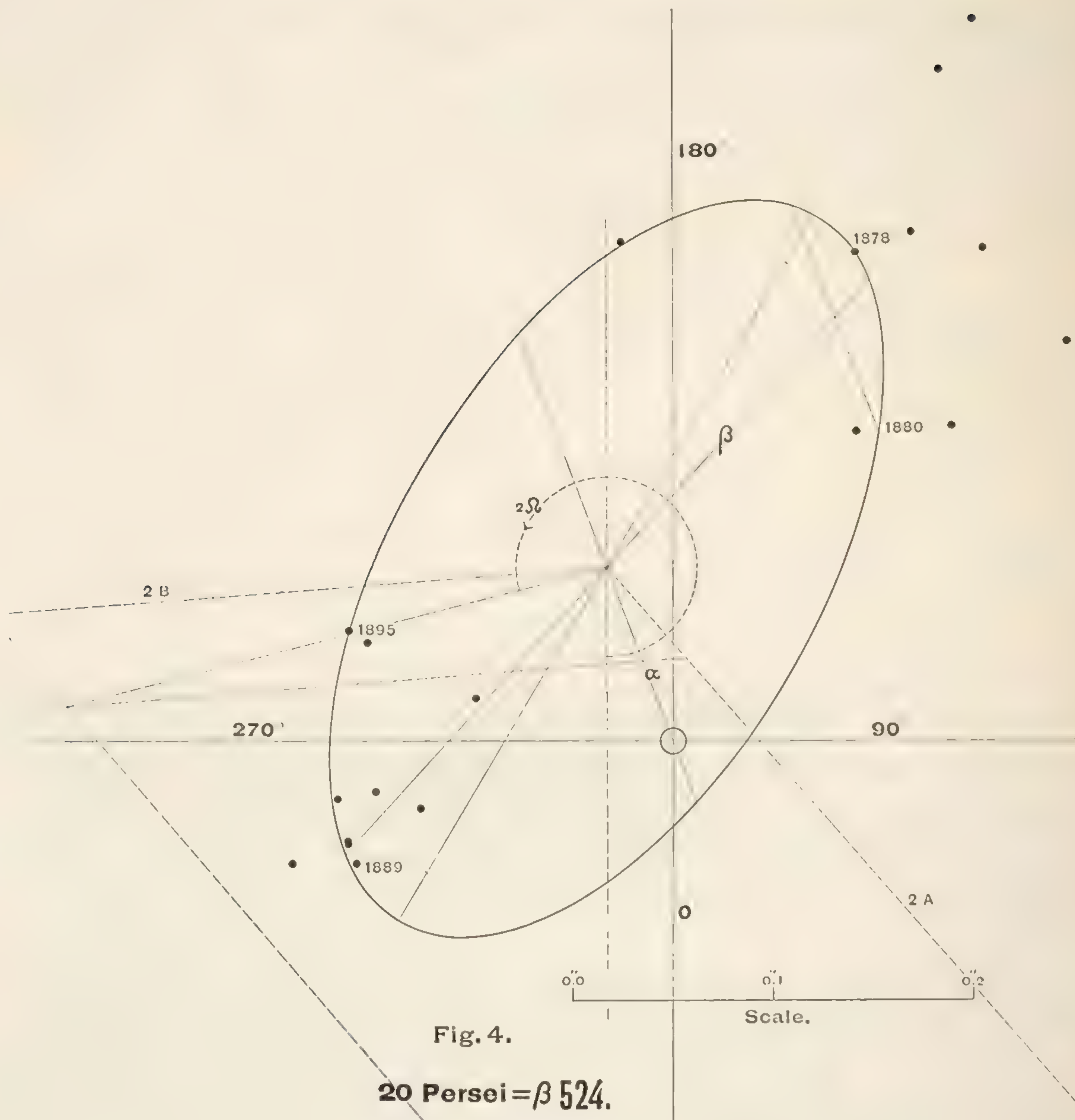
$$P = \frac{2\pi a'b'(t_2 - t_1)}{\int_{t_1}^{t_2} \rho^2 \frac{d\theta}{dt} dt}. \quad (10)$$

In case the period is computed from the change in the mean anomalies, we have

$$n = \frac{M_2 - M_1}{t_2 - t_1} \quad ; \quad P = \frac{360^\circ}{n} \quad (11)$$

The periastron passage is given by $T = t_1 - \frac{M_1}{n}$, or it may be found from the principle of areas, in the same manner as the period. Thus, since the double areal velocity is known, we simply determine the double area included between a given radius vector and the periastron, and ascertain the intervening time. This interval is to be added to or subtracted from the time of observation, according as the date chosen is before or after the epoch of periastron passage.

To find the node by graphical construction we draw from the centre of the ellipse lines whose position-angles are $2A$ and $2B$; then, parallels to these at distances related as $a^2\beta^2$ to $b^2\alpha^2$. Connect the intersection of the parallel lines with the centre, and this will give a line whose position-angle is 2Ω . This construction is easily deduced from (4), and in practice will be found extremely exact. The graphical method is highly practicable, and in the present state of double-star Astronomy is the one which should generally be preferred. The possible inaccuracies of the method are greatly inferior to the uncertainty still attaching to the best orbits. The principal difficulty experienced by computers consists in the finding of a satisfactory apparent orbit.



The apparent orbit of 20 *Persei* = $\beta 524$ is shown above. We find by the figure $e = 0.738$,

$$\frac{b^2 a^2}{a^2 \beta^2} = 0.194; \quad A = 20^\circ.5; \quad B = 137^\circ.3; \quad \Omega = 142^\circ.2; \quad i = 67^\circ.9; \\ \lambda = 103^\circ.1; \quad n = -9^\circ.0; \quad P = 40.0 \text{ years}; \quad a = 0''.290; \quad T = 1884.40.$$

To obtain the apparent orbit it is best to make use of both angles and distances. If the precession has a sensible effect upon the position angles, it is desirable to refer the observations to a common epoch by applying the formula

$$\Delta\theta = n \sin \alpha \sec \delta (t - t_0). \quad (12)$$

where $n = 20''.04987$, and t_0 is the date of observation, t the epoch adopted. We then combine the individual measures of the best observers into suitable annual means, and plot the resulting positions on a convenient scale. The approximate normal places thus defined are subject to two conditions:

(1) That the areas swept over by the radius vector shall be proportional to the times;

(2) That the apparent ellipse which satisfies the law of areas shall conform also to the observed distances.

The ellipse which satisfies these conditions must be found by trial. Fine planimeter measurement renders the approximation comparatively rapid, and when a satisfactory ellipse has been obtained we derive the elements and compare the computed with the observed places.

We first determine e , then compute the ratio $\frac{b^2 a^2}{a^2 \beta^2}$, and find the node by graphical construction; it is then easy to find i, λ, P, T , and a , as explained in the foregoing method. If further refinement of the elements be desired, recourse must be had to differential formulae.

It is to be remarked, however, that the assumption of constant areal velocity is equivalent to postulating the absence of unseen bodies or other disturbing influences, and as this is not yet fully established, the orbits which best represent the angular motion are not necessarily correct, as may be seen in the case of 70 *Ophiuchi*. If it is necessary to violate the distances in a conspicuous manner in order to preserve the law of the areas, the result must be looked upon with suspicion. In the present state of double-star Astronomy most of our orbits must be regarded as tentative, but when they shall finally be improved there is no doubt that, if the motion is really undisturbed, both angles and distances will be well represented.

If it is desired to compute ρ and θ from the elements, we may employ the formulae

$$\tan(\theta - \Omega) = \tan(\lambda + v) \cos i \quad ; \quad \rho = a(1 - e \cos E) \frac{\cos(\lambda + v)}{\cos(\theta - \Omega)}.$$

The element λ is counted from the node between 0° and 180° , in the direction of the motion; in case of retrograde motion the formula for θ becomes

$$\tan(\Omega - \theta) = \tan(\lambda + v) \cos i.$$

Graphical Method of Finding the Apparent Orbit of a Double Star.

It is frequently desirable to project the apparent orbit of a double star from the elements; this interesting and useful result may be effected in a

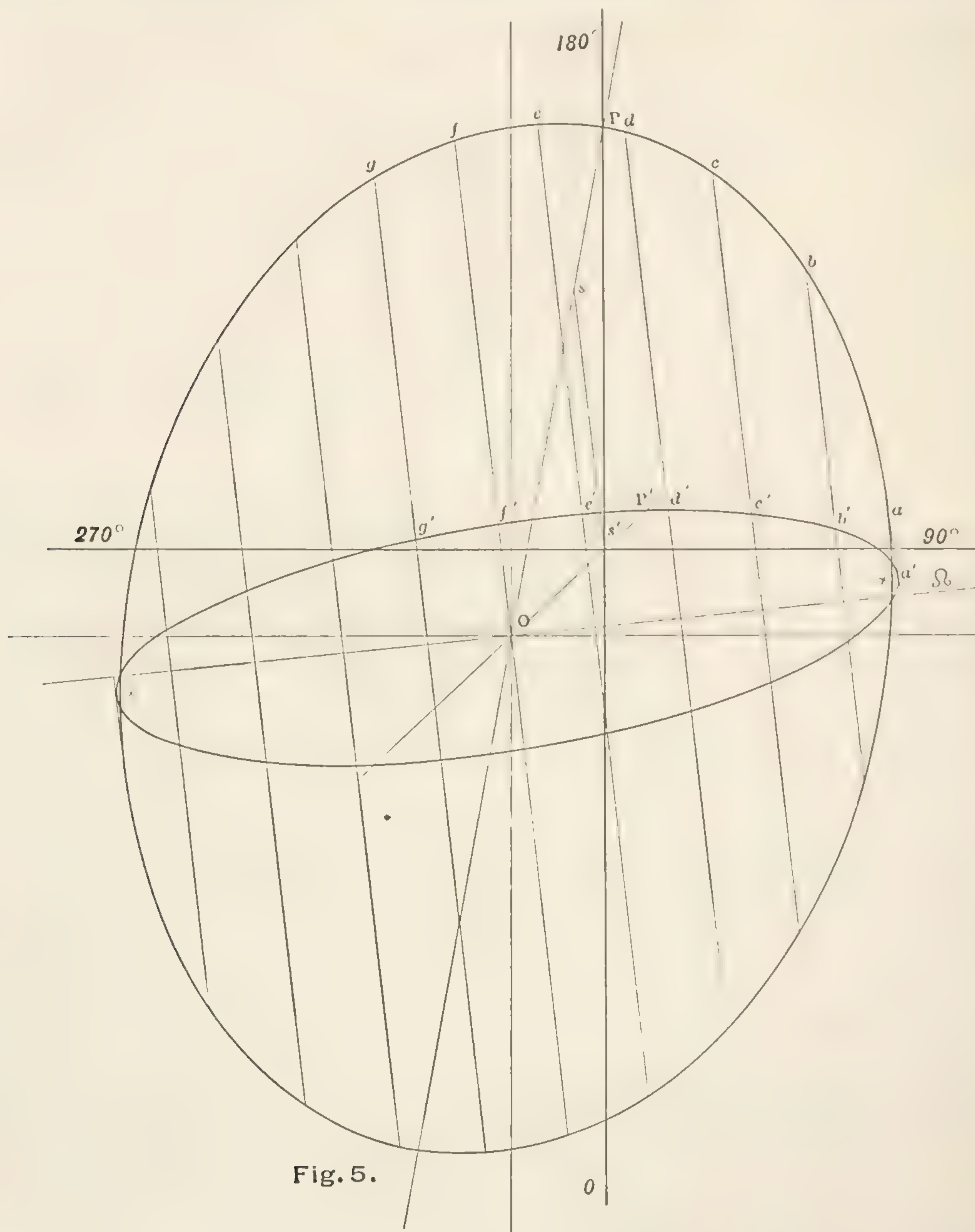


Fig. 5.

very simple manner. In order to make the process more intelligible we shall apply it to a particular case, and for this purpose we select the orbit of *9 Argûs* = β 101.

The elements required for this purpose are the following:

Eccentricity,	$e = 0.700 \pm 0.02$
Major semi-axis,	$a = 0''.6549$
Node,	$\Omega = 95^\circ.5$
Inclination,	$i = 77^\circ.72$
Node to periastron,	$\lambda = 75^\circ.28$

We lay down on suitable drawing paper two lines which intersect each other at right angles, and thus mark the four quadrants of position-angle. The intersection of these lines will be the centre of the real orbit and also the centre of the apparent orbit. The line of nodes is then drawn through the centre, having a position-angle of $95^\circ.5$. In like manner we lay down the line whose position-angle is $\Omega + \lambda = 170^\circ.78$, and this will be the major axis of the real ellipse.

We now adopt a convenient scale, which will give a length on the drawing paper of 10 or 12 inches for the major axis.

With close stars $0''.1$ may represent one or two inches of the scale, so that the work can be done with the highest degree of accuracy. From the centre the length of the major semi-axis ($0''.6549$) is laid down on the line just drawn, and the distance of the foci of the ellipse from the centre will be ae ($0''.6549 \times 0.70$). The ellipse is then drawn in the usual manner.

We now lay off points on the line of nodes at equal distances from the centre of the ellipse, and through these points draw lines aa' , bb' , cc' , dd' etc., perpendicular to the line of nodes. The lengths of these lines on either side are found in seconds of arc by the scale used, and then multiplied by the cosine of the inclination ($\cos 77^\circ.72 = 0.214$); the resulting values are marked on the corresponding lines at a' , b' , c' , d' , e' , f' , etc., on both sides of the line of nodes.

The points thus determined will lie on the arc of the true ellipse as seen from the Earth, and when we pass the curve through them, we have the apparent orbit of the double star.

To find the position of the star in the apparent ellipse, we multiply the distance of the focus of the real ellipse from the line of nodes by the cosine of the inclination, and thus find the point s' , which will be the position of the central star in the projected orbit. A line $Os'P'$, drawn from the centre through this point to intersect the arc of the apparent ellipse, gives the position-angle of the real major axis, and the position of the real periastron.

Having thus obtained the position of the central star in the apparent orbit, it only remains to draw through the principal star lines parallel to those inter-

secting at the centre and marking the four quadrants, which may now be erased. In the figure the lines which mark the four quadrants are somewhat heavier than the rest, so that they are easily recognized.

Thus a very simple process of projection enables us to trace the outline of the apparent orbit of any star when the required elements are given; and from the observed positions it is possible to see at a glance whether the apparent orbit represents the observations satisfactorily. It only remains to add that in the case of retrograde motion, the angle λ (which should always be counted in the direction of motion, while the ascending node should be taken between 0° and 180°) must for purposes of graphical representation be taken as negative, and the position-angle of the major axis of the real ellipse becomes $\Omega - \lambda$, whereas for direct motion the angle is $\Omega + \lambda$, as in the case of *9 Argûs*.

§ 11. *Formulae for the Improvement of Elements.*

The foregoing graphical method, when judiciously applied, will give elements having all the accuracy which can be desired in the present state of double-star Astronomy. But as some improvement of a very refined character will ultimately be possible, we shall present the differential formulae which may be employed to effect these slight variations of the elements.

The formulae for finding the position-angle θ from the elements are

$$\begin{aligned} M &= n(t-T) = E - e' \sin E, \\ \tan \frac{1}{2} v &= \sqrt{\frac{1+e}{1-e}} \tan \frac{1}{2} E, \\ \tan(v+\lambda) \cos i &= \tan(\theta - \Omega). \end{aligned}$$

Since θ is a function of the six elements, Ω , i , λ , e , T , n , we have

$$d\theta = \frac{\partial F(\theta)}{\partial \Omega} d\Omega + \frac{\partial F(\theta)}{\partial i} di + \frac{\partial F(\theta)}{\partial \lambda} d\lambda + \frac{\partial F(\theta)}{\partial e} de + \frac{\partial F(\theta)}{\partial T} dT + \frac{\partial F(\theta)}{\partial n} dn.$$

When the variations of the elements are finite, but small, we have the approximate formula,

$$\theta_0 - \theta_s = \Delta\theta = A\Delta\Omega + B\Delta i + C\Delta\lambda + D\Delta e + G\Delta T + H\Delta n,$$

where A , B , C , D , G and H , denote the partial differential coefficients.

From the equations which enable us to compute θ we obtain these coef-

ficients by partial differentiation with respect to the several elements. Thus we find

$$\begin{aligned} A &= +1; \\ B &= -\cos^2(\theta - \Omega) \tan(v + \lambda) \sin i, \\ C &= \sec^2(v + \lambda) \cos^2(\theta - \Omega) \cos i, \\ D &= \left(\frac{2 \tan \frac{1}{2} E}{(1-e)\sqrt{1-e^2}} + \sqrt{\frac{1+e}{1-e}} \frac{\sec^2 \frac{1}{2} E \sin E}{1-e \cos E} \right) \cos^2 \frac{1}{2} v \cdot C, \\ G &= -\sqrt{\frac{1+e}{1-e}} \frac{\sec^2 \frac{1}{2} E \cos^2 \frac{1}{2} v \cdot C}{1-e \cos E}, \\ H &= \frac{(t-T')}{n} \cdot G. \end{aligned}$$

The formulae usually employed by astronomers for effecting the differential corrections of the elements thus take the form

$$\begin{array}{l} A_1\Delta\Omega + B_1\Delta i + C_1\Delta\lambda + D_1\Delta e + G_1\Delta T + H_1\Delta n - \Delta\theta_1 = 0 , \\ A_2\Delta\Omega + B_2\Delta i + C_2\Delta\lambda + D_2\Delta e + G_2\Delta T + H_2\Delta n - \Delta\theta_2 = 0 , \\ . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ A_v\Delta\Omega + B_v\Delta i + C_v\Delta\lambda + D_v\Delta e + G_v\Delta T + H_v\Delta n - \Delta\theta_v = 0 . \end{array}$$

There are six quantities to be deduced from this system of equations ; a solution by the method of Least Squares will generally ensure the best results. In the above form of the equations it is tacitly assumed that the residuals in angle represent absolute displacements of the companion in space, regardless of its distance from the central star, which is evidently inexact. The importance of a given error in angle increases in proportion to the length of the radius vector, and as the distance of the companion is generally very unequal in different parts of the apparent orbit, the formulae should be so modified as to render the absolute displacements of the observed positions a minimum. This improvement can be effected as follows. We shall assume that the major axis can be best determined from the apparent orbit, which serves as an interpolating curve analogous to that recommended by SIR JOHN HERSCHEL, and hence this element need not be regarded as variable. It is, therefore, required to compute the slight variations for the other six elements.

Let us suppose that the value of ρ corresponding to the position-angle θ_o is ρ_a ; this value may be computed or measured graphically from the diagram. Let the corrected angle and distance be θ_c and ρ_c respectively. Then it is easy to see that the displacement of a point on the apparent orbit due to the correction of the elements will be given by

$$\Delta s = \sqrt{\left(\frac{\rho_a + \rho_c}{2}\right)^2 (\theta_o - \theta_c)^2 + (\rho_a - \rho_c)^2}.$$

In case the length of the radius vector in the apparent orbit is practically constant, the last term of the radical becomes insensible, and the displacement in space at a given distance is proportional to the displacement in angle. But as many of the orbits are very eccentric and highly inclined, and the radius vector therefore changes rapidly, the best result can be obtained only by the use of the complete residuals expressed above. In computing these values numerically we may express $(\rho_a - \rho_c)$ in degrees by the formula $2\left(\frac{\rho_a - \rho_c}{\rho_a + \rho_c}\right) 57^\circ.3$; and since $(\theta_o - \theta_c)$ is already given in degrees, we must express the coefficient as an abstract number in units of the major semi-axis, in order to give the displacements in angle weight proportional to the length of the radius vector:

Since the second term of the resulting expression under the radical sign

$$\Delta\theta^\circ = \sqrt{\left[\left(\frac{\rho_a + \rho_c}{2a}\right)(\theta_o - \theta_c)^\circ\right]^2 + \left[\frac{2(\rho_a - \rho_c)}{(\rho_a + \rho_c)} \cdot 57^\circ.3\right]^2}$$

will often be very small, it will frequently be sufficient to use the first term only; or in other words, to assign the residuals in angle weights proportional to the lengths of the radii vectores.

This method of improving the elements will be found very much shorter than that involved in the process of correcting both angles and distances by separate differential formulae, and will lead to the same results without loss of accuracy.

§12. *A General Method for Facilitating the Solution of Kepler's Equation by Mechanical Means.**

The standard works on planetary motion, such as GAUSS' *Theoria Motus*, OPPOLZER'S *Bahnbestimmung*, and WATSON'S *Theoretical Astronomy*, give methods for solving KEPLER'S Equation which are very satisfactory when the eccentricity of the orbit is small, and also when this element is large, as in the case of most of the periodic comets. When the eccentricity is small, an expansion in series, usually by LAGRANGE'S Theorem, enables us to find the eccentric anomaly with the desired facility. The series frequently employed has the form

$$E_0 = M + e'' \sin M + e'' \left(\frac{e}{2}\right) \sin 2M + \dots$$

* *Monthly Notices*, June, 1895; also Note in *Monthly Notices* for December, 1895.

To the approximate value E_0 , obtained from a few terms of this series, we apply a correction resulting from the expansion by TAYLOR'S Theorem:

$$E = E_0 + \frac{dE_0}{dM_0} dM_0 + \dots$$

The equation of KEPLER gives

$$\frac{dM_0}{dE_0} = 1 - e \cos E_0;$$

and since

$$dM_0 = M - M_0,$$

we find two terms of the series to be

$$E = E_0 + \frac{M - M_0}{1 - e \cos E_0}.$$

Successive applications of this formula will readily yield the true value of the eccentric anomaly. But when the eccentricity is considerable the expansion in series fails to converge with the desired rapidity. On the other hand, when the orbits differ but little from parabolas, the solution can readily be found by means of special tables, such as those given by GAUSS, WATSON and OPPOLZER.

It is very remarkable that among the many solutions of KEPLER'S Equation discovered by mathematicians there is not one, so far as I am aware, which has come into general use among astronomers that is applicable to ellipses of all possible eccentricities.

The method to which I desire to direct attention is a modification of the graphical method originally invented by J. J. WATERSTON (*Monthly Notices*, 1849-50, p. 169), and subsequently rediscovered by DUBOIS (*Astronomische Nachrichten*, no. 1404). The method was afterwards discussed by KLINKERFUES in his *Theoretische Astronomie*, p. 17; but so far as I am aware* it never came into practical use until employed in the investigations embodied in this work.

Suppose we construct, on a convenient scale, a semi-circumference of the curve of sines, $y = \sin x$. In practice it is desirable to use millimetre paper, and a convenient scale is obtained by taking one degree of the arc as five millimetres, so that the scale may easily be read to $0^\circ.1$. The origin of the arc is taken at the origin of coördinates; and as the scale along the axis of abscissae extends from 0° to 180° , it will have a length of 90 centimetres.

In the figure let OM represent the mean anomaly, and suppose from M

* *Monthly Notices*, December, 1895.

we draw a right line making an angle ψ with the axis of abscissae, the angle ψ being defined by the equation

$$\tan \psi = \frac{1}{e}.$$

Let the abscissa of the point C , determined by the intersection of the right line MC with the sine curve, be denoted by E . Then we evidently have

$$OE - ME = OM.$$

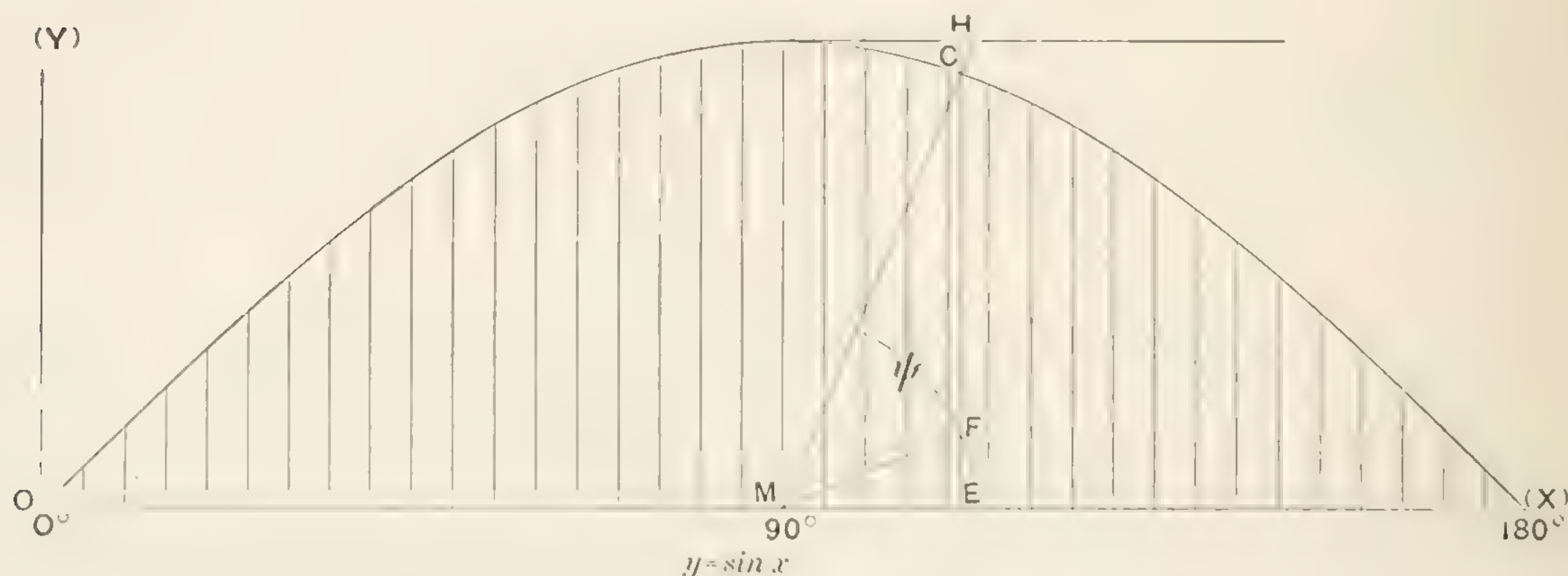


Fig. 6

Thus, denoting the arc OE by E , and observing that $e \sin \psi = \cos \psi$, we find that $e \sin \psi = ME$, the radius in the case of $\sin \psi$ being such that $\sin \psi$ is always equal to $\sin E$.

Hence we get

$$OE - ME = OM,$$

or

$$E - e \sin E = M,$$

which is the Equation of KEPLER.

Therefore we conclude that if for an orbit of given eccentricity we construct a triangle CME (in practice this may be made of cardboard) and apply the vertex M of the triangle to the successive mean anomalies, the base coinciding with the x -axis, the intersection of the hypotenuse with the curve of sines will give at once abscissae which are the corresponding eccentric anomalies. Any actual diagram such as we have described will be subject to slight inaccuracies of construction, owing to the transcendental nature of the sines, and hence we cannot obtain solutions of absolute precision. But it is entirely possible to get approximate solutions exact to 0.1 , and this work can be done with the greatest rapidity. It is merely necessary to slide the base of the

triangle along the x -axis, placing the vertex M at the points corresponding to the different values of the mean anomaly, and reading off the corresponding eccentric anomalies.

This triangle device is rendered possible by virtue of the fact that ψ is constant in $\tan \psi = \frac{1}{e}$; and we may observe that in case of elliptic orbits the angle ψ can vary only from 45° in the case of a parabola to 90° in the case of a circle. This method is therefore directly applicable to ellipses of every possible eccentricity, and the accuracy of the solution is always substantially the same. In the case of parabolic motion, however, the method fails, since when $\psi = 45^\circ$ the hypotenuse MC is tangent to the sine curve at the origin. But for $e < 1$ the hypotenuse MC intersects the curve $y = \sin x$, and the intersection will be well defined except when e approaches unity and M is very small. In such cases it is best to use the Special Tables or the Theory of Parabolic Motion. Solutions exact to $0^\circ.1$ are often sufficient in the present state of double-star observation, and we readily see how great is the practical value of this method in comparing a long series of observations with a given set of elements. One hundred approximate solutions of KEPLER'S Equation, accurate to $0^\circ.1$, may be obtained by this method in less than half an hour; while if e lies between 0.35 and 0.85 probably a skilled computer could not obtain the same results by the ordinary method in less than a day. Thus the time and labor required for this work is much diminished, and it is clear that the chances of large error are correspondingly reduced.

If a curve of sines were engraved on a metallic plate it would be an easy matter to devise a movable protractor which could be set at any angle; such a piece of apparatus would serve for every possible elliptic orbit, and would last for an indefinite time. Considering the immense labor devolving upon astronomers in the computation of the motion of the heavenly bodies, it would seem that such a labor-saving device might be advantageously employed in the offices of the astronomical ephemerides. However, as several astronomers have prepared tables for facilitating the solution of KEPLER'S Equation in the case of orbits which are not very eccentric, such an apparatus would be useful chiefly in work on the more eccentric asteroids, the double stars, and the periodic comets. In dealing with the motions of these bodies the labor saved would be very considerable, and we might hope that the apparatus here suggested would come into actual use. But in case this instrument of precision could not be successfully manufactured, owing to its limited commercial use, it is easy for a working astronomer to construct a curve of sines on millimetre paper.

This can be mounted on a suitable wooden board, and a triangle of cardboard will give the solutions of KEPLER'S Equation for any given orbit.

Thus, while the graphical method, originally proposed by WATERSTON, afterwards independently discovered by DUBOIS, and subsequently discussed by KLINKERFUES, was suggested many years ago, it does not appear that it has yet come into general use; and therefore it deserves the careful attention of astronomers. It is worthy of remark that a method of such great practical importance should rest in comparative oblivion during half a century, at a time when astronomers were constantly working on the motions of periodic comets and double stars; but it is probable that neither WATERSTON nor DUBOIS recognized the great generality and high value of the method in practical work. Since writing the paper which I communicated to the Royal Astronomical Society in June, 1895, I have had occasion to make great use of the method in revising the orbits of double stars, and have found it not only the easiest and most rapid process yet invented, but one altogether so satisfactory that we may predict its universal adoption by astronomers. The simplicity and generality of the method and the rapidity and accuracy with which solutions can be obtained, invite the inference that in the nature of the case the method is probably ultimate, and is not likely to be improved upon in any future age.

While this method is of special importance in dealing with the motions of double stars, owing to the wide range of their eccentricities, it will evidently be almost, if not quite, equally important in the case of periodic comets and the asteroids. But in dealing with comets and planets, where we desire very exact solutions of KEPLER'S Equation, it will be necessary to correct the approximate values by the formula

$$\Delta E_0 = \frac{M - M_0}{1 - e \cos E_0},$$

where M_0, E_0 are the approximate values of the mean and eccentric anomalies. A second correction will ensure all the accuracy desirable in planetary and cometary ephemerides.*

*Among the other means for solving KEPLER'S Equation we mention especially the tables of ASTRAND (ENGLEMANN, Leipzig); DOBERCK, *A.N.*, Bd. 139; and a graphical method by MR. H. C. PLUMMER, *Monthly Notices*, March, 1896.

CHAPTER II.

ON THE ORBITS OF FORTY BINARY STARS.

Introductory Remarks.

THE present chapter is occupied with detailed researches on the motions of the forty stars whose orbits can be best determined at this epoch. The material presented for each star has been collected from all available sources and is very complete. It is highly improbable that any important records have been overlooked, and since we have drawn the material almost wholly from original sources, future investigators will have little need to repeat the labor involved in collecting observations of these stars prior to 1895.

In some cases we have not used all of the available measures, either because the observations appeared to be defective, or because good observations were obtained too late to be incorporated in the discussions, which were not changed unless the elements adopted were found to be inconsistent with the new material. In the main, our choice of observations has been guided by the assumption that it is possible to find an orbit which is consistent with undisturbed elliptical motion. The observations have justified a violation of this principle only in the case of 70 *Ophiuchi*, which presented anomalies too large to be attributed to errors of observation. If the course of time should show that other stars also are perturbed, it will become apparent that we have not always made the best choice of the material now available.

In the determination of these orbits a number of distinguished astronomers have contributed their observations in advance of publication. They have not only sent manuscript copies of valuable measures, but have offered their work with a generosity which merits my most grateful acknowledgement. Among those to whom we return thanks are: M. G. BIGOURDAN, National Observatory, Paris; PROFS. G. C. COMSTOCK and A. S. FLINT, Washburn Observatory, Madison; PROF. S. DE GLASENAPP, Director of the Observatory, Imperial University, St. Petersburg; PROF. G. W. HOUGH, Director of the Dearborn Observatory, Evanston, Ill.; PROF. V. KNORRE, Royal Observatory, Berlin; T. LEWIS, Esq., Royal Observatory, Greenwich; M. W. MAW, Esq., Private

Observatory, London; PROF. G. V. SCHIAPARELLI, Director of the Royal Observatory, Milan; PROF. W. SCHUR, Director of the Royal Observatory, Göttingen; JOHN TEBBUTT, Esq., Private Observatory, Windsor, N.S. Wales; DR. H. C. WILSON, Goodsell Observatory, Northfield, Minn.

I have also had the constant coöperation of PROFESSORS BURNHAM and BARNARD, who have made valuable suggestions in addition to contributing important observations, some of which were secured expressly for this work. In the investigation of the individual orbits my friends MR. GEO. K. LAWTON, MR. ERIC DOOLITTLE, and MR. F. R. MOULTON have at different times rendered valuable assistance in the execution of a large part of the computations. Without such assistance, uniformly characterized by both zeal and enthusiasm, it would have been impossible to have completed the determination of so many orbits in so short a time. To these gentlemen I acknowledge my deep and lasting obligations. Besides aiding me in the preparation of Chapter I, MR. MOULTON has assisted in arranging the manuscript for the printer, and in reading the proofs, and thus not only expedited the work but also ensured greater accuracy than otherwise would have been possible.

While no effort has been spared to ensure exactness in the computations and in the drawings, it can scarcely be hoped that in dealing with so great a mass of material all errors have been avoided. There is reason, however, to believe that such errors as may exist in the work will have no appreciable effect upon the final results.

A number of the orbits embodied in this Chapter have been published in the *Astronomical Journal*, the *Astronomische Nachrichten*, and the *Monthly Notices* of the Royal Astronomical Society; references to these sources will be found in the appropriate places.

Abbreviations of the Names of Observers.

A.C. = Alvan Clark.	Brw. = Brünnow.	Dur. = Durham Observers.
A.G.C. = Alvan G. Clark.	Cal. = Callandreau.	Ek. = Eneke.
Adh. = Adolph.	Cin. = Cincinnati Observers.	El. = Ellery.
Au. = Auwers.	Col. = Collins.	En. = Englemann.
β . = Burnham.	Com. = Comstock.	Fer. = Ferrari.
Bar. = Barnard.	Cop. = Copeland.	Fl. = Flammarion.
Be. = Bessel.	Da. = Dawes.	Fli. = Flint.
Bh. = Bruhns.	Dav. = Davidson.	Flt. = Fletcher.
Big. = Bigourdan.	Dem. = Dembowski.	Fö. = Förster.
Bo. = Bond.	Dk. = Doberek.	Fr. = Franz.
Bö. = Börgen.	Du. = Dunér.	Ga. = Galle.

Gia. = Giacomelli.	Ma. = Main.	Sec. = Secchi.
Gl. = Gledhill.	Mä. = Mädler.	See = T. J. J. See.
Glas. = Glasenapp.	Mac. = Maclear.	Sel. = Sellors.
Go. = Goldeny.	Maw = M. W. Maw.	Sh. = Schur.
H ₁ . = W. Herschel.	Mi. = Miller.	Sl. = Selander.
H ₂ . = J. F. W. Herschel.	Mit. = Mitchell.	Sm. = Smith.
Hi. = Hind.	Ml. = Moulton.	So. = South.
Hl. = Hall.	New. = Newcomb.	Sr. = Searle.
Ho. = Hough.	No. = Nobile.	St. = O. Stone.
Hol. = Holden.	Pei. = Peirce.	T. = Tebbutt.
Hv. = Harvard Observers.	Per. = Perrotin.	Tar. = Tarrant.
Ja. = Jacob.	Pet. = Peters.	Tj. = Tietjen.
Jed. = Jedrzejewicz.	Ph. = Philpot.	Vo. = Vogel.
Jo. = Jones.	Pl. = Plummer.	Wdo. = Waldo.
Ka. = Kaiser.	Po. = Powell.	Wh. = Wichman.
Kn. = Knott.	Pr. = Pritchett.	Ws. = J. M. Wilson.
Knr. = Knorre.	Rad. = Radcliffe Observers.	H.C.W. = H. C. Wilson.
Kü. = Küstner.	Rus. = Russell.	W. & S. = Wilson & Seabroke.
Ley. = Leyton Observers.	Σ. = W. Struve.	Well. = Wellmann.
Lin. = Lindstedt.	H Σ. = H. Struve.	Winn. = Winnecke.
Lov. = Lovett.	O Σ. = O. Struve.	Wlk. = Winlock.
Ls. = Lewis.	Sch. = Schiaparelli.	Wr. = Wrottesley.
Lu. = Luther.	Scl. = Schlüter.	Y. = Young.
Lv. = Leavenworth.	Sea. = Seabroke.	

Σ 3062.

$\alpha = 0^h 1^m$; $\delta = +57^\circ 53'$.
6.9, yellowish ; 7.5, bluish white.

Discovered by Sir William Herschel, August 25, 1782.

OBSERVATIONS.

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1782.65	319.4	—	1	Herschel	1842.80	207.3	0.87	1	Mädler
1783.05	319.1	—	1	Herschel	1843.58	208.7	0.92	3	Mädler
1823.81	36.7	1.25 ±	1	Struve	1843.80	210.0	0.94	1	Dawes
1831.71	85.7	0.82	2	Struve	1844.49	213.7	0.85	5	Mädler
1833.71	108.6	0.56	3	Struve	1846.42	220.3	0.97	2	O. Struve
1835.66	132.6	0.41	5	Struve	1847.53	225.1	1.12	5	Mädler
1836.61	146.4	0.47	5	Struve	1848.22	229.7	1.14	2	O. Struve
1840.32	186.5	0.65	4	O. Struve	1848.87	228.8	1.16	1	Dawes
1840.78	186.9	0.8 ±	3-2	Dawes	1849.19	232.5	1.09	3	O. Struve
1841.58	193.6	0.89	7	Mädler	1850.04	233.9	1.17	3	O. Struve
1841.86	193.4	0.95	2	Dawes	1850.71	232.3	1.31	3	Mädler
					1850.93	235.2	—	1	Dawes

l	θ_0	ρ_0	n	Observers	l	θ_0	ρ_0	n	Observers
1851.16	235.7	1.35	2	O. Struve	1871.57	283.8	1.39	7	Dembowski
1851.18	236.9	1.16	8	Mädler	1871.60	284.0	1.6	1	Gledhill
1851.75	234.5	1.27	2	Mädler	1872.63	285.7	1.47	6	Dembowski
1852.49	238.4	1.23	3	O. Struve	1872.80	286.3	1.45	1	W. & S.
1854.11	243.5	1.48	4	O. Struve	1873.63	287.6	1.45	9	Dembowski
1854.32	244.3	1.28	3	Dawes	1873.80	297.8	0.91	1	Leyton Obs.
1854.99	249.9	Sep.	6	Dembowski	1873.82	287.8	1.45	1	W. & S.
1855.05	242.7	1.38	3	O. Struve	1873.84	288.0	1.55	2	Gledhill
1855.80	249.4	1.3	8	Dembowski	1874.64	289.8	1.40	6	Dembowski
1855.91	247.9	1.33	3	Morton	1874.72	299.1	1.08	1	Leyton Obs.
1856.57	245.5	1.41	1	Winnecke	1874.86	291.2	1.37	1	W. & S.
1856.62	250.6	1.2	4	Dembowski	1874.91	291.1	1.35	2	Gledhill
1856.66	247.8	1.40	2	O. Struve	1875.67	292.2	1.47	6	Dembowski
1856.80	248.8	1.13	1	Mädler	1875.69	292.9	1.49	5	Dunér
1857.37	250.4	1.50	3	O. Struve	1876.74	293.3	1.61	1	O. Struve
1857.60	253.4	1.25	3	Secchi	1876.67	294.5	1.46	5	Dembowski
1857.71	252.2	1.2	4	Dembowski	1876.87	294.5	1.60	3-2	Doberek
1858.54	252.4	1.2	2	Dembowski	1876.93	298.8?	1.41	1	W. & S.
1859.16	255.3	1.46	3	O. Struve	1876.99	294.5	1.46	5-4	Plummer
1861.79	265.2	1.21	2	Mädler	1877.61	295.8	1.46	4	Dembowski
1862.18	261.7	1.51	2	O. Struve	1877.74	297.3	1.49	4	Doberek
1862.79	263.6	1.46	11	Dembowski	1878.60	299.1	1.51	4	Dembowski
1862.83	266.1	1.29	2	Mädler	1878.90	302.3	1.39	5	Doberek
1863.80	266.0	1.43	9	Dembowski	1879.45	301.9	1.50	8	Hall
1863.86	265.6	1.40	1	Dawes	1879.77	303.2	1.33	5	Doberek
1864.73	268.7	1.40	7	Dembowski	1880.60	304.5	1.50	6	Burnham
1865.70	271.2	1.35	6	Dembowski	1880.88	304.3	1.55	4	Doberek
1865.71	269.9	1.43	3	Knott	1881.14	301.0	1.44	3-2	Jedrzejewicz
1865.71	271.9	1.14	2-3	Leyton Obs.	1881.60	307.8	1.60	3	Burnham
1866.20	270.4	1.47	2	O. Struve	1881.81	306.5	1.97	2-1	Bigourdan
1866.64	270.3	1.46	3	Leyton Obs.	1881.83	305.5	1.40	4	Hall
1866.72	275.5	1.13	3	Harvard	1882.11	304.9	1.29	7	Jedrzejewicz
1866.74	273.4	1.44	5	Dembowski	1882.70	312.3	1.62	1	O. Struve
1866.97	270.0	1.34	1	Secchi	1882.82	308.1	1.52	4-3	Doberek
1867.74	275.2	1.41	7	Dembowski	1883.60	309.8	1.69	9	Englemann
1868.67	277.5	1.38	4	Dembowski	1883.94	312.8	1.44	3	Hall
1868.75	268.3	1.66	3-1	Leyton Obs.	1884.47	311.7	1.26	2	Seabroke
1868.98	276.5	1.59	2	O. Struve	1885.80	316.1	1.46	5	Hall
1869.75	279.9	1.48	6	Dembowski	1886.20	315.2	1.43	3-2	Seabroke
1870.18	279.2	1.48	2	O. Struve	1886.92	314.6	1.46	5	Hall
1870.44	281.0	1.5	1	Gledhill	1887.06	315.5	1.36	6-3	Schiaparelli
1870.64	280.6	1.63	-	Leyton Obs.	1887.10	310.7	1.50	3	Tarrant
1870.67	282.2	1.43	7	Dembowski					

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1888.09	317.7	1.40	1	Schiaparelli	1892.94	323.7	1.62	1	Jones
1888.94	319.4	1.36	4	Hall	1892.99	328.5	1.47	2	Schiaparelli
1888.96	319.5	1.46	6	Schiaparelli	1893.83	327.8	1.58	2	Comstock
1889.57	321.1	1.45	3	Burnham	1893.96	330.9	1.45	2	Schiaparelli
1889.86	323.0	1.45	4	Hall	1894.28	330.6	1.70	3-2	Bigourdan
1889.94	320.5	1.38	1	Seabroke	1894.64	331.99	1.86	1	Glasenapp
1890.76	321.8	1.61	1	Bigourdan	1895.10	331.2	1.58	1	Davidson
1890.79	325.2	1.34	5	Hall	1895.14	330.3	1.61	7-6	Bigourdan
1890.93	323.5	1.52	1	Schiaparelli	1895.15	327.4	1.16	3	Hough
1891.48	322.4	1.5±	1	See	1895.18	331.9	1.46	2-1	Comstock
1891.95	327.3	1.47	2	Schiaparelli	1895.73	334.3	1.53	4	See
1892.71	329.1	1.47	3	Comstock	1895.74	334.5	1.40	2	Moulton
1892.86	325.6	1.52	2	Collins					

When HERSHEY discovered this pair he measured the angle and repeated his observation the following year, without finding any sensible change.* Beginning with 1823, STRUVE followed the star for ten years; and from the measures thus secured he discovered that the system is a binary in rapid orbital motion. Since STRUVE's time the star has been carefully measured by many of the best observers, so that there is abundant material upon which to base an orbit which seems likely to be substantially correct.

Having collected all the published observations of Σ 3062 from original sources, I have formed for each year a mean position which is the arithmetical mean of the mean results obtained severally by the best observers. In accordance with the experience of STRUVE, OTTO STRUVE, DEMBOWSKI, and BURNHAM these yearly means may be held to furnish the most trustworthy basis for the elements of an orbit. The following is a table of the orbits hitherto published for this star:

P	T	e	a	Ω	i	λ	Authority	Source
94.765	1837.414	0.4496	1.255	15.04	35.53	135.46	Mädler, 1840	Dorp. Obs. IX, 180
146.83	1834.01	0.57536	0.9982	77.35	38.6	42.2	Mädler, 1847	Die Fixst.-Syst.
105.64	1836.60	0.4151	1.446	47.6	46.3	93.87	von Fuss, 1867	Mel. Acad. St. Petersburg.,
112.644	1835.196	0.50090	1.310	32.2	29.97	97.52	Schur, 1867	A.N. 1636 [1867, p. 128]
104.415	1834.88	0.4612	1.27	38.6	32.2	92.1	Doberck, 1877	A.N. 2156
102.943	1835.508	0.4472	1.270	39.15	32.2	92.1	Doberck, 1879	A.N. 2277

By the method of KLINKERFUES we find the following elements:

$$\begin{aligned}
 P &= 104.61 \text{ years} & \Omega &= 47^\circ.15 \\
 T &= 1836.26 & i &= 43^\circ.85 \\
 e &= 0.450 & \lambda &= 90^\circ.90 \\
 a &= 1''.3712 & n &= +3^\circ.441355
 \end{aligned}$$

* *Astronomische Nachrichten*, 3292.

Apparent orbit:

Length of major axis	= 2".526
Length of minor axis	= 1".984
Angle of major axis	= 45°.7
Angle of periastron	= 138°.4
Distance of star from centre	= 0".446

It will be seen that these elements are very similar to those derived by VON FUSS in 1867. The following comparison of the computed and observed places shows that the above elements are highly satisfactory, and that the true elements of this remarkable binary will hardly differ sensibly from the values here obtained.

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1782.65	319.4	315.7	-	1.44	+3.7	-	2	Herschel
1823.81	36.7	45.3	1.25 ±	1.16	-8.6	+0.09	1	Struve
1831.71	85.7	85.1	0.82	0.72	+0.6	+0.10	2	Struve
1833.73	108.6	105.3	0.56	0.61	+3.3	-0.05	3	Struve
1835.66	132.6	130.5	0.41	0.55	+2.1	-0.14	5	Struve
1836.61	146.4	143.8	0.47	0.55	+2.6	-0.08	5	Struve
1840.55	186.7	188.8	0.72	0.71	-2.1	+0.01	7-6	OΣ 4; Dawes 3 2
1841.72	193.5	197.6	0.92	0.79	-4.1	+0.13	9	Mädler 7; Dawes 2
1842.80	207.3	204.7	0.87	0.86	+2.6	+0.01	1	Mädler
1843.69	209.3	209.5	0.93	0.91	-0.2	+0.02	4	Mädler 3; Dawes 1
1844.49	213.7	213.6	0.85	0.96	+0.1	-0.11	5	Mädler
1846.42	220.3	222.2	0.97	1.07	-1.9	-0.10	2	O. Struve
1847.53	225.1	226.1	1.12	1.11	-1.0	+0.01	5	Mädler
1848.54	229.2	229.7	1.15	1.16	-0.5	-0.01	3	OΣ 2; Dawes 1
1849.19	232.5	231.9	1.09	1.18	+0.6	-0.09	3	O. Struve
1850.56	233.8	236.1	1.24	1.23	-2.3	+0.01	7-6	OΣ 3; Mädler 3; Dawes 1-0
1851.36	235.7	238.3	1.26	1.25	-2.6	+0.01	12	OΣ 2; Mädler 8; Mädler 2
1852.49	238.4	241.6	1.23	1.29	-3.2	-0.06	3	O. Struve
1854.47	245.9	246.7	1.38	1.33	-0.8	+0.05	13-7	OΣ 4; Dawes 3; Dembowski 6-0
1855.58	246.6	249.4	1.34	1.35	-2.8	-0.01	14	OΣ 3; Dembowski 8; Mo. 3
1856.69	249.1	251.5	1.31	1.37	-2.4	-0.06	7	Dembowski 4; OΣ 2; Mädler 1
1857.56	251.6	254.0	1.32	1.38	-2.4	-0.06	10	OΣ 3; Seabroke 3; Dembowski 4
1858.54	252.4	256.3	1.2	1.39	-3.9	-0.19	2	Dembowski
1859.16	255.3	257.3	1.46	1.40	-2.0	+0.06	3	O. Struve
1861.79	265.2	263.4	1.21	1.42	+1.8	-0.21	2	Mädler
1862.60	263.8	265.2	1.43	1.43	-1.4	0.00	15	OΣ 2; Dembowski 11; Mädler 2
1863.83	265.8	267.7	1.41	1.43	-1.9	-0.02	10	Dembowski 9; Dawes 1
1864.73	268.7	269.7	1.40	1.43	-1.0	-0.03	7	Dembowski
1865.70	270.5	271.8	1.39	1.44	-1.3	-0.05	9	Dembowski 6; Knott 3
1866.60	271.3	273.6	1.42	1.44	-2.3	-0.02	8	OΣ 2; Dembowski 5; Sea. 1
1867.74	275.2	276.1	1.41	1.44	-0.9	-0.03	7	Dembowski
1868.82	277.0	278.2	1.48	1.44	-1.2	+0.04	6	Dembowski 4; OΣ 2
1869.75	279.9	280.6	1.48	1.44	-0.7	+0.04	6	Dembowski
1870.43	280.8	281.5	1.47	1.44	-0.7	+0.03	10	OΣ 2; Gledhill 1; Dembowski 7
1871.58	283.9	283.8	1.49	1.45	+0.1	+0.04	8	Dembowski 7; Gledhill 1
1872.71	286.0	286.1	1.46	1.44	-0.1	+0.02	7	Dembowski 6; W. & S. 1
1873.76	287.8	288.3	1.48	1.44	-0.5	+0.04	12	Dembowski 9; W. & S. 1; Gl. 2
1874.80	290.7	290.4	1.37	1.44	+0.3	-0.07	9	Dembowski 6; W. & S. 1; Gl. 2
1875.68	292.5	292.2	1.48	1.44	+0.3	+0.04	11	Dembowski 6; Dunér 5



t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1876.84	294.5	294.6	1.51	1.44	-0.1	+0.07	13-11	Dembowski 5; Dk. 3-2; Pl. 5-4
1877.68	296.5	296.2	1.48	1.44	+0.3	+0.04	8	Dembowski 4; Doberek 4
1878.75	300.7	298.4	1.45	1.44	+2.3	+0.01	9	Dembowski 4; Doberek 5
1879.61	302.5	300.2	1.41	1.44	+2.3	-0.03	13	Hall 8; Doberek 5
1880.74	304.4	302.5	1.52	1.43	+1.9	+0.09	10	β 6; Doberek 4
1881.59	305.2	304.3	1.60	1.43	+0.9	+0.17	12-10	Jed. 3-2; β 3; Big. 2-1; Hall 4
1882.46	306.5	306.1	1.41	1.43	+0.4	-0.02	11-10	Jed. 7; Doberek 4-3
1883.77	311.3	307.7	1.56	1.43	+3.6	+0.13	12	Englemann 9; Hall 3
1884.47	311.7	310.2	1.26	1.43	+1.5	-0.17	2	Seabroke
1885.80	316.1	312.9	1.45	1.43	+3.2	+0.02	5	Hall
1886.56	314.9	314.4	1.44	1.43	+0.5	+0.01	8-7	Seabroke 3-2; Hall 5
1887.08	313.1	315.4	1.43	1.43	-2.3	0.00	9-6	Schiaparelli 6-3; Tarrant 3
1888.66	318.9	317.5	1.41	1.43	+1.4	-0.02	11	Sch. 1; Hall 4; Sch. 6
1889.79	321.5	320.9	1.43	1.44	+0.6	-0.01	8	β 3; Hall 4; Seabroke 1
1890.86	324.3	323.1	1.43	1.44	+1.2	-0.01	6	Hall 5; Schiaparelli 1
1891.71	324.9	323.8	1.48	1.44	+1.1	+0.04	3	See 1; Schiaparelli 3
1892.89	326.7	327.2	1.52	1.44	-0.5	+0.08	8	Com. 3; Col. 2; Jo. 1; Sch. 2
1893.90	329.3	329.2	1.51	1.44	+0.1	+0.07	4	Comstock 2; Schiaparelli 2
1894.46	331.3	330.3	1.70	1.45	+1.0	+0.35	4-2	Glasenapp 1-0; Bigourdan 3-2
1895.30	332.3	332.1	1.44	1.45	+0.2	-0.01	16-14	Big. 7-6; Ho. 0-3; Com. 2-1; See 4

EPIHEMERIS.

t	θ_c	ρ_c	t	θ_c	ρ_c
1896.50	334.8	1.45	1902.50	346.8	1.46
1897.50	336.8	1.45	1903.50	348.8	1.46
1898.50	338.8	1.45	1904.50	350.8	1.46
1899.50	340.8	1.45	1905.50	352.8	1.46
1900.50	342.8	1.46	1906.50	354.8	1.46
1901.50	344.8	1.46			

It will be seen that there are occasional systematic errors both in the angles and in the distances, and in some cases these deviations appear to be rather more extensive than we should expect in the work of the best observers; but the star has some peculiar difficulties, especially as regards the distance, and on the whole the measures are fairly accordant for so close an object.

This star deserves the careful attention of observers, as the next 20 years will give the material which will make the orbit exact to a very high degree. It may be pointed out that the system has a considerable proper motion in space, in $\alpha + 0''.346$, in $\delta + 0''.020$; and therefore the chances are that it has a sensible parallax. If the parallax could be determined it would give us the absolute dimensions of the system and the combined mass of the components — two elements of the highest interest in the study of the stellar systems.

η CASSIOPEAE = Σ 60.

$\alpha = 0^h 42^m.9$; $\delta = +57^\circ 18'$.
4, yellow ; 7, purple.

Discovered by Sir William Herschel, August 17, 1779.

OBSERVATIONS.

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1779.81	$70^\circ \pm$	11.09	1	Herschel	1850.19	106.8	7.96	15-14	Mädler
1780.52	—	11.46	1	Herschel	1850.61	105.5	8.32	2	Johnson
1782.45	62.1	—	1	Herschel	1850.72	106.5	8.01	6-7	Mädler
1803.11	70.8	—	1	Herschel	1850.84	105.6	8.16	5	Jacob
1814.10	78.5	9.70	1	Bessel	1851.45	106.6	8.17	7-6	Fletcher
1820.16	81.1	10.68	5	Struve	1851.76	107.7	7.72	3	Mädler
1827.21	85.6	10.2	1	Struve	1851.84	108.0	8.04	3	O. Struve
1830.75	86.2	10.07	5	Bessel	1851.89	106.9	8.12	4	Miller
1831.75	88.7	9.69	1	Herschel	1851.89	106.4	8.04	3	Jacob
1832.05	87.6	9.78	5	Struve	1852.61	108.5	7.65	7-8	Mädler
1832.87	88.7	9.74	2	Dawes	1853.39	108.4	7.57	5	Mädler
1834.76	89.6	9.80	1	Bessel	1853.51	109.2	7.98	7	Jacob
1835.26	91.2	9.52	3	Struve	1853.90	110.1	7.52	3	Mädler
1836.46	91.1	10.83	2-1	Mädler	1853.92	109.4	—	6	Powell
1836.74	92.1	9.39	4	Struve	1854.00	109.6	7.91	1	Dawes
1840.14	95.8	8.98	37-29 obs.	Kaiser	1854.56	112.0	7.97	4	O. Struve
1841.34	98.1	9.21	3	O. Struve	1854.80	110.6	7.60	2	Mädler
1841.57	98.3	9.50	4	Mädler	1854.91	111.9	7.80	7	Dembowski
1841.80	95.7	9.33	1	Dawes	1854.94	111.5	—	6	Powell
1842.41	98.3	8.76	2-1	Mädler	1854.95	110.0	8.12	2	Morton
1842.65	96.4	9.09	7	Schlüter	1855.24	110.9	7.95	3	Winnecke
1843.07	98.4	8.97	3	Schlüter	1855.52	111.0	7.60	4-3	Mädler
1844.56	100.1	8.48	6-5	Mädler	1855.79	110.2	7.89	2	Secchi
1845.44	101.1	8.44	8	Mädler	1855.93	112.5	7.63	9-1	Powell
1845.86	97.2	8.85	1	Jacob	1855.94	113.2	7.57	4	Dembowski
1846.41	100.5	8.89	2	Jacob	1855.96	112.4	7.80	3	Morton
1846.66	102.5	8.57	12	Mädler	1856.07	112.4	7.57	4	Jacob
1846.72	101.5	8.71	2	Jacob	1856.51	112.9	7.22	2-1	Mädler
1847.34	102.7	8.28	6-7	Mädler	1856.55	117.3	8.34	3	Luther
1847.40	101.8	8.48	5	O. Struve	1856.86	114.6	7.33	4	Dembowski
1848.12	102.5	8.60	2-1	Jacob	1857.06	112.9	7.49	3	Jacob
1849.66	105.0	8.26	4	O. Struve	1857.22	114.1	7.57	2	O. Struve
					1857.23	114.5	7.09	5	Mädler
					1857.87	115.8	7.14	4	Dembowski
					1858.06	115.1	7.42	3	Jacob
					1858.19	115.9	7.12	4	Mädler
					1858.62	115.8	7.24	3	Dembowski

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1859.27	115.7	6.96	2-1	Mädler	1872.01	—	6.0 $\pm$	1	Seabroke
1859.72	116.6	7.02	6-1	Powell	1872.18	140.8	5.94	2	O. Struve
1859.94	117.0	7.08	2	Morton	1872.50	140.5	6.02	7	Dunér
1860.68	119.8	7.17	2	O. Struve	1872.63	139.1	5.97	6	Dembowski
1860.97	118.3	6.99	7-6	Powell	1872.65	137.8	6.10	4	Knott
1861.58	119.8	7.37	5	Auwers	1872.77	144.0	5.94	1	Main
1861.70	117.9	7.08	5	Mädler	1872.86	124.4	6.32	—	Leyton Obs.
1861.82	118.2	6.44	6	Main	1873.06	142.3	—	1	W. & S.
1861.95	120.6	6.7	3-2	Powell	1873.53	144.6	5.68	3	O. Struve
1862.71	120.6	6.85	8	Mädler	1873.66	140.7	5.97	7	Dembowski
1862.86	121.3	7.00	12	Dembowski	1873.68	143.7	6.03	2	Gledhill
1862.88	120.4	7.15		Leyton Obs.	1873.83	144.7	6.33	1	W. & S.
1863.80	123.4	6.87	9	Dembowski	1873.86	141.2	5.66	1	Leyton Obs.
1864.00	123.1	6.65	4-3	Powell	1873.98	143.6	—	6	Nobile
1864.80	125.0	6.76	9	Dembowski	1874.22	144.9	5.82	1	Dunér
1865.59	125.5	6.52	6	Englemann	1874.63	143.1	5.83	7	Dembowski
1865.62	126.4	6.67	8	Dembowski	1874.90	146.0	5.8	1	W. & S.
1865.69	125.7	6.75	3	Knott	1875.15	148.6	5.58	2	O. Struve
1865.76	123.9	6.43	2-1	Leyton Obs.	1875.51	146.7	5.77	10	Dunér
1866.22	132.6	6.44	2	O. Struve	1875.66	146.5	5.67	7	Dembowski
1866.63	124.7	6.38	3	Leyton Obs.	1875.78	146.1	5.78	1	Main
1866.65	123.9	6.66	1	Searle	1875.94	147.7	—	2	Doberck
1866.72	128.5	6.58	7	Dembowski	1876.61	149.3	5.59	7	Dembowski
1866.84	126.0	—	1	Winlock	1876.79	149.1	5.18	6	Plummer
1866.86	127.7	6.79	4	Secchi	1876.86	149.3	4.72	1	Leyton Obs.
1867.15	130.1	6.55	1	Searle	1877.19	152.8	5.44	1	O. Struve
1867.65	130.0	6.31	1	Main	1877.69	151.5	5.48	6	Dembowski
1867.74	130.4	6.48	7	Dembowski	1877.76	150.4	5.77	5	Doberck
1868.37	131.8	6.38	5	Dunér	1878.19	154.6	5.25	2	O. Struve
1868.53	132.9	6.43	3	O. Struve	1878.58	153.7	5.42	5	Dembowski
1868.67	132.1	6.33	4	Dembowski	1878.83	153.9	5.51	1	Goldney
1868.90	124.3	6.21	1	Leyton Obs.	1878.90	155.1	5.28	5	Doberck
1869.67	132.4	6.12	1	Main	1879.20	154.7	5.16	2	O. Struve
1869.72	124.8	6.58	1	Leyton Obs.	1879.01	156.8	5.35	7	Hall
1869.75	134.0	6.20	6	Dembowski	1879.80	158.3	5.21	3	Doberck
1869.93	135.2	6.16	4	Dunér	1879.96	161.9	5.60	5	Franz
1870.07	133.4	6.39	5-4	Powell	1880.14	159.9	5.32	7	Jedrzejewicz
1870.18	136.2	6.28	2	O. Struve	1880.60	161.1	5.26	5	Doberck
1870.67	135.3	6.16	7	Dembowski	1881.10	164.1	5.32	2-1	Doberck
1870.72	135.8	6.09	3	Gledhill	1881.14	162.8	5.10	3-2	Jedrzejewicz
1871.10	137.4	5.90	2-1	Powell	1881.16	162.0	5.26	3	O. Struve
1871.65	137.6	6.08	6	Dembowski	1881.72	161.4	5.18	2	Pritchett
1871.70	138.0	6.03	2	Gledhill	1881.90	163.1	5.30	4	Hall
1871.93	140.9	—	1	W. & S.					

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1882.15	165.5	5.08	3	Jedrzejewicz	1890.79	188.4	5.07	5	Hall
1882.70	166.8	5.28	1	O. Struve	1891.48	191.7	5.02	5-4	See
1882.76	166.3	5.11	6-5	Doberek	1891.74	191.8	4.79	4-3	Maw
1882.87	165.7	5.15	6	Englemann	1892.77	191.1	4.92	3	Comstock
1883.94	168.8	5.12	3	Hall	1892.85	197.3	4.90	2	Collins
1885.23	172.8	5.27	1	Seabroke	1892.95	197.4	4.75	1	Jones
1885.81	173.4	5.06	5	Hall	1893.84	196.0	4.88	1	Comstock
1886.07	176.3	4.92	5	Englemann	1893.97	198.2	5.12	1	Lovett
1886.20	176.6	4.78	3-2	Seabroke	1894.05	201.6	4.89	1	Comstock
1886.95	175.3	4.99	5	Hall	1894.1	200.2	4.96	1	Maw
1886.97	178.6	4.71	7	Tarrant	1895.16	204.8	4.97	3	Hough
1887.35	180.6	4.6	1	Smith	1895.17	203.8	5.01	3	Comstock
1888.48	181.3	4.69	2	Seabroke	1895.29	203.4	4.84	3	See
1888.54	183.9	4.83	5	Maw	1895.73	204.3	4.78	2	See
1888.97	183.2	4.88	4	Hall	1895.73	205.9	4.74	2	Moulton
1889.10	185.9	4.64	3	Seabroke					
1889.86	185.4	4.98	4	Hall					

At the date of discovery SIR WILLIAM HERSCHEL found the distance* of the component to be $11''.09$, and estimated the angle at 70° . At the epoch 1780.52 he found the distance $11''.46$, but made no measure of the angle of position until 1782.45, when it proved to be $62^\circ.07$. HERSCHEL observed the angle to be $70^\circ.8$, in 1803, but made no measure of the distance. The earliest observation of both angle and distance is a rough measure by BESSEL, in 1814; and although his angle is nearly correct, it is evident from the subsequent work of STRUVE that the distance is much too small. Since the time of STRUVE η Cassiopeae has been followed by nearly all of the best observers; so that we have good material upon which to base an investigation of the orbit.

Although the observations of η Cassiopeae do not suffice to fix all the elements so well as might be desired, yet it appears that the range of uncertainty is comparatively unimportant, except in the case of the periodic time, which may possibly differ several years from the value here derived. Some of the orbits found for η Cassiopeae by previous computers are indicated in the following Table of Elements.

P	T	e	a	Ω	i	λ	Authority	Source
181 ^{yrs.}	1896.0	0.77083	10.335	25.55	57.98	243.65	Powell	M.N., vol. XXI, p. 66
176.37	1924.78	0.6268	10.68	50.80	68.5	245.9	Dunér	Mes. Micro., p. 166
222.435	1909.24	0.5763	9.83	39.95	53.83	223.33	Doberek	A.N. 2091
195.235	1901.25	0.6244	8.639	33.33	48.3	229.45	Grüber	A.N. 2111
167.4	1904.0	0.622	8.702	41.02	52.09	233.1	Coit	M.N., vol. XLII, p. 359
208.1	1908.9	0.500	8.45	47.1	47.6	214.2	Lewis	M.N., vol. LV, p. 20
190.50	1906.12	0.547	8.2047	43.0	46.08	222.02	See	A.J. 343

* *Astronomical Journal*, 343; and *Astronomical Journal*, 355.

We find the following elements for this celebrated binary :

$P = 195.76$ years	$\Omega = 46^\circ.1$
$T = 1907.84$	$i = 45^\circ.95$
$e = 0.5142$	$\lambda = 217^\circ.87$
$a = 8''.2128$	$n = +1^\circ.83899$

Apparent orbit :

Length of major axis	= $15''.80$
Length of minor axis	= $10''.24$
Angle of major axis	= $55^\circ.8$
Angle of periastron	= $254^\circ.5$
Distance of star from centre	= $3''.80$

The table of computed and observed places shows that these elements are highly satisfactory. But the rapid orbital motion near periastron will make it possible to effect a slight improvement in about ten years.

The parallax of the system recently determined by DR. HERMANN S. DAVIS of Columbia College seems to be entitled to great weight; and yet the value is so large that with these elements the mass is only 0.166 that of the sun. The distance of the system is 464540 times the distance of the earth from the sun, and the semi-major axis of the orbit is 18.54 astronomical units. This mass is very small for the size of the system, and if the parallax of $0''.43$ be confirmed, say, by Heliometer measures, our ideas of the nature of the stellar systems will have to be considerably modified. The parallax of $0''.154$ found by OTTO STRUVE in 1856, from measures with the micrometer, gives a distance for the system of 1339400 astronomical units. The semi major axis comes out 53.33 times the distance of the earth from the sun, and the combined mass proves to be 3.96.

The companion is at present near the line of nodes, and its relative motion in the line of sight is near its maximum value. The brightness and width of this pair is such as to justify an application of the spectroscopic method for determining parallax developed in § 5, Chapter I.

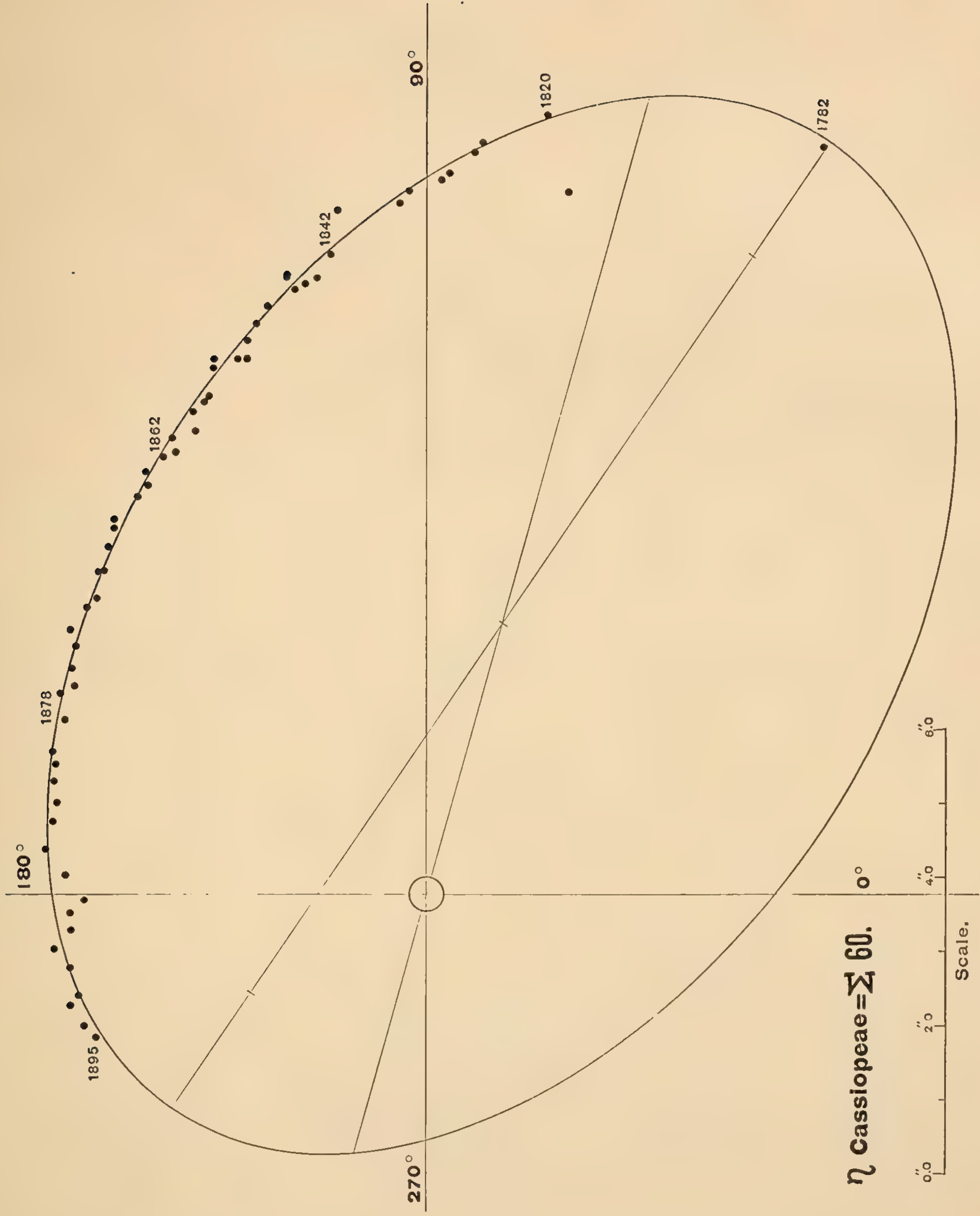
In this connection we may point out the great importance of the determination of the parallaxes of *double* rather than of *single* stars. The parallaxes of single stars are of comparatively little interest, since they give us only the distance and hence the velocity perpendicular to the line of vision, and the radiation compared to that of the sun. On the other hand, the parallaxes of double stars whose orbits are known give us, besides these data, the absolute dimensions of the orbits and the combined masses of the components — two elements of the highest importance in the study of the systems of the universe. η Cassiopeae is remarkable for the great angular distance of the components,

and for the rapid proper motion of the system. Both of these circumstances support the belief that the star is comparatively near to us in space, and render it certain that the parallax is sensible.

In 1881 MR. LUDWIG STRUVE discussed the relative motion of the components about the common center of gravity of the system; and from his investigation it follows that $\frac{M_2}{M_1} = 0.268$, or the masses of the two stars, according to OTTO STRUVE'S parallax, are respectively 2.90 and 1.06 times the combined mass of the sun and earth. The companion is therefore more massive than the sun and moves in an ellipse nearly twice the size of the orbit of *Neptune*; but the eccentricity is so large that in periastron the companion would come considerably within the orbit of the outer planet, while at apastron it would recede to more than three times that distance.

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1779.81	70 $\pm$	57.2	11.09	11.33	+12.8 $\pm$	-0.24	1	Herschel
1780.52	—	57.6	11.46	11.36	—	+0.10	1	Herschel
1782.45	62.1	58.7	—	11.42	+ 3.4	—	1	Herschel
1803.11	70.8	70.3	—	11.41	+ 0.5	—	1	Herschel
1814.10	78.5	76.7	9.70	11.00	+ 1.8	-1.30	1	Bessel
1820.16	81.1	80.5	10.68	10.67	+ 0.6	+0.01	5	Struve
1827.21	85.6	85.4	10.2	10.21	+ 0.2	-0.01	1	Struve
1830.75	86.2	87.9	10.07	9.94	- 1.7	+0.13	5	Bessel
1831.75	88.7	88.6	9.69	9.87	+ 0.1	-0.18	1	Herschel
1832.46	88.1	89.1	9.76	9.82	- 1.0	-0.06	7	Σ . 5; Dawes 2
1835.26	91.2	91.4	9.52	9.58	- 0.2	-0.06	3	Struve
1836.74	92.1	92.6	9.39	9.44	- 0.5	-0.05	4	Struve
1841.57	97.4	96.9	9.35	9.02	+ 0.5	+0.33	8	$O\Sigma$. 3; Mädler 4; Dawes 1
1842.41	98.3	97.8	8.76	8.91	+ 0.5	-0.15	2-1	Mädler
1844.56	100.1	99.7	8.48	8.73	+ 0.4	-0.25	6-5	Mädler
1845.65	99.2	100.7	8.64	8.62	- 1.5	+0.02	9	Mädler 8; Jacob 1
1846.60	101.5	101.7	8.72	8.51	- 0.2	+0.21	16	Mädler 12; Jacob 4
1847.37	102.3	102.5	8.38	8.44	- 0.2	-0.06	11-12	Mädler 6-7; $O\Sigma$. 5
1848.12	102.5	103.4	8.60	8.37	- 0.9	+0.23	2-1	Jacob
1849.66	105.0	105.0	8.26	8.25	$\pm$ 0.0	+0.01	4	O. Struve
1850.87	106.4	106.4	8.04	8.12	$\pm$ 0.0	-0.08	26	Mädler 21; Jacob 5
1851.80	107.8	107.5	7.88	8.00	+ 0.3	-0.12	6	Mädler 3; $O\Sigma$. 3
1852.61	108.5	108.5	7.65	7.91	$\pm$ 0.0	-0.25	7-8	Mädler
1853.68	109.3	109.8	7.69	7.81	- 0.5	-0.12	21-15	Mä. 8; Ja. 7; Po. 6-0
1854.76	111.5	111.2	7.79	7.69	+ 0.3	+0.10	13	$O\Sigma$. 4; Mä. 2; Dem. 7 [Mo. 3
1855.81	111.9	112.5	7.70	7.59	- 0.6	+0.11	22-16	Mä. 4-3; Sec. 2; Po. 9-4; Dem. 4;
1856.45	113.4	113.8	7.37	7.48	- 0.4	-0.11	10-9	Ja. 4; Mä. 2-1; Dem. 4
1857.34	114.1	114.8	7.32	7.40	- 0.7	-0.08	14	Ja. 3; $O\Sigma$. 2; Mä. 5; Dem. 4
1858.29	115.6	116.4	7.26	7.30	- 0.8	-0.04	10	Ja. 3; Mä. 4; Dem. 3
1859.60	116.4	118.3	7.02	7.14	- 1.9	-0.12	10-7	Mä. 2-1; Po. 6-4; Mo. 2
1860.68	119.8	119.4	7.17	7.09	+ 0.4	+0.08	2	O. Struve
1861.82	119.2	121.4	6.89	6.95	- 2.2	-0.06	8-7	Mädler 5; Powell 3-2
1862.78	120.9	122.9	6.92	6.87	- 2.0	+0.05	20	Mädler 8; Dembowski 12
1863.80	123.4	124.7	6.87	6.75	- 1.3	+0.12	9	Dembowski



t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1864.40	124.1	125.8	6.70	6.68	— 1.7	+0.02	13-12	Powell 4-3; Dembowski 9
1865.63	125.9	127.8	6.65	6.57	— 1.9	+0.08	17	En. 6; Dem. 8; Kn. 3
1866.60	129.6	229.4	6.61	6.49	+ 0.2	+0.12	13	OY. 2; Dem. 7; Sec. 4
1867.44	130.3	131.2	6.51	6.39	— 0.9	+0.12	8	Searle 1; Dembowski 7
1868.52	132.3	132.9	6.37	6.31	— 0.6	+0.06	12	Du. 5; OY. 3; Dem. 4
1869.84	134.6	135.6	6.18	6.17	— 1.0	+0.01	10	Dembowski 6; Dunér 4
1870.41	135.2	136.6	6.23	6.13	— 1.1	+0.10	17-16	Po. 5-4; OY. 2; Dem. 7; Gl. 3
1871.48	137.3	138.6	6.00	6.05	— 1.3	—0.05	10-9	Po. 2-1; Dem. 6; Gl. 2
1872.49	139.5	140.7	6.01	5.96	— 1.2	+0.05	19	OY. 2; Du. 7; Dem. 6; Kn. 4
1873.62	143.3	143.3	6.00	5.86	$\pm$ 0.0	+0.14	19-13	W.&S. 2-1; OY. 3; Dem. 7; Gl. 2;
1874.58	144.7	145.5	5.82	5.79	— 0.8	+0.03	9	Du. 1; Dem. 7; W.&S. 1 [No. 6-0
1875.54	147.4	147.6	5.67	5.72	— 0.2	—0.05	21-19	OY. 2; Du. 10; Dem. 7; Dk. 2-0
1876.70	149.2	150.2	5.53	5.64	— 1.0	—0.11	13	Dembowski 7; Plummer 6
1877.73	151.0	152.6	5.62	5.57	— 1.6	+0.05	11	Dembowski 6; Doberck 5
1878.77	154.2	155.1	5.40	5.51	— 0.9	—0.11	11	Dem. 5; Gold. 1; Dk. 5
1879.59	159.0	157.4	5.39	5.44	+ 1.6	—0.05	15	Hall 7; Doberck 3; Franz 5
1880.37	160.5	159.2	5.29	5.41	+ 1.3	—0.12	12	Jedrzejewicz 7; Doberck 5
1881.46	162.8	162.1	5.22	5.37	+ 0.7	—0.15	11-9	Dk. 1; Jed. 3-2; Pr. 2; Hl. 4
1882.59	165.8	165.3	5.11	5.30	+ 0.5	—0.19	15-14	Jed. 3; Dk. 6-5; En. 6
1883.94	168.8	168.9	5.12	5.24	— 0.1	—0.12	3	Hall
1885.52	173.1	172.8	5.16	5.17	+ 0.3	—0.01	6	Seabroke 1; Hall 5
1886.55	176.7	174.9	4.85	5.12	+ 1.8	—0.27	20-19	En. 5; Sea. 3-2; Hl. 5; Tar. 7
1887.35	180.6	178.4	4.6	5.08	+ 2.2	—0.28	1	Smith
1888.66	182.8	182.1	4.80	5.03	+ 0.7	—0.23	11	Seabroke 2; Maw 5; Hall 4
1889.48	185.6	184.6	4.81	5.00	+ 1.0	—0.19	7	Seabroke 3; Hall 4
1890.79	188.4	188.5	5.07	4.95	— 0.1	+0.12	5	Hall
1891.61	191.7	191.2	4.90	4.92	+ 0.5	—0.02	9-7	See 5-4; Maw 4-3
1892.86	196.3	195.0	4.82	4.87	+ 1.3	—0.05	6	Com. 3; Col. 2; Jo. 1
1893.90	197.1	198.5	5.00	4.84	— 1.4	+0.16	2	Comstock 1; Lovett 1
1894.07	200.9	199.0	4.92	4.83	+ 1.9	+0.09	2	Comstock 1; Maw 1
1895.29	203.4	202.9	4.84	4.79	+ 0.5	+0.05	3	See

EPIHEMERIS.

t	θ_c	ρ_c	t	θ_c	ρ_c
1896.50	207.6	4.73	1899.50	217.2	4.55
1897.50	210.1	4.68	1900.50	221.1	4.46
1898.50	213.7	4.62			

γ ANDROMEDAE BC = OY 38.

$\alpha = 1^h 57^m.8$; $\delta = +41^\circ 51'$.
5.5, bluish ; 7, bluish.

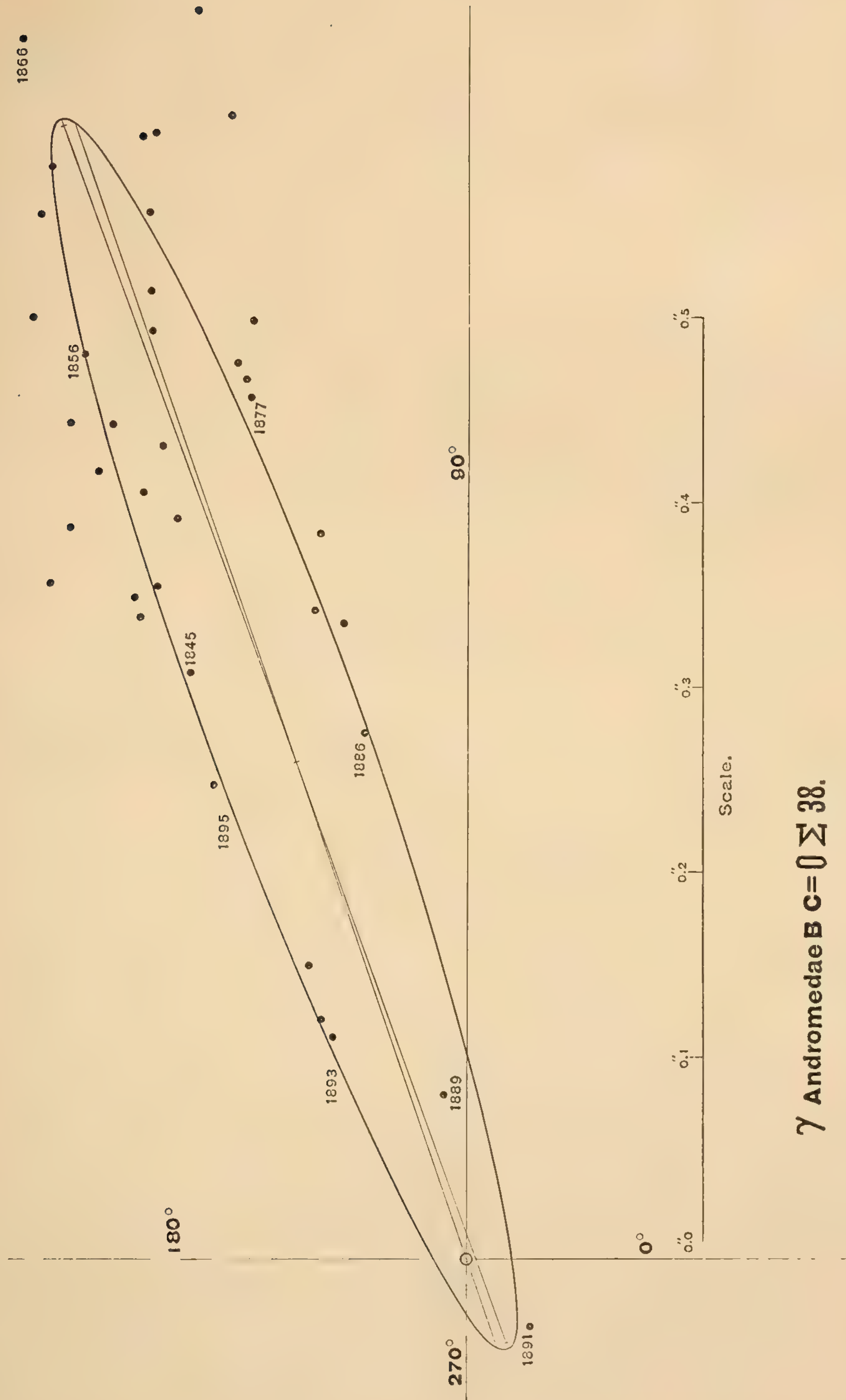
Discovered by Otto Struve in 1842.

OBSERVATIONS.

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1843.00	119.7	0.45 $\pm$	2	Dawes	1846.64	111.3	0.43	7-3	Mitchel
1843.19	119.8	0.35	2-1	Mädler	1847.13	117.9	0.52	5	O. Struve
1843.55	125.5	0.48	3	O. Struve	1847.82	111.3	0.6 $\pm$	4	Dawes
1845.15	116.9	0.39	4	Mädler	1849.69	114.9	0.47	4	O. Struve

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1851.19	116.6	0.40	4	Mädler	1869.84	107.0	0.63	3	O Struve
1852.21	114.5	0.48	2	Mädler	1869.95	105.6	0.5 $\pm$	13	Dembowski
1852.78	111.3	0.5 $\pm$	2	Jacob	1871.01	110.6	0.63	15	Dunér
1853.23	116.0	0.47	3	Mädler	1872.83	101.5	0.63	4-2	Brünnnow
1853.79	108.5	0.55 $\pm$	4	Dawes	1872.92	91.8	0.5 $\pm$	2-1	W. & S.
1853.94	106.8	0.4 $\pm$	4	Jacob	1873.17	105.4	0.63	5	O. Struve
1854.75	112.0	0.61	1	Dawes	1874.00	109.3	0.53	1	Newcomb
1855.02	119.4	—	1	Mädler	1874.53	96.3	0.51	2	Gledhill
1855.09	109.8	0.40	1	Secchi	1876.79	105.7	—	1	W. & S.
1856.12	116.7	0.5 $\pm$	1	Jacob	1877.05	104.1	0.48	6	Schiaparelli
1856.20	116.5	0.45	1	Mädler	1877.71	103.9	—	1	Doberck
1856.21	121.7	0.41	2	Winnecke	1877.94	102.4	0.84	1	Seabroke
1856.84	113.0	0.67	3	O. Struve	1878.21	101.0	0.36	8	Hall
1856.90	109.7	0.47	3	Secchi	1878.65	102.1	0.43	2	Burnham
1857.23	115.4	0.45	3-1	Mädler	1880.06	107.9	0.36	1	Burnham
1858.06	114.0	—	2	Jacob	1880.11	106.7	—	2	Seabroke
1858.22	115.4	—	2	Mädler	1880.12	94.1	—	8	Jedrzejewicz
1858.99	108.9	0.45	3	Secchi	1882.05	104.0	0.49	6-1	Bigourdan
1859.81	108.7	0.53	1	Dawes	1883.15	93.1	0.29	7	Englemann
1862.55	115.2	0.50	4-2	Mädler	1883.16	106.7	—	1	Seabroke
1863.27	108.5	0.45 $\pm$	8	Dembowski	1883.87	103.1	0.40	2	Perrotin
1863.86	107.7	0.59	1	Dawes	1884.18	113.3	—	3	Seabroke
1863.99	107.6	0.61	—	Romberg	1884.65	117.6	0.35	1	Perrotin
1865.67	107.1	0.59	4	Knott	1886.83	101.0	0.29	1	Newcomb
1865.68	106.9	0.60	1	Dawes	1889.51	98.2	0.09	1	Burnham
1865.76	106.3	0.58	2-1	Leyton Obs.	1891.72	312.6	0.05 $\pm$	3	Burnham
1866.21	110.0	0.70	3	O. Struve	1893.79	121.8	0.14	3	Barnard
1866.74	132.3	—	1	Winlock	1894.56	121.6	0.15	3	Barnard
1866.74	107.2	—	1	Searle	1895.63	118.5	0.18	3	Barnard
1866.74	100.4	—	1	Winlock	1895.72	121.2	0.29	3	See
1866.85	104.2	0.64	1	Leyton Obs.	1895.72	115.3	elongated	1	Moulton
1867.79	104.3	0.5 $\pm$	1	Newcomb					
1868.82	102.0	0.69	6-5	Brünnnow					

Since OTTO STRUVE'S discovery of this extraordinary binary in 1842 the companion has described nearly an entire revolution, but as the orbit is very eccentric and highly inclined nearly all the observations lie in the narrow region included between position-angle 120° and 100° . Only in recent years has it been possible for observers to prove the reality of orbital motion; some ten years ago the object was found to be getting more and more difficult, and



γ Andromedae B C = 0 Σ 38.

hence it became clear that the distance was diminishing. In 1886 NEWCOMB found the distance $0''.29$ and the angle 101° ; in small telescopes the star appeared single. When BURNHAM examined the object in 1889 he found it exceedingly difficult even with the 36-inch refractor of the Lick Observatory, and during 1890 the companion was wholly invisible. When the star was examined in 1891 it was found that the companion had changed to the opposite quadrant, the angle being 312.6 and the distance so excessively small that it was estimated at $0''.05 \pm$. BARNARD's examination of the object in 1893 gave the key to the situation. The companion had swept rapidly round to 121.8 , thus passing over about 320° of position angle since the measure in 1889. BURNHAM at once undertook an investigation of the orbit, and obtained a very satisfactory set of elements. His paper, in the *Monthly Notices* for December, 1893, contains an illustration of the apparent orbit, and a complete list of measures down to 1893. We have added the measures made since that date, and derived a set of elements very similar to that found by BURNHAM. His elements are:

$$\begin{array}{ll} P = 54.8 \text{ years} & \Omega = 113^\circ.5 \\ T = 1892.1 & i = 78^\circ.9 \\ e = 0.875 & \lambda = 200^\circ.8 \\ a = 0''.37 \end{array}$$

We find the following elements of γ *Andromedae*:

$$\begin{array}{ll} P = 54.0 \text{ years} & \Omega = 113^\circ.4 \\ T = 1892.1 & i = 77^\circ.85 \\ e = 0.857 & \lambda = 200^\circ.1 \\ a = 0''.3705 & n = -6^\circ.6667 \end{array}$$

Apparent orbit:

$$\begin{array}{ll} \text{Length of major axis} & = 0''.706 \\ \text{Length of minor axis} & = 0''.084 \\ \text{Angle of major axis} & = 109^\circ.9 \\ \text{Angle of periastron} & = 289^\circ.0 \\ \text{Distance of star from centre} & = 0''.298 \end{array}$$

The table of computed and observed places shows a good agreement for an object of this difficulty. The residuals are easily within the limits of the errors of observation. The orbit is remarkable for its great eccentricity and high inclination. Both of these elements are well defined, and the values given above will never be materially altered. Thus the error in the eccentricity can hardly surpass ± 0.02 , while a variation of one year in the period is to be regarded as improbable. In regard to the shape of the real orbit, γ *Andromedae* takes its place between γ *Virginis* and γ *Centauri*. These three remarkable systems are also similar as regards the relative brightness of their components,

which in each case are nearly equal. Since the companion of γ *Andromedae* is now within the reach of ordinary telescopes the accompanying ephemeris will be useful to astronomers.

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1843.25	121.6	116.6	0.43	0.34	+ 5.0	+0.09	7-6	Dawes 2; Mädler 2-1; O Σ . 3
1845.15	116.9	115.1	0.39	0.41	+ 1.8	-0.02	4	Mädler
1846.64	111.3	114.3	0.43	0.45	- 1.0	-0.02	7-3	Mitchell
1847.47	114.6	113.9	0.56	0.48	+ 0.7	+0.08	9	O Σ . 5; Dawes 4
1849.69	114.9	113.0	0.47	0.53	+ 1.9	-0.06	4	O Σ .
1851.19	116.6	112.5	0.40	0.56	+ 4.1	-0.16	4	Mädler
1852.49	112.9	112.1	0.49	0.58	+ 0.8	-0.09	4	Mädler 2; Jacob 2
1853.65	110.4	111.8	0.47 $\pm$	0.59	- 1.4	-0.12	11	Mädler 3; Dawes 4; Jacob 4
1854.75	112.0	111.5	0.61	0.60	+ 0.5	+0.01	1	Dawes
1855.05	114.6	111.4	0.4 $\pm$	0.61	+ 3.2	-0.21	2-1	Mädler 1-0; Secchi 1
1856.18	118.3	111.1	0.45	0.62	+ 7.2	-0.17	4	Jacob 1; Mädler 1; Winn. 2
1856.99	112.7	110.9	0.53	0.63	+ 1.8	-0.10	9-7	O Σ . 3; Secchi 3; Mädler 3-1
1858.42	112.8	110.6	0.45	0.64	+ 1.2	-0.19	7-3	Jacob 2-0; Mädler 2-0; Secchi 3
1859.81	108.7	110.2	0.53	0.65	- 1.5	-0.12	1	Dawes
1862.55	115.2	109.6	0.50	0.66	+ 5.6	-0.16	4-2	Mädler
1863.71	107.9	109.3	0.55	0.65	- 1.4	-0.10	9	Dem. 8; Dawes 1; Romberg
1865.70	106.8	108.9	0.59	0.64	- 2.1	-0.05	7-6	Knott 4; Dawes 1; Leyton 2-1
1866.21	110.0	108.7	0.70	0.64	+ 1.3	+0.06	3	O Σ .
1867.79	104.3	108.3	0.5 $\pm$	0.63	- 4.0	-0.13	1	Newcomb
1868.82	102.0	108.1	0.69	0.62	- 6.1	+0.07	6-5	Brünnow
1869.90	106.0	107.8	0.57	0.61	- 1.8	-0.04	16	O Σ . 3; Dembowski 13
1871.01	110.6	107.5	0.63	0.60	+ 3.1	+0.03	15	Dunér
1872.83	101.5	107.0	0.63	0.58	- 5.5	+0.05	4-2	Brünnow
1873.17	105.4	106.9	0.63	0.57	- 1.5	+0.06	5	O Σ .
1874.26	102.8	106.5	0.52	0.55	- 3.7	-0.03	3	Newcomb 1; Gledhill 2
1876.79	105.7	105.6	-	0.51	+ 0.1	-	1-0	Wilson and Seabroke
1877.05	104.1	105.5	0.48 $\pm$	0.50	- 1.4	-0.02	6	Schiaparelli
1878.43	101.6	104.9	0.40	0.47	- 3.3	-0.07	10	Hall 8; β 2
1880.10	102.9	104.1	0.36	0.43	- 1.2	-0.07	11-1	β 1; Seabroke 2-0; Jed. 8-0
1882.05	104.0	102.9	0.49	0.38	+ 1.1	+0.11	6-1	Bigourdan
1883.39	100.9	101.9	0.35	0.34	- 1.0	+0.01	10-9	Englemann 7; Sea. 1-0; Per. 2
1884.41	115.4	100.9	0.35	0.30	+14.5	+0.05	4	Seabroke 3; Perrotin 1
1886.83	101.0	96.8	0.29	0.19	+ 4.2	+0.10	1	Newcomb
1889.51	98.2	79.7	0.09	0.07	+18.5	+0.02	1	Burnham
1891.72	312.6	300.5	0.05 $\pm$	0.05	+12.1	$\pm$ 0.00	3	Burnham
1893.79	121.8	125.6	0.14	0.11	- 3.8	+0.03	3	Barnard 3
1894.56	121.6	121.4	0.15	0.16	+ 0.2	-0.01	3	Barnard
1895.63	118.5	118.8	0.18	0.23	- 0.3	-0.05	5	Barnard
1895.72	118.2	118.6	0.29	0.24	- 0.4	+0.05	4-3	See 3; Moulton 1-0

EPHEMERIS.

t	θ_c	ρ_c	t	θ_c	ρ_c
1896.70	117.2	0.30	1899.70	114.70	0.42
1897.70	116.2	0.35	1900.70	114.4	0.44
1898.70	115.5	0.39			

α CANIS MAJORIS = SIRIUS = A.G.C. 1.

$\alpha = 6^h 40^m.4$; $\delta = -16^\circ 34'$.
1, white ; 10, yellow.

Discovered by Alvan G. Clark, January 31, 1862.

OBSERVATIONS.

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1862.08	$85^\circ \pm$	$10 \pm$	1	Alvan Clark	1868.02	73.2	10.25	2	Searle
1862.19	84.6	10.07	3	Bond	1868.04	72.1	—	1	Peirce
1862.20	85.0	10.09	5	Rutherfurd	1868.23	70.3	11.25	7	Vogel
1862.23	84.5	10.42	2	Chacornac	1868.24	69.6	11.35	5	Bruhns
1862.28	83.8	(4.92)	1	Lassell	1868.26	71.7	10.95	5	Englemann
1863.15	88.4	7.63	1	Secchi	1869.10	74.7	10.26	7-4	Brünnnow
1863.21	82.5	10.15	2	O. Struve	1869.15	73.6	11.23	3	Vogel
1863.21	81.3	9.54	6	Rutherfurd	1869.20	68.7	11.17	1	Dunér
1863.23	84.9	10.00	1	Dawes	1869.20	68.6	11.07	2	Winlock
1863.27	82.8	—	1	Bond	1869.23	69.4	10.93	1	Peirce
1864.14	79.4	10.60	3	Marth	1870.13	68.1	11.16	12-4	Peirce
1864.18	80.1	9.60	1-3	Lassell	1870.17	65.9	11.06	7-5	Winlock
1864.22	78.6	10.70	4-2	Bond	1870.24	65.1	12.06	5	Vogel
1864.22	74.8	10.92	6-3	O. Struve	1871.16	65.9	10.75	3	Secchi
1864.23	84.9	—	1	Dawes	1871.20	70.3	11.19	2-1	Peirce
1864.24	79.7	10.08	1	Winnecke	1871.23	64.1	11.11	2	Dunér
1865.10	76.8	—	3	Lass.&Mar.	1871.25	60.1	12.10	4-3	Pechtile
1865.21	77.6	10.59	2	O. Struve	1872.18	59.8	11.05	2	Dunér
1865.22	75.5	9.59	8	Secchi	1872.21	66.6	10.69	3	Börgen
1865.23	77.8	10.77	5-4	Foerster	1872.24	62.4	11.50	1	Newcomb
1865.25	76.9	—	3	Tietjen	1872.24	64.3	11.46	6	Hall
1865.26	76.0	—	—	Bond	1872.26	61.3	—	3	Skinner
1865.26	76.9	(9.0)	1	Englemann	1873.20	65.8	11.12	1	Hall
1866.07	77.2	10.43	2-1	Knott	1873.22	60.8	10.57	1-4	Dunér
1866.21	—	10.74	1	Bruhns	1873.23	70.0	9.80	1	Börgen
1866.21	75.2	10.93	3	O. Struve	1873.23	66.3	10.42	1	Bruhns
1866.22	73.9	10.97	2-1	Tietjen	1873.93	65.0	11.29	1	W. & S.
1866.23	74.1	11.29	3-1	Foerster	1874.16	59.0	11.46	7	Newcomb
1866.23	74.0	10.21	2-3	Hall	1874.19	58.7	10.99	2-1	Holden
1866.23	74.9	10.57	3	Newcomb	1874.23	58.0	11.10	2	Hall
1866.25	78.3	10.34	1	Tuttle	1874.83	57.5	—	1	Burton
1866.26	74.7	10.09	3	Eastmann	1875.19	57.1	10.73	4	Dunér
1866.29	71.3	10.11	3	Secchi	1875.21	56.6	11.41	2	Newcomb
1867.02	74.2	11.15	7-6	Winlock	1875.21	55.9	11.89	5-4	Holden
1867.10	73.8	10.66	6-5	Searle	1875.28	56.4	11.08	4	Hall
1867.22	72.1	10.98	1	O. Struve					
1867.24	72.3	—	2	Foerster					
1867.27	74.9	9.92	2-1	Eastmann					

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1876.03	57.8	11.12	1	Watson	1881.99	43.6	9.38	11	Burnham
1876.05	54.6	11.45	1	Peters	1882.13	43.1	9.30	9	Hough
1876.09	54.9	11.82	6	Holden	1882.13	42.4	9.76	4-3	Bigourdan
1876.14	55.0	11.55	4	Russell	1882.18	42.2	9.95	6	Frisby
1876.22	55.2	11.19	6	Hall	1882.23	42.5	9.67	7	Hall
1877.11	52.8	11.19	4-3	Cincinnati	1882.54	44.0	—	6	Englemann
1877.16	52.8	11.35	4	Holden	1883.10	40.1	9.05	10	Burnham
1877.26	53.4	10.95	5	Hall	1883.10	39.0	9.41	1	Young
1877.97	52.4	10.83	8	Burnham	1883.12	39.7	9.02	11	Hough
1878.07	50.5	11.07	4	Holden	1883.14	41.3	—	4	Wilson
1878.15	51.0	10.71	9	Cincinnati	1883.17	41.4	9.75	7	Frisby
1878.19	54.4	11.24	5	Pritchett	1883.19	39.9	9.10	2-1	Bigourdan
1878.22	53.2	11.4	—	Eastmann	1883.21	39.1	9.26	6	Hall
1878.24	51.7	10.76	5	Hall	1884.05	36.0	9.67	6	Perrotin
1878.70	50.0	10.61	20-14	Cincinnati	1884.17	35.3	8.79	3-1	Bigourdan
1879.05	50.7	10.44	10	Burnham	1884.18	36.7	8.51	11	Hough
1879.12	47.8	11.35	5	Holden	1884.19	36.4	8.39	10	Burnham
1879.15	50.3	10.78	5	Pritchett	1884.23	37.7	8.81	8	Hall
1879.20	50.1	10.55	6	Hall	1884.27	36.3	8.70	5	Young
1879.75	46.5	10.29	1	Cincinnati	1885.11	34.1	8.09	8	Young
1880.00	48.8	10.55	1	Russell	1885.20	32.7	7.96	10	Hough
1880.10	47.1	10.48	4	Holden	1885.27	34.7	8.06	8	Hall
1880.11	48.3	10.00	11	Burnham	1886.05	29.8	7.59	4	Young
1880.17	49.6	9.87	3	Hough	1886.14	28.7	7.21	12	Hough
1880.18	46.7	9.92	6-4	Bigourdan	1886.22	30.6	7.39	6	Hall
1880.22	51.1	—	1	Smith	1887.14	25.4	7.08	4	Young
1880.25	47.8	10.30	8	Hall	1887.19	23.7	6.78	7	Hough
1880.28	48.6	10.38	2	Frisby	1887.23	24.2	6.51	4	Hall
1881.07	46.3	9.77	8	Burnham	1888.24	23.3	5.78	5	Hall
1881.12	43.3	10.83	2	Holden	1889.97	13.9	5.27	5	Burnham
1881.14	44.3	10.62	5-3	Bigourdan	1890.27	359.7	4.19	3	Burnham
1881.17	46.9	10.11	6	Frisby					
1881.18	46.5	9.81	7	Young					
1881.26	45.3	9.60	5	Hough					
1881.26	45.3	10.00	6	Hall					

The discovery of the companion of *Sirius* is one of the justly celebrated events of modern Astronomy. It extended to the regions of the fixed stars the principle of theoretical prediction which has proved so admirable in the solar system, and which in the hands of LEVERRIER and ADAMS had led to the discovery of *Neptune*. BESSEL had occasion to make a careful examination of the proper motions of a considerable number of stars, including *Sirius* and *Procyon*. The two dog stars, instead of moving uniformly on the arcs of

great circles, seemed to trace out irregular sinuous paths across the sky, and a further study of these anomalies convinced BESSEL that the two stars were perturbed by invisible bodies. In 1844 he wrote, in a letter to HUMBOLDT: "I adhere to the conviction that *Procyon* and *Sirius* form real binary systems, consisting of a visible and an invisible star. There is no reason to suppose luminosity an essential quality of cosmical bodies. The visibility of countless stars is no argument against the invisibility of countless others."

In 1857 the suggestion of BESSEL was taken up by PETERS, who made an investigation of the observed inequalities, and found the following elements for the orbit described by *Sirius* about the common centre of gravity of the system:

Periastron passage	= 1791.431
Mean yearly motion	= $7^{\circ}.1865$
Period	= 50.01 years
Eccentricity	= 0.7994

In 1861 the question was again examined by SAFFORD, who transmitted to BRÜNNOW an investigation which assigned to the companion a position-angle of $83^{\circ}.8$ for the epoch 1862.1. A short time afterwards, on Jan. 31, 1862, MR. ALVAN G. CLARK was trying the new 18-inch object glass of the Dearborn telescope, and on pointing the instrument on *Sirius* exclaimed: "Why, father it has a companion!" And sure enough the faint but massive disturbing body announced by BESSEL was seen within a few degrees of the place assigned by the theoretical astronomers. It now became a matter of great interest to ascertain from the motion of the new companion whether it was really the disturbing body; a few years showed that it had sensibly the required motion, and left no doubt of the identity of the two objects. In 1864 AUWERS undertook a new determination of the elements based on all the observations, and found:

Periastron passage	= 1793.890
Mean annual motion	= $7^{\circ}.28475$
Period	= 49.418 years
Eccentricity	= 0.6010

A definitive determination afterwards published gave the following results:

P = 49.399 years	Ω = $61^{\circ}.96$
T = 1843.275	i = $47^{\circ}.14$
e = 0.6148	λ = $18^{\circ}.91$
a = $2''.331$	

When the micrometrical measures began to accumulate, various computers made new investigations of the orbit. The following table of elements is very

complete. The last set credited to Dr. AUWERS were based on all the observations up to 1892.

P	T	e	a	Ω	i	λ	Authority	Source
^{yrs.} 49.6	1891.8	0.58	8.41	42.4	57.1	—	Colbert, 1885	Dearborn Report
58.47	1896.47	0.4055	8.58	50.0	55.4	216.3	Gore, 1889	M.N., XLIX, no. 8
51.22	1890.55	0.945	—	188	—	—	Mann	
49.46	1893.18	0.7512	8.31	10.2	53.	—	Mann	
57.02	1894.17	0.538	8.50	40.75	51.43	48.58	Howard	A.J. 235
49.399	1844.216	0.6292	7.568	37.51	42.43	39.94	Auwers, 1892	A.N. 3084
51.97	1893.5	0.568	8.31	40.3	50.8	135.4	Burnham, 1893	Pub. Lick Obs. II, p. 239
51.101	1893.759	0.6131	7.77	37.06	44.6	223.61	Zwiers, 1895	A.N. 3336

During 1890 the distance of the companion became so small that it was lost in the rays of the large star, even when viewed with the 36-inch refractor of the Lick Observatory. As it was evident that no further observations could be made until the object emerged on the other side, BURNHAM collected all the measures with great care and embodied them in his important paper in the *Monthly Notices* for April, 1891.

The orbit which we have given in this work is very similar to that found by BURNHAM, except that the eccentricity is higher and more nearly in accord with the value of this element found by AUWERS. The orbit is based wholly on the micrometrical measures, and the data used in deriving the mean places have been very carefully selected.

We find the following elements of the orbit of *Sirius*:

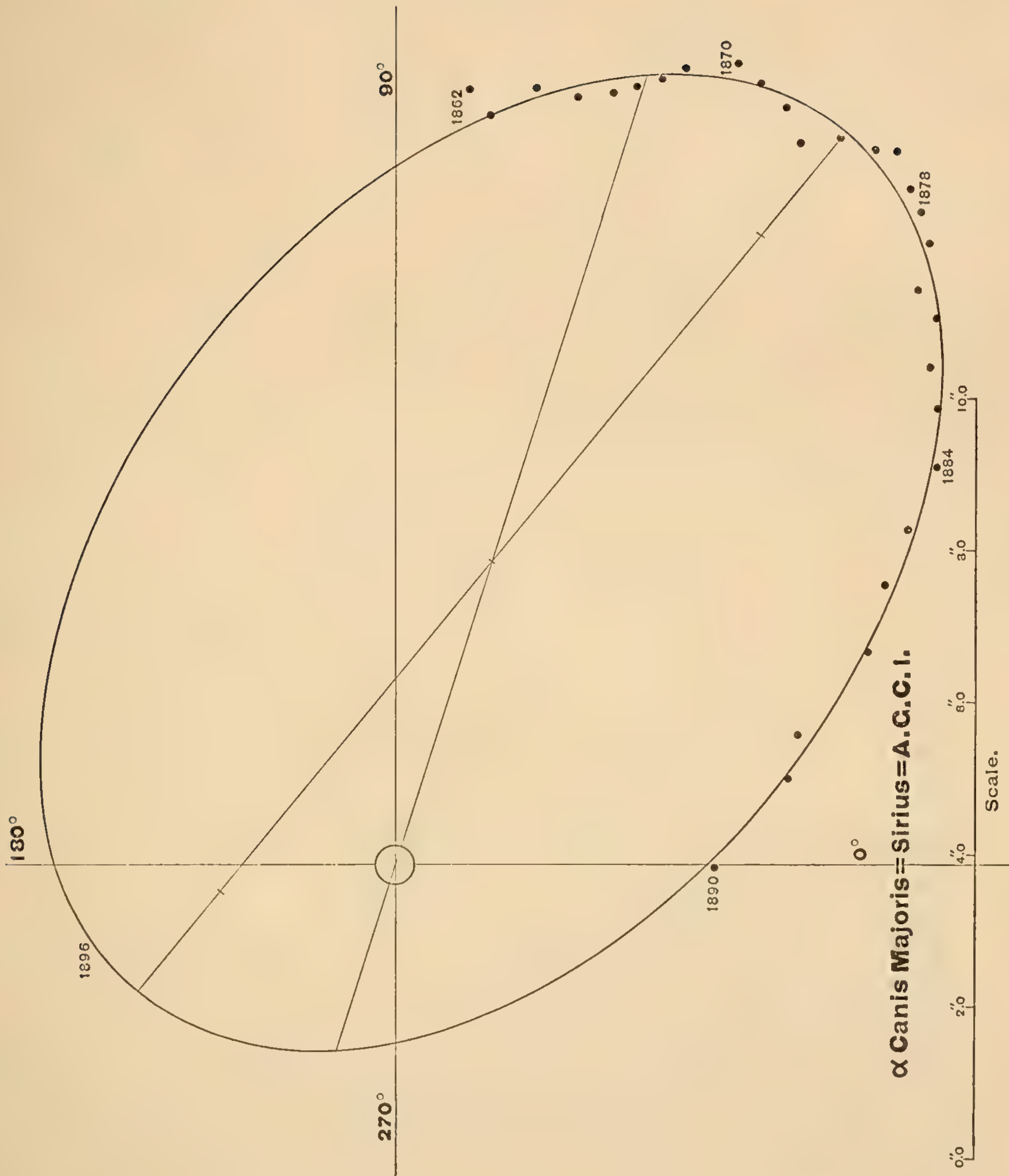
$$\begin{aligned}
 P &= 52.20 \text{ years} & \Omega &= 34^\circ.3 \\
 T &= 1893.50 & i &= 46^\circ.77 \\
 e &= 0.620 & \lambda &= 131^\circ.03 \\
 a &= 8''.0316 & n &= -6^\circ.89655
 \end{aligned}$$

Apparent orbit:

$$\begin{aligned}
 \text{Length of major axis} &= 14''.63 \\
 \text{Length of minor axis} &= 9''.50 \\
 \text{Angle of major axis} &= 50^\circ.7 \\
 \text{Angle of periastron} &= 252^\circ.4 \\
 \text{Distance of star from centre} &= 4''.16
 \end{aligned}$$

EPHEMERIS.

t	θ_c	ρ_c	t	θ_c	ρ_c
1896.20	193.9	4.12	1899.20	158.9	4.97
1897.20	180.8	4.44	1900.20	149.5	5.25
1898.20	169.0	4.72			



COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1862.21	84.7	84.1	10.19	9.78	+0.6	+0.41	10	Bond 2; Rutherford 5; Chacornac 2
1863.21	82.9	81.6	9.90	10.03	+1.3	-0.13	10-9	OΣ. 2; Rutherford 6; Dawes 1; Bond 1-0
1864.20	79.9	79.4	10.36	10.25	+0.5	+0.11	16-12	Mar. 3; Las. 1-3; Bond 4-2; OΣ. 6-3; Da. 1-0; Winn 1
1865.22	76.8	77.1	10.35	10.48	-0.3	-0.13	22-15	Las. 3-0; OΣ. 2; Sec. 8; Fö. 5-4; Tj. 3; Bd.—; En. 1-0
1866.22	74.4	75.0	10.52	10.67	-0.6	-0.15	21-20	Kn. 2-1; Brh. 1; OΣ. 3; Tj. 2-1; Fö. 3-1; Hl. 2-3; N. 3;
1867.14	72.8	73.0	10.68	10.83	-0.2	-0.15	9-13	Wk. 0-6; Sr. 6-5; OΣ. 1; Fö. 2 0 [Tut. 0-1; East. 3; Sec. 2
1868.16	71.4	71.0	10.90	10.97	+0.4	-0.07	20-19	Searle 2; Peirce 1-0; Vl. 7; Brulins 5; Englemann 5
1869.19	70.1	68.9	11.10	11.09	+1.2	+0.01	7	Vl. 3; Dunér 1; Winnecke 2; Peirce 1
1870.18	67.0	67.0	11.42	11.20	±0.0	+0.22	19-14	Peirce 12-4; Winnecke 7-5; Vl. 0-5
1871.21	65.1	65.1	11.28	11.27	±0.0	+0.01	11-9	Secchi 3; Peirce 2-1; Dunér 2; Pech. 4-3
1872.23	62.9	63.1	11.17	11.31	-0.2	-0.14	15-13	Dunér 3; Börgen 3; N. 1; Hall 6; Doberek 3-0
1873.36	60.8	61.0	10.85	11.32	-0.2	-0.47	1-7	Hall 0-1; Dunér 1-4; Bruhns 0-1; W. & S. 0-1
1874.19	58.6	59.5	11.18	11.29	-0.9	-0.11	11-10	N. 7; Holden 2-1; Hall 2
1875.34	56.3	57.3	11.28	11.22	-1.0	+0.06	16-14	Bur. 1-0; Dunér 4; N. 2; Holden 5-4; Hall 4
1876.11	54.9	55.7	11.43	11.14	-0.8	+0.29	17-18	Watson 0-1; Peters 1; Holden 6; Rus. 4; Hall 6
1877.18	53.0	53.7	11.16	11.02	-0.7	+0.14	13-12	Cin. 4-3; Holden 4; Hall 5
1878.14	51.4	51.8	11.00	10.84	-0.4	+0.16	26-31	β. 8; Holden 4; Cin. 9; Pr. 0-5; East. 0-1; Hall 5
1879.04	49.6	50.0	10.75	10.68	-0.4	+0.07	46-40	Cin. 20-14; β. 10; Holden 5; Pritchett 5; Hall 6
1880.15	47.9	47.5	10.22	10.39	+0.4	-0.17	36-34	Cin. 1; Rus. 1; Hol. 4; β. 11; Ho. 3; Big. 6-4; Hl. 8; Frs. 2
1881.17	45.4	45.2	10.11	10.08	+0.2	+0.03	39-37	β. 8; Holden 2; Big. 5-3; Frs. 6; Y. 7; Hough 5; Hall 6
1882.20	42.9	42.7	9.60	9.72	+0.2	-0.12	43-36	β. 11; Hough 9; Big. 4-3; Frs. 6; Hall 7; Englemann 6
1883.15	40.1	39.8	9.32	9.23	+0.3	+0.09	41-36	β. 10; Y. 1; Hough 11; Ws. 4-0; Frs. 7; Big. 2-1; Hl. 6
1884.18	36.4	37.2	8.81	8.80	-0.8	+0.01	43-41	Perrotin 6; Big. 3-1; Hough 11; β. 10; Hall 8; Young 5
1885.19	33.2	33.9	8.04	8.24	-0.7	-0.16	26	Young 8; Hough 10; Hall 8
1886.14	29.7	30.4	7.40	7.63	-0.7	-0.23	20	Young 4; Hough 12; Hall 6
1887.19	24.4	25.5	6.79	6.85	-1.1	-0.06	15	Young 4; Hough 7; Hall 4
1888.53	17.9	17.7	5.53	5.75	+0.2	-0.22	4-5	Hall 3; β. 1-2
1889.06	12.7	13.6	5.26	5.24	-0.9	+0.02	3	Burnham
1890.27	359.7	0.2	4.19	4.09	-0.5	+0.10	3	Burnham

The comparison of the computed with the observed places shows an extremely satisfactory agreement, and we are led to believe that the elements given above will prove to be near the truth. The differences between these elements and those found by AUWERS are not greater than might be expected from the material used in the two cases. Adopting the foregoing elements and GILL's parallax of $0''.38$, we find the mass of the system to be 3.473 times that of the sun and earth; the major semi-axis comes out 21.136 astronomical units. Thus the system of *Sirius* is a magnificent one, having 3.47 times the mass of the planetary system, and slightly larger dimensions than the orbit of the planet *Uranus*. The masses, according to AUWERS, are in the ratio 1:2.119; or, in units of the sun's mass, 1.113 and 2.360 respectively. The future observation of this star is a matter of the highest interest. There is some reason to suppose that *Sirius* is very much expanded, more nearly resembling a nebula than the sun; if this inference be true, the action of the companion will raise enormous bodily tides in the mass of *Sirius*. Since the height of the tides varies inversely as the cube of the distance, it will follow that the tidal eleva-

tion at periastron will be about 80 times higher than at apastron. There would thus arise a periodic disturbance in the mass of *Sirius* depending on the revolution of the companion. It seems probable that high tides would increase the radiation of *Sirius*, and hence if it were possible to make photometric measures of absolute accuracy, or of such a character that the brightness could be compared at intervals of 25 years, it might some day be possible to detect the alteration in brightness arising from the tidal action of the companion.

The excessive faintness of this massive body is an extraordinary anomaly which is not easily explained. From the shape of the orbit, however, we may believe that the system has been formed by the usual process, and for some reason the companion has rapidly become obscure. As the companion is apparently still self-luminous, its darkness is not so conspicuous as the excessive brilliancy of *Sirius*. The change in the color of *Sirius* since ancient times is even more remarkable.

9 ARGÛS = β 101.

$\alpha = 7^h 47^m.1$; $\delta = -13^\circ 38'$.
5.7, yellow ; 6.3, yellow.

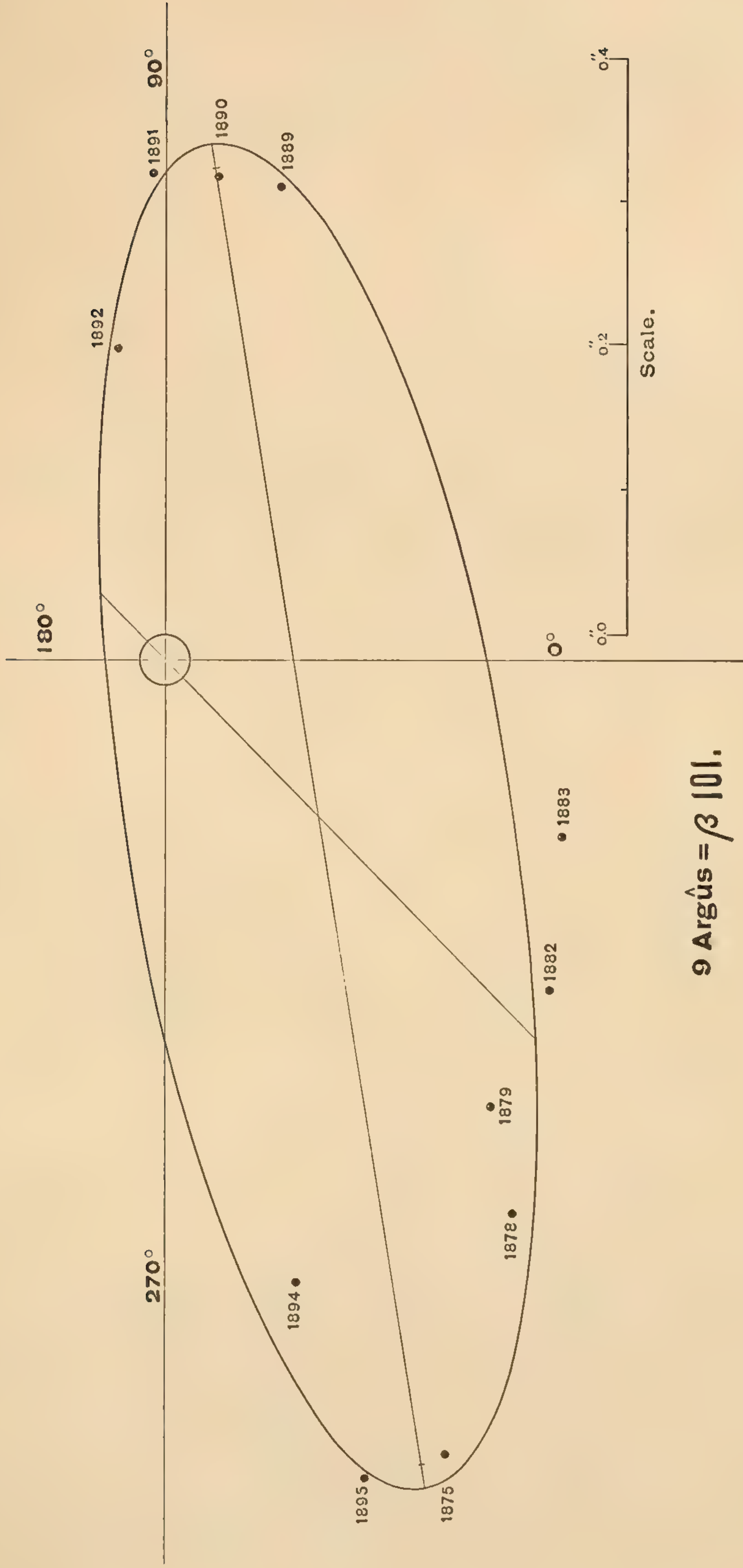
Discovered by Burnham with his celebrated six-inch Clark Refractor, March 11, 1873.

OBSERVATIONS.

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1873.19	double	—	1	Burnham	1891.06	91.5	0.34	4	β . & Sch.
1875.24	289.7	0.58	2	Dembowski	1892.05	98.7	0.22	3	Burnham
1878.50	302.2	0.45	4	St. & β .	1893.94	282.1	0.44	3	Barnard
1879.68	306.2	0.38	2	Hall	1894.18	282.0	0.42	3	Barnard
1882.21	319.7	0.35	4	Schiaparelli	1894.25	286.6	0.35	3	Comstock
1883.11	336.2	0.30	1	Burnham	1894.85	287.3	0.63	5-4	Barnard
1889.08	76.4	0.34	4	Burnham	1895.21	285.2	0.42	2	Comstock
1890.22	83.8	0.34	6	Burnham	1895.25	285.4	0.59	5	Barnard
					1895.30	283.8	0.58	3	See

The first investigation of the orbit was made by GLASENAPP and published in the *Monthly Notices* for June, 1892. His elements are:

$$\begin{array}{ll}
 P = 40.54 \text{ years} & \Omega = 116^\circ.7 \\
 T = 1844.02 & i = 59^\circ.2 \\
 e = 0.090 & \lambda = 251^\circ.3 \\
 a = 0''.45 & z = +8^\circ.880
 \end{array}$$



$$9 \operatorname{Arg} z = 101^\circ.$$

BURNHAM revised this orbit, in May, 1893, and by relying on the distances as well as the angles, arrived at an apparent ellipse of very different character, from which we derived the following elements (*Astronomy and Astrophysics*, June, 1893):

$$\begin{array}{ll} P = 23.377 \text{ years} & \Omega = 95^\circ.75 \\ T = 1892.706 & i = 76^\circ.87 \\ e = 0.68 & \lambda = 73^\circ.92 \\ a = 0''.612 & n = +15^\circ.3998 \end{array}$$

It did not take long to decide which set of elements was to be preferred.* BARNARD examined the star with the 36-inch refractor of the Lick Observatory in December, 1893, and found that since 1892.05 the radius vector of the companion had swept over about 180° , so that the small star was in the fourth quadrant. I took occasion recently, while measuring double stars with the 26-inch refractor of the Leander McCormick Observatory of the University of Virginia, to measure 9 *Argûs* on three good nights. The observations confirm those of BARNARD, and show that BURNHAM'S apparent orbit is not far from the truth. With the new measures, it seemed worth while to re-investigate the orbit; accordingly, from a consideration of all the observations, I find the following elements of 9 *Argûs*:

$$\begin{array}{ll} P = 22.00 \text{ years} & \Omega = 95^\circ.5 \\ T = 1892.30 & i = 77^\circ.72 \\ e = 0.70 & \lambda = 75^\circ.28 \\ a = 0''.6549 & n = +16^\circ.3636 \end{array}$$

Apparent orbit:

$$\begin{array}{ll} \text{Length of major axis} & = 0''.941 \\ \text{Length of minor axis} & = 0''.267 \\ \text{Angle of major axis} & = 99^\circ.2 \\ \text{Angle of periastron} & = 134^\circ.5 \\ \text{Distance of star from centre} & = 0''.152 \end{array}$$

It is confidently believed that these elements will prove to be nearly correct, in spite of the small number of observations upon which they are based.

COMPARISON OF THE COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1875.24	289.7	291.7	0.58	0.58	-2.0	0.00	2	Dembowski
1878.50	302.2	302.5	0.45	0.47	-0.3	-0.02	4	Cincinnati and Burnham
1879.68	306.2	305.4	0.38	0.44	+0.8	-0.06	2	Hall
1882.21	319.7	324.5	0.35	0.31	-4.8	+0.04	4	Schiaparelli
1883.11	336.2	335.7	0.30	0.26	+0.5	+0.04	1	Burnham
1889.08	76.4	73.6	0.34	0.33	+2.8	+0.01	4	Burnham
1890.22	83.8	82.8	0.34	0.36	+1.0	-0.02	6	Burnham
1891.06	91.5	90.1	0.34	0.34	+1.4	0.00	4	Burnham and Schiaparelli
1892.05	98.7	107.0	0.22	0.16	-8.3	+0.06	3	Burnham
1893.94	282.1	276.8	0.44	0.42	+5.3	+0.02	3	Barnard
1895.30	283.8	283.6	0.58	0.57	+0.2	+0.01	3	See

* *Astronomische Nachrichten*, 3297.

It will be seen that the residuals are very small for such a close and difficult star; and it is evident that future observations will not change the present orbit materially, although it is desirable to secure additional exact measures which will improve the elements as much as possible. If adequate attention is given to this object, its orbit will soon be one of the best in the heavens. A short ephemeris is:

t	θ_e	ρ_e	t	θ_e	ρ_e
1896.3	285.8	0.59	1899.3	295.2	0.55
1897.3	288.8	0.60	1900.3	299.0	0.51
1898.3	291.9	0.59			

As the eccentricity of the orbit is well determined by the rapid motion of the companion round the periastron, the established conspicuous magnitude of this element must be regarded as the most remarkable phenomenon of the system.

For the next few years the star will be relatively easy, and double-star observers should give it particular attention.

ζ CANCRI AB = γ 1196.

$\alpha = 8^h 6^m.2$; $\delta = +17^\circ 58'$.
5.5, yellow ; 6.2, yellow.

Discovered by Sir William Herschel, November 21, 1781.

OBSERVATIONS.

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1781.90	363.5	—	1	Herschel	1835.30	28.8	—	1	Mädler
1825.27	57.8	1.09	—	South	1835.31	20.2	1.14	3	Struve
1826.22	57.6	1.14	3	Struve	1835.60	15.7	—	3	Mädler
1828.80	38.4	1.04	2	Struve	1836.27	15.4	1.20	3	Struve
1831.16	31.8	1.34	5-3	Herschel	1836.31	15.1	—	5	Mädler
1831.28	29.8	1.05	6	Struve	1836.68	16.1	—	4	Dawes
1831.30	30.8	1.09	3	Dawes	1840.15	6.1	1.24	35-23 obs.	Kaiser
1832.12	27.9	—	8	Herschel	1840.20	4.4	1.19	8	Dawes
1832.12	27.0	—	7	Dawes	1840.29	7.5	1.00	7	O. Struve
1832.19	31.3	1.32	5	Bessel	1841.16	0.9	1.18	5	Dawes
1832.28	27.5	1.15	4	Struve	1841.31	1.0	1.05	6-4	Mädler
1833.13	26.3	—	9	Herschel	1842.22	356.3	1.18	6	Dawes
1833.21	26.2	1.19	9	Dawes	1842.26	358.9	1.07	6	Mädler
1833.27	22.1	1.15	3	Struve	1842.29	359.3	1.29	4	O. Struve

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1843.18	355.0	1.12	8	Dawes	1856.07	304.2	1 $\pm$	7	Dembowski
1843.19	356.9	1.06	4	Mädler	1856.21	306.3	1.21	4-3	Jacob
1843.30	354.3	1.17	3	O. Struve	1856.23	309.4	1.16	2	Morton
1844.28	350.3	1.16	4	O. Struve	1856.25	307.2	0.77	2	Secchi
1844.39	354.4	1.02	10	Mädler	1856.28	307.5	1.00	2	Mädler
1845.25	350.4	1.05	13	Mädler	1856.31	307.3	1.01	10-7	Winnecke
1845.31	347.9	0.97	3	O. Struve	1856.93	296.5	1.03	3	Dembowski
1845.83	349.4	1.2	1	Jacob	1857.27	298.4	0.98	3	O. Struve
1846.27	347.5	1.02	16	Mädler	1857.29	304.5	0.96	3-2	Mädler
1846.29	344.8	0.95	3	O. Struve	1857.29	303.9	0.78	6	Secchi
1846.29	344.4	—	1	Jacob	1857.90	299.7	1.14	3-1	Jacob
1847.18	344.6	1.09	4	Mädler	1858.18	294.2	1 $\pm$	7	Dembowski
1847.33	342.2	0.96	5	O. Struve	1858.20	297.6	1.05	3	Mädler
1848.13	338.5	1.05	1	Dawes	1858.28	295.5	0.98	1	O. Struve
1848.24	338.1	1.06	6	Dawes	1859.27	294.9	0.98	8	Mädler
1848.25	342.8	1.0	1	W. C. Bond	1859.30	286.5	0.91	2	O. Struve
1848.28	340.0	1.03	7-6	Mädler	1860.26	282.9	—	—	Döllen
1848.30	337.7	0.91	5	O. Struve	1860.26	283.3	—	—	Wagner
1849.29	334.2	1.11	5	Dawes	1860.26	281.0	0.70	1	Dawes
1849.32	336.1	0.80	4	O. Struve	1860.26	284.8	—	—	Schiaparelli
1850.29	332.9	0.94	3	O. Struve	1860.27	281.3	0.81	2	O. Struve
1850.71	330.0	1.03	1	Mädler	1860.28	279.9	—	—	Döllen
1851.18	333.5	1.1 $\pm$	3	Fletcher	1860.28	282.0	—	—	Wagner
1851.21	329.0	1.05	9	Mädler	1860.28	283.4	—	—	Schiaparelli
1851.28	327.2	1.02	3	O. Struve	1860.28	285.0	—	—	Winnecke
1851.25	327.9	1.01	7	Dawes	1860.30	286.0	1.02	5-4	Mädler
1852.16	329.0	1.0 $\pm$	3	Fletcher	1861.14	282.8	—	5	Powell
1852.23	324.4	1.06	3	Dawes	1861.26	282.2	0.97	2	Mädler
1852.25	326.9	1.06	6	Mädler	1861.27	275.3	0.87	3	O. Struve
1852.32	321.7	0.89	2	O. Struve	1862.31	267.5	0.74	2	O. Struve
1853.20	322.0	1.22	3	Jacob	1862.32	274.4	0.97	4	Mädler
1853.24	323.5	1.06	8-7	Mädler	1863.13	263.1	0.74	15	Dembowski
1853.30	319.8	0.97	2	O. Struve	1863.25	267.3	0.95	—	Leyton Obs.
1854.20	315.3	0.98	3	Dawes	1863.25	262.5	0.67	1	Dawes
1854.27	318.6	1.08	10-9	Mädler	1863.30?	268.1	0.70	1	Knott
1854.29	320.2	1.02	1	Morton	1864.15	255.0	0.55	10	Dembowski
1854.37	321.9	—	12	Powell	1864.29	253.2	0.71	2	Dawes
1855.10	308.6	1 $\pm$	7	Dembowski	1864.31	350.0 $\pm$	0.60	1	Englemann
1855.19	312.4	1.07	3	Secchi	1864.30	253.3	0.72	2	O. Struve
1855.26	310.6	1.06	4	Mädler	1865.21	245.7	0.50	12	Dembowski
1855.31	310.3	0.91	3	O. Struve	1865.30	243.4	0.63	3-2	Dawes
1855.31	305.9	1.04	7-6	Winnecke	1865.33	245.3	0.64	2	Secchi
					1865.36	241.4	0.61	3	Knott
					1865.30	244.0	0.86	4	Englemann

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1866.19	238.4	0.52	9	Dembowski	1877.17	108.7	0.68	7	Dembowski
1866.27	237.8	0.70	1	O. Struve	1877.23	107.9	0.79	7	Schiaparelli
1866.28	234.6	0.40	2	Secchi	1877.23	110.3	0.81	3-6	Plummer
1866.31	233.3	0.78	4	Knott	1877.24	108.1	0.87	3-2	Dobereck
1866.37	231.5	0.72	1	Leyton Obs.	1877.27	108.0	0.72	3	O. Struve
1866.94	228.3	0.66	1	Knott	1877.32	107.3	0.74	1	Pritchett
1867.08	229.7	0.59	3-1	Harvard	1878.16	104.1	1.01	1-2	Dobereck
1867.22	224.4	obl.	9	Dembowski	1878.18	100.3	0.66	6	Dembowski
1868.20	210.9	0.5	7	Dembowski	1878.26	100.8	0.7	7	Jedrzejewicz
1868.28	214.7	0.72	2	O. Struve	1878.29	99.1	0.76	3	O. Struve
1869.26	197.6	0.64	1	Peirce	1878.32	102.3	0.81	3	Hall
1869.32	198.4	0.62	2	O. Struve	1879.27	93.1	0.87	6	Schiaparelli
1869.37	203.6	0.48	4	Dunér	1879.29	91.8	0.74	3	O. Struve
1870.08	188.1	0.64	5-2	Harvard	1880.21	85.2	0.61	5	Hall
1870.15	187.3	0.5	9	Dembowski	1880.22	89.8	0.89 $\pm$	6	Jedrzejewicz
1870.28	186.3	0.66	4	O. Struve	1880.24	88.9	—	2	Dobereck
1870.30	188.3	0.43	3-4	Dunér	1880.29	85.2	0.73	6	Burnham
1870.56	181.0	0.2	2	Gledhill	1881.24	81.1	0.91 $\pm$	4	Jedrzejewicz
1871.15	175.5	Contatto	7	Dembowski	1881.24	84.9	0.84	5	Dobereck
1871.26	175.1	0.2	2	Gledhill	1881.28	86.8	0.88	3	O. Struve
1871.29	178.2	0.55	3	Dunér	1881.30	79.0	0.71	3	Hall
1871.30	169.4	—	—	Scharnhorst	1881.30	80.2	0.92	6	Schiaparelli
1871.31	171.3	0.59	3	O. Struve	1881.31	73.7	0.77	2	Pritchett
1872.11	166.7	0.6	2	Knott	1882.09	75.7	0.74	1	Bigourdan
1872.21	167.5	0.70	3	Wilson	1882.20	73.3	0.79	4	Hall
1872.23	162.8	Contatto	7	Dembowski	1882.22	76.2	1.05	6	Englemann
1872.31	163.0	0.58	3	O. Struve	1882.25	75.1	0.98	6	Schiaparelli
1872.33	163.3	0.69	2	Dunér	1882.26	75.0	0.94 $\pm$	4	Jedrzejewicz
1873.19	150.2	0.5	10	Dembowski	1883.24	72.4	1.05	6	Englemann
1873.22	150.9	0.5 $\pm$	4	W. & S.	1883.29	69.3	1.00	6	Schiaparelli
1873.28	152.0	0.61	3	O. Struve	1883.31	66.4	0.82	4	Hall
1873.63	149.3	0.55	2	Gledhill	1884.19	62.7	1.06	3	Perrotin
1874.09	141.6	0.74	7	Dembowski	1884.22	61.9	—	8	Bigourdan
1874.13	140.1	0.45 $\pm$	2	Gledhill	1884.25	63.9	0.98	7	Schiaparelli
1874.18	141.3	0.58	3 2	W. & S.	1884.26	60.6	0.98	3	O. Struve
1874.28	144.5	0.64	3	O. Struve	1884.27	61.5	0.88	5	Hall
1874.29	142.8	0.62	2	Dunér	1884.28	67.0	0.94	4	Englemann
1875.14	130.1	0.74	8	Dembowski	1884.38	61.4	—	3	Sea. & Smith
1875.26	128.9	0.70	6	Schiaparelli	1885.27	59.0	1.25	2	Seabroke
1875.28	132.4	0.62	3	O. Struve	1885.29	58.0	1.04	5	Schiaparelli
1875.29	133.3	0.77	2	W. & S.	1885.29	59.4	1.05	4	Englemann
1875.33	129.5	0.59	5	Dunér	1886.08	57.2	1.09	4	Tarrant
1876.14	119.4	0.72	6	Dembowski	1886.24	51.4	1.06	2-1	Sea. & Smith
1876.26	120.7	—	6	Dobereck	1886.28	55.0	1.03	4	Hall
1876.29	119.45	0.66	2	O. Struve	1886.29	51.2	0.98	3	Jedrzejewicz
					1886.30	56.3	1.08	5	Englemann

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1887.24	50.4	0.89	4	Hall	1891.22	35.7	1.04	5	Hall
1887.26	48.4	0.97	11	Schiaparelli	1891.24	34.1	1.14	3	Bigourdan
1887.35	46.0	1.21	4-1	Sea. & Smith	1892.24	31.6	1.09	3	Maw
1888.25	46.5	1.03	4	Hall	1892.25	31.3	1.26	2-3	Knorre
1888.26	49.2	—	3	Smith	1892.26	30.1	1.11	11	Schiaparelli
1888.27	43.7	1.04	9	Schiaparelli	1892.28	30.4	1.10	6	Bigourdan
1888.33	45.8	1.09	2	O. Struve	1892.89	28.7	0.99	3	Jones
1888.36	41.4	1.13	1	Maw	1893.20	27.2	0.98	2	Comstock
1889.17	42.0	1.20	4	Sea.&Hodges	1893.22	26.4	1.07	3	Maw
1889.19	40.3	1.05	3	Leavenworth	1893.24	27.6	1.12	13	Schiaparelli
1889.21	40.7	1.08	12	Schiaparelli	1894.15	26.0	1.47	1	Ebell
1889.21	43.4	—	2	Glaserapp	1894.16	23.8	1.24	3	H. C. Wilson
1889.23	43.6	0.99	5	Hall	1894.23	22.9	0.93	3	Comstock
1889.28	43.7	1.23	2	O. Struve	1894.24	23.5	1.08	13	Schiaparelli
1889.29	40.9	1.07	3	Maw	1894.24	25.0	1.05	4	Maw
1890.23	37.2	11.1	9-7	Schiaparelli	1894.39	23.2	1.39	5-4	Bigourdan
1890.26	36.4	0.95	2	Comstock	1895.23	21.9	1.22	2	Lewis
1890.28	36.9	0.99	4	Hall	1895.23	20.9	1.01	3	Comstock
1891.05	32.3	1.04	5-4	Flint	1895.27	17.1	1.09	1	Davidson
1891.21	34.3	1.14	9-10	Schiaparelli	1895.28	22.8	1.13	4	See

The closer components of this ternary (or quarternary) system have been found to revolve rapidly in a period of about sixty years, while the remote component moves much more slowly, and probably will complete its orbit in six or seven centuries. Both stars move retrograde, and the system thus made up is one of great interest to the physical astronomer. From the time of WILLIAM STRUVE the observations are both abundant and exact, and hence the orbit of the close pair can now be determined with a high degree of precision. We shall treat only of the close binary, neglecting the remote companion and the dark body which PROFESSOR SEELIGER supposes to attend it. It is evident that the third component will exercise a considerable disturbing influence upon the close pair, but PROFESSOR SEELIGER has shown that this influence is probably obscured by the large errors incident to the measurement of a system which is never much wider than one second of arc. Assuming that the motion will be sensibly undisturbed, we shall deduce the orbit of the closer pair by the same process which is employed in the case of other binaries. The motion of this system has been investigated by numerous computers; the following list of orbits is fairly complete:

P	T	e	a	Ω	i	λ	Authority	Source
58.91	1853.37	0.2346	1.292	1.47	63.3	266.0	Mädler, 1840	Dorpat obs. IX, p. 177
58.27	1816.687	0.444	0.892	33.67	24.01	133.01	Mädler, 1848	Fixt.-Syst. I, p. 248
42.501	1805.67	0.4743	1.013	10.52	65.65	227.15	Villargeau 1849	A.N. 967
58.94	1815.53	0.256	1.030	18.4	48.6	141.9	Winnecke 1855	
58.23	1872.44	0.3023	0.908	150.3	36.24	171.78	Plummer, 1871	M.N. XXXI, p. 195
60.45	1869.9	0.365	0.908	107.5	23.5	85.3	Flam., 1873	Catal. d. ét. doub. p. 49
62.4	1869.3	0.353	0.908	109.0	20.7	199.0	O. Struve, 1874	C.R. LXXIX, p. 1467
59.486	1870.82	0.3318	0.886	358.05	18.52	188.55	Dobereck, 1880	A.N. 2322 [1881
60.3	1866.0	0.391	0.853	81.55	15.53	109.73	Seeliger, 1881	Wien. Akad. LXXXIII,
59.11	1868.112	0.3819	0.853	80.18	11.13	109.73	Seeliger, 1888	Akad. d. Wiss., Münch. '88

An examination of all the measures led to the mean places given in the accompanying table; from these we find the following elements:

$$\begin{aligned}
 P &= 60.0 \text{ years} & \Omega &= 88^\circ.7 \\
 T &= 1870.40 & i &= 7^\circ.4 \\
 e &= 0.340 & \lambda &= 264^\circ.0 \\
 a &= 0''.8579 & n &= -6^\circ.000
 \end{aligned}$$

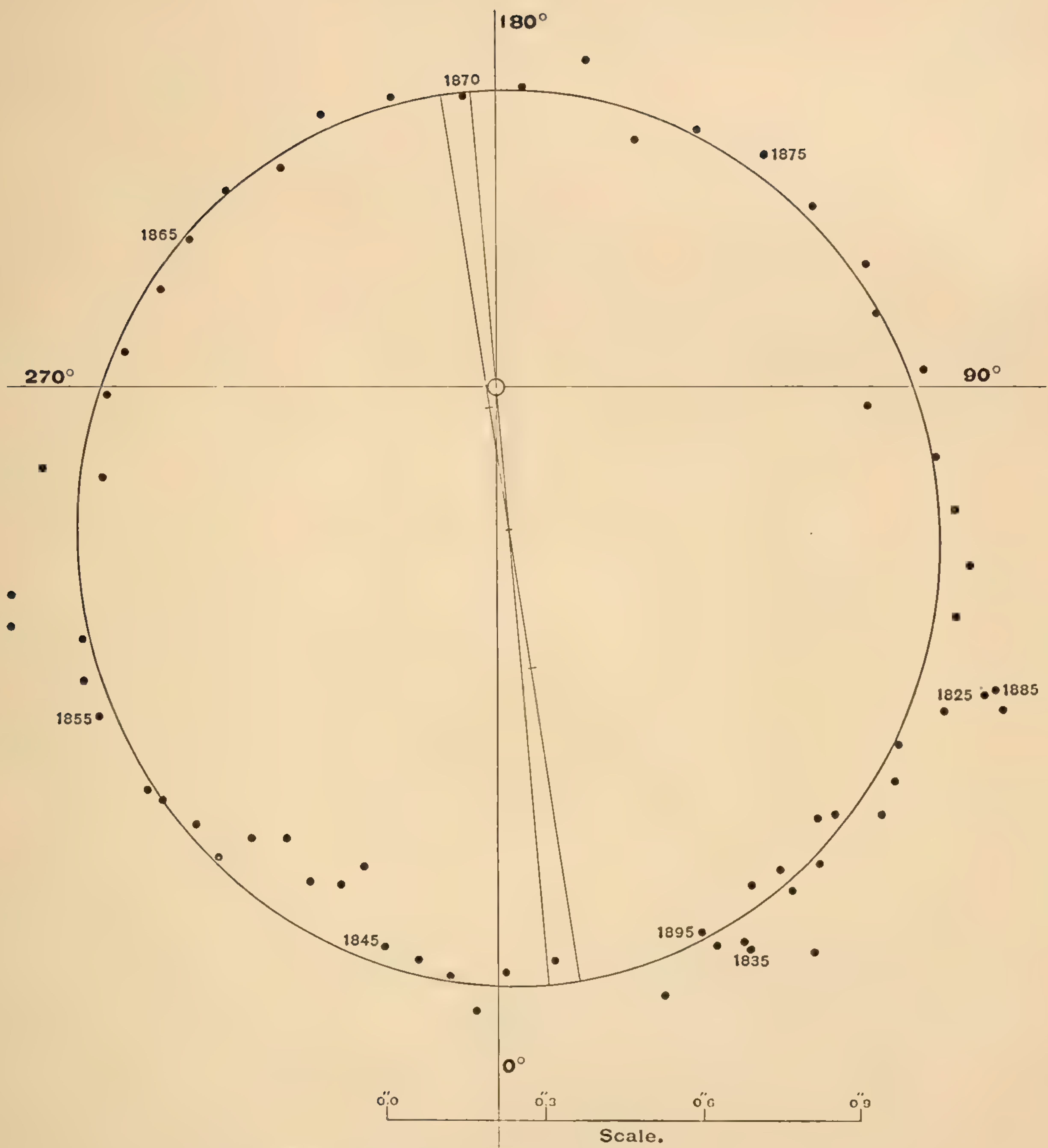
Apparent orbit:

$$\begin{aligned}
 \text{Length of major axis} &= 1''.704 \\
 \text{Length of minor axis} &= 1''.632 \\
 \text{Angle of major axis} &= 8^\circ.8 \\
 \text{Angle of periastron} &= 184^\circ.9 \\
 \text{Distance of star from centre} &= 0''.290
 \end{aligned}$$

The comparison of the computed with the observed places shows a good agreement, and indicates that no radical change in the above elements is to be expected. The period is perhaps uncertain by half a year, while the eccentricity can hardly be varied by more than ± 0.03 . The motion extends over more than one revolution, and is well represented by the above elements in all parts of the orbit. The apparent ellipse is remarkable for its circularity, and the small inclination renders the motion almost the same in the apparent as in the real orbit. The general interest thus attaching to this system is greatly enhanced by problems arising from the perturbations of the third star and its theoretical companion.

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
	$^\circ$	$^\circ$	$''$	$''$	$^\circ$	$''$		
1781.90	363.5	359.6	—	1.14	+3.9	—	1	Herschel
1825.27	57.8	59.0	1.09	0.96	-1.2	+0.13	—	South
1826.22	57.6	55.0	1.14	0.98	+2.6	+0.16	3	Struve
1828.80	38.4	44.1	1.04	1.03	-5.7	+0.01	2	Struve
1831.29	30.3	34.9	1.07	1.07	-4.6	± 0.00	9	Struve 6; Dawes 3
1832.23	29.4	30.9	1.23	1.09	-1.5	+0.14	9	Bessel 5; Struve 4
1833.24	24.2	28.0	1.17	1.10	-3.8	+0.07	12	Dawes 9; Struve 3



t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
	$^{\circ}$	$^{\circ}$	$''$	$''$	$^{\circ}$	$''$		
1835.40	21.6	21.5	1.14	1.12	+0.1	+0.02	7-3	Mädler 1; Σ . 3; Mädler 3
1836.42	15.5	17.4	1.20	1.13	-1.9	+0.07	12-3	Σ . 3; Mädler 5-0; Dawes 4-0
1840.24	6.0	5.2	1.09	1.14	+0.8	-0.05	15	Dawes 8; O Σ . 7
1841.23	0.9	2.1	1.11	1.14	-1.2	-0.03	11-9	Dawes 5; Mädler 6-4
1842.25	358.2	358.8	1.18	1.14	-0.7	+0.04	16	Dawes 6; Mädler 6; O Σ . 4
1843.22	355.4	355.8	1.12	1.13	-0.4	-0.01	15	Dawes 8; Mädler 4; O Σ . 3
1844.33	352.4	352.2	1.09	1.12	+0.2	-0.03	14	O Σ . 4; Mädler 10
1845.57	348.6	348.1	1.08	1.12	+0.5	-0.04	4	O Σ . 3; Jacob 1
1846.29	344.6	345.7	0.95	1.11	-1.1	-0.16	4-3	O Σ . 3; Jacob 1-0
1847.31	342.6	342.3	0.99	1.10	+0.3	-0.11	7	Mädler 2; O Σ . 5
1848.24	339.4	339.2	1.01	1.09	+0.2	-0.08	20-19	Dawes 1; Dawes 6; Bond 1; Mädler 7-6; O Σ . 5
1849.31	335.1	335.3	0.95	1.07	-0.2	-0.12	9	Dawes 5; O Σ . 4
1850.50	331.4	330.9	0.98	1.06	+0.5	-0.08	4	O Σ . 3; Mädler 1
1851.23	329.4	328.3	1.04	1.04	+1.1	$\pm$ 0.00	22	Fletcher 3; Mädler 9; O Σ . 3; Dawes 7
1852.24	325.5	323.7	1.00	1.02	+1.8	-0.02	14	Fletcher 3; Dawes 3; Mädler 6; O Σ . 2
1853.25	321.8	320.3	1.01	1.00	+1.5	+0.01	13-9	Jacob 3-10; Mädler 8-7; O Σ . 2
1854.28	319.0	316.0	1.00	0.98	+3.0	+0.02	26-4	Dawes 3; Mädler 10-0; Mo. 1; Powell 12-0
1855.23	309.6	311.8	0.98	0.96	-2.2	+0.02	24-17	Dem. 7; Secchi 3-0; Mädler 4-0; O Σ . 3; Winnecke 7-6
1856.33	305.5	306.6	0.96	0.93	-1.1	+0.03	30-21	Dem. 7; Ja. 4-0; Mo. 2-0; Sec. 2; Mä. 2; Winn. 10-7;
1857.44	301.6	301.0	0.91	0.90	+0.6	+0.01	15-17	O Σ . 3; Mädler 3-2; Secchi 6; Jacob 3-1 [Dem. 3
1858.22	295.8	296.7	0.99	0.88	-0.9	+0.11	11-8	Dem. 7; Mädler 3-0; O Σ . 1
1859.28	290.7	290.9	0.95	0.85	-0.2	+0.10	10	Mädler 8; O Σ . 2
1860.28	282.8	284.6	0.76	0.82	-1.8	-0.06	8-3	Dawes 1; O Σ . 2; Mädler 5-0
1861.22	280.1	278.6	0.87	0.79	+1.5	+0.08	10-3	Powell 5-0; Mädler 2; O Σ . 3
1862.31	270.9	270.9	0.86	0.75	$\pm$ 0.0	+0.11	6-2	O Σ . 2; Mädler 4
1863.23	264.6	263.4	0.70	0.72	+1.2	-0.02	17	Dembowski 15; Dawes 1; Knott 1
1864.21	253.8	255.0	0.66	0.69	-1.2	-0.03	14	Dembowski 10; Dawes 2; Englemann 1; O Σ . 2
1865.30	244.0	245.2	0.60	0.65	-1.2	-0.05	24-19	Dembowski 12; Dawes 3-2; Secchi 2; Knott 3; En. 4
1866.39	233.9	233.8	0.63	0.62	+0.1	+0.01	18-13	Dem. 9; O Σ . 1; Secchi 2; Knott 4-0; Ley. 1-0; Knott 1
1867.15	224.4	225.3	0.59	0.61	-0.9	-0.02	9-1	Harvard 3-1; Dembowski 9-0
1868.24	212.8	212.4	0.61	0.58	+0.4	+0.03	9-7	Dembowski 7; O Σ . 2-0
1869.32	199.9	199.1	0.58	0.57	+0.8	+0.01	7-6	Peirce 1-0; O Σ . 2; Dunér 4
1870.27	186.2	186.7	0.56	0.56	-0.5	$\pm$ 0.00	23-21	Harvard 5-2; Dembowski 9; O Σ . 4; Dunér 3-4; Gl. 2
1871.25	175.0	173.7	0.57	0.56	+1.3	+0.01	15-6	Dembowski 7; Gledhill 2-0; Dunér 3; O Σ . 3
1872.24	164.6	161.3	0.64	0.58	+3.3	+0.06	17-10	Knott 2; Wilson, 3; Dembowski 7-0; O Σ . 3; Dunér 2
1873.33	150.6	147.8	0.54	0.59	+2.8	-0.05	19	Dembowski 10; W. & S. 4; O Σ . 3; Gledhill 2
1874.19	142.1	138.1	0.61	0.62	+3.0	-0.01	17-16	Dembowski 7; Gledhill 2; W. & S. 3-2; O Σ . 3; Dunér 2
1875.26	130.8	126.5	0.68	0.65	+4.3	+0.03	24	Dembowski 8; Sch. 6; O Σ . 3; W. & S. 2; Dunér 5
1876.23	119.8	117.4	0.69	0.68	+2.4	+0.01	13-7	Dembowski 5; Doberck 6-0; O Σ . 2
1877.24	108.4	108.6	0.74	0.72	-0.2	+0.02	24-26	Dem. 7; Sch. 7; Plummer 3-6; Dk. 3-2; O Σ . 3; Pr. 1
1878.24	101.3	100.4	0.73	0.74	+0.9	-0.01	20-19	Doberck 1-0; Dembowski 6; Jed. 7; O Σ . 3; Hall 3
1879.28	92.4	92.7	0.81	0.78	-0.3	+0.03	9	Schiaparelli 6; O Σ . 3
1880.24	87.3	86.4	0.75	0.81	+0.9	-0.06	19-17	Hall 5; Jedrzejewicz 6; Doberck 2-0; β . 6
1881.28	80.9	79.9	0.84	0.84	+1.0	$\pm$ 0.00	23	Jed. 4; Doberck 5; O Σ . 3; Hall 3; Sch. 6; Pritchett 2
1882.20	75.1	74.4	0.90	0.87	+0.7	+0.03	21	Bigourdan 1; Hall 4; Englemann 6; Sch. 6; Jed. 4
1883.28	69.3	68.8	0.96	0.90	+0.5	+0.06	16	Englemann 6; Schiaparelli 6; Hall 4
1884.26	63.6	63.8	0.97	0.93	-0.2	+0.04	33-22	Per. 3; Big. 8-0; Sch. 7; O Σ . 3; Hl. 5; En. 4; S. & S. 3-0
1885.28	58.8	59.2	1.11	0.95	+1.7	+0.16	11	Seabroke 2; Schiaparelli 5; Englemann 4
1886.24	54.2	54.8	1.05	0.98	-0.6	+0.07	18-17	Tarrant 4; S. & S. 2-1; Hall 4; Jed. 3; Englemann 5
1887.28	48.3	50.2	1.02	1.00	-1.9	+0.02	19-16	Hall 4; Schiaparelli 11; S. & S. 4-1
1888.29	45.3	46.4	1.07	1.02	-1.1	+0.05	19-16	Hall 4; Smith 3-0; Schiaparelli 9; O Σ . 2; Maw 1
1889.22	42.1	42.5	1.10	1.04	-0.4	+0.06	31-29	Sea. 4; Leav. 3; Hl. 5; O Σ . 2; Maw 3; Sch. 12; Gl. 2-0
1890.26	36.8	38.5	1.02	1.06	-1.7	-0.04	16-14	Schiaparelli 9-7; Comstock 2; Hall 4
1891.18	34.1	35.2	1.09	1.07	-1.1	+0.02	22	Flint 5-4; Schiaparelli 9-10; Hall 5; Bigourdan 3
1892.38	30.4	30.9	1.11	1.09	-0.5	+0.02	25-26	Maw 3; Knott 2-3; Schiaparelli 11; Bigourdan 6; Jo. 3
1893.22	27.1	27.1	1.06	1.10	$\pm$ 0.0	-0.04	18	Comstock 2; Maw 3; Schiaparelli 13
1894.23	24.0	24.6	1.16	1.11	-0.6	+0.05	29-28	Eb. 1; H.C.W. 3; Com. 3; Sch. 13; Maw 4; Big. 5-4
1895.25	20.7	21.3	1.11	1.12	-0.6	-0.01	10	Lewis 2; Comstock 3; Davidson 1; See 4

A more critical investigation of these problems will commend itself to the attention of astronomers; the best results will depend upon the reduction of exact observations by the refined methods of analysis. In the present state of micrometrical measurement, a very refined treatment is seriously embarrassed by the errors of observation; but the methods of physical Astronomy ought eventually to enable us to improve the theory of the motion of the system, which is here taken as undisturbed.

The following is a short ephemeris for the use of observers:

t	θ_c	ρ_c	t	θ_c	ρ_c
1896.25	18.0	1.13	1899.25	8.4	1.13
1897.25	14.8	1.13	1900.25	5.3	1.14
1898.25	11.6	1.13			

Σ 3121.

$\alpha = 9^h 12^m.1$; $\delta = +29^\circ 0'$.
7.2, white ; 7.5, yellowish.

Discovered by William Struve in 1831.

OBSERVATIONS.

t	θ_o	ρ_o	n		t	θ_o	ρ_o	n	
1832.31	20.0	0.85	3	Struve	1868.30	27.6	0.81	2	O. Struve
1840.31	246.5	0.40 ±	3-1	O. Struve	1869.31	26.1	0.88	1	O. Struve
1844.28	193.5	0.33	2-1	O. Struve	1870.33	206.9	0.65	2	Dunér
1846.29	27.6	0.55	1	O. Struve	1870.44	210.4	0.5 ±	1	Gledhill
1847.34	214.2	0.54	1	O. Struve	1871.20	212.7	0.5 ±	1	Gledhill
1848.25	33.0	0.53	1	O. Struve	1871.27	208.2	0.75	3	Dunér
1849.32	43.3	0.48	1	O. Struve	1871.30	35.3	0.79	2	O. Struve
1850.30	228.6	0.42	1	O. Struve	1871.44	211.0	0.57	5	Dembowski
1851.26	59.7	0.33	1	O. Struve	1872.09	209.3	0.68	1	Dunér
1861.29	Double vers le Norde		1	O. Struve	1872.31	36.4	0.68	1	O. Struve
1861.30	8.9	0.67	1	O. Struve	1873.69	214.2	obl.	8	Dembowski
1863.11	194.8	0.7	1	Dembowski	1873.70	214.5	0.5 ±	1	Gledhill
1864.30	13.0	0.71	1	O. Struve	1874.24	220.	<0.3	2	Dunér
1865.77	206.8	0.80	2	Englemann	1874.28	46.7	0.53	2	O. Struve
1867.65	201.3	0.70	5	Dembowski	1875.20	225.	0.2 ±	1	Dunér
					1875.29	250.1	obl.	1	O. Struve
					1875.29	65.2	0.30	4	Schiaparelli
					1875.31	251.9	ovale	2	Dembowski

t	θ_o	ρ_o	n		t	θ_o	ρ_o	n	
1877.25	183.0	oblong	1	O. Struve	1885.30	215.8	0.4 ±	3	Schiaparelli
1878.21	185.2	0.25 ±	1	Burnham	1886.33	221.2	0.27	4	Englemann
1879.21	193.0	0.40	2	Burnham	1887.27	250.4	0.22 ±	9	Schiaparelli
1879.33	186.8	0.43	1	O. Struve	1888.27	286.3	0.22 ±	7	Schiaparelli
1879.57	200.4	0.43	5	Schiaparelli	1889.30	132.3	0.23 ±	7	Schiaparelli
1880.26	200.3	0.35	3	Hall	1890.29	152.9	0.27 ±	4	Schiaparelli
1880.31	199.8	0.50	1	Burnham	1891.26	163.3	0.35	4	Hall
1881.29	198.0	0.61	1	O. Struve	1891.32	166.7	0.33 ±	2	Schiaparelli
1881.34	205.3	0.46	2	Schiaparelli	1892.26	175.3	0.41 ±	7	Schiaparelli
1882.25	194.8	0.31	4	Englemann	1893.25	182.3	0.47	7-2	Schiaparelli
1882.31	205.8	0.45	4	Schiaparelli	1893.25	185.9	0.44	1	Comstock
1882.34	205.2	0.53	1	O. Struve	1894.18	185.9	0.49	1	Wilson
1883.22	221.2	0.39	6	Englemann	1894.21	186.6	0.58	3	Bigourdan
1883.28	213.8	0.52	3	Schiaparelli	1894.24	183.3	0.45	3	Comstock
1883.31	215.7	0.45	3	Hall	1894.25	186.3	0.48 ±	5	Schiaparelli
1884.27	218.9	0.42	1	O. Struve	1895.23	190.5	0.65	3	Lewis
1884.39	222.7	0.38	4	Schiaparelli	1895.26	8.8	0.50	3	Comstock
1884.61	225.6	0.30	4	Englemann	1895.31	12.6	0.55	2	See

WILLIAM STRUVE rated the magnitudes of the components of this pair at 7.5 and 7.8* respectively. Recent observations with the 26-inch refractor of the Leander McCormick Observatory of the University of Virginia convince the writer that the brightness of the components has been over-estimated by at least a whole magnitude. The star is close and very faint, and the natural difficulty of the object will doubtless account for the rather large discordances in some of the observations.

As Σ 3121 has been observed for many years, and the pair revolves with great rapidity, several orbits have been determined by previous investigators. The following is believed to be a complete list of the elements hitherto published:

P	T	e	a	Ω	i	λ	Authority	Source
39.18	1850.0	0.3471	0.696	19.94	52.4	143.3	Fritsche, 1866	{ Bulletin de l'Acad. de St. Pétersbourg, t. X A.N. 2156 A.N. 2808
40.62	1850.0	0.3725	0.715	23.5	54.11	141.6	Fritsche, 1866	
37.03	1842.78	0.26	0.71	16.0	74.25	149.5	Doberck, 1877	
34.642	1878.52	0.3086	0.6725	24.85	75.43	129.45	Celoria, 1887	

* *Astronomical Journal*, 349.

From an investigation of all the observations, I find the following elements:

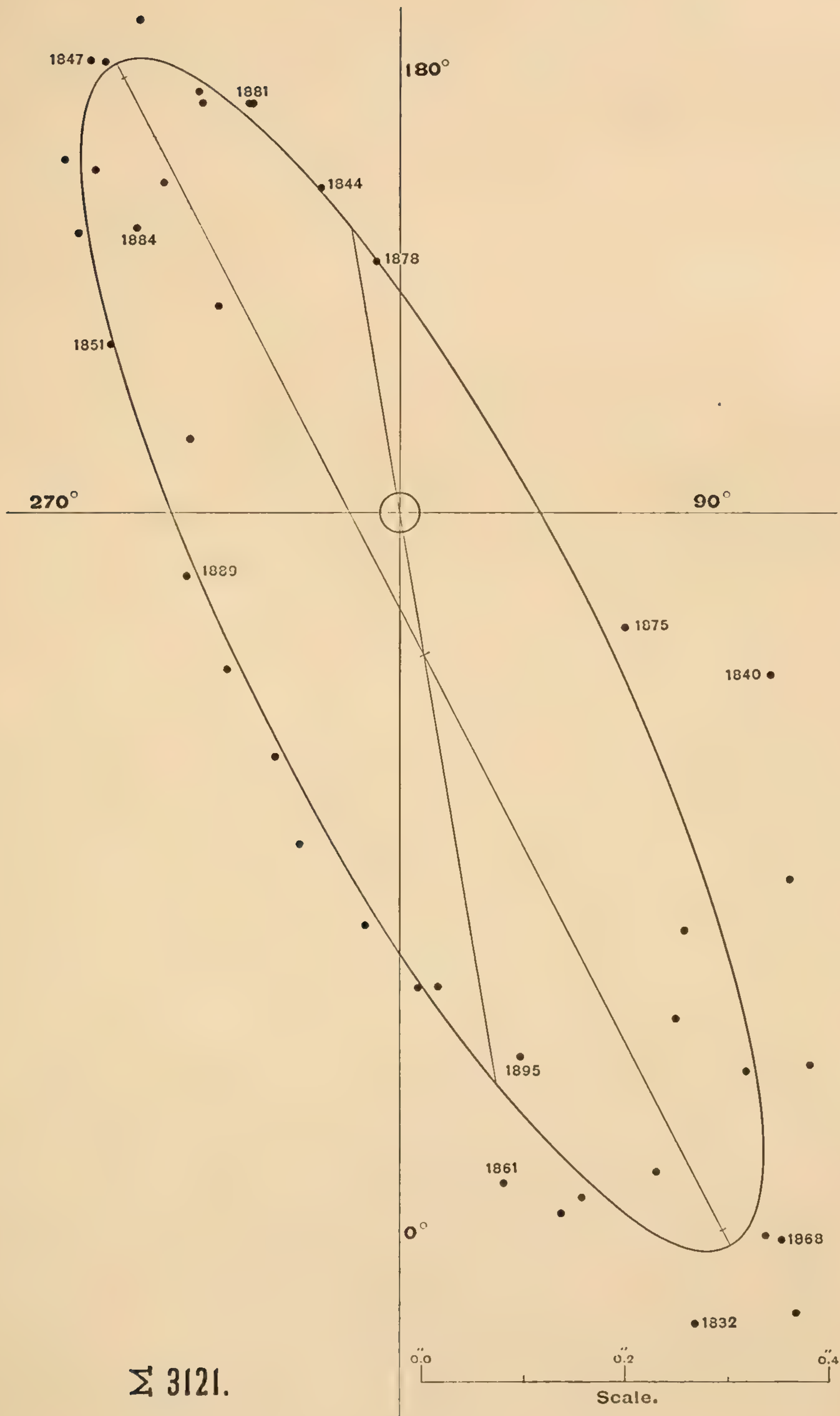
$$\begin{aligned}
 P &= 34.00 \text{ years} & \Omega &= 28^\circ.25 \\
 T &= 1878.30 & i &= 75^\circ.00 \\
 e &= 0.330 & \lambda &= 127^\circ.52 \\
 a &= 0''.6692 & n &= +10^\circ.5883
 \end{aligned}$$

Apparent orbit:

$$\begin{aligned}
 \text{Length of major axis} &= 1''.318 \\
 \text{Length of minor axis} &= 0''.319 \\
 \text{Angle of major axis} &= 27^\circ.4 \\
 \text{Angle of periastron} &= 189^\circ.6 \\
 \text{Distance of star from center} &= 0''.142
 \end{aligned}$$

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1832.31	20.0	22.3	0.85	0.79	- 2.3	+0.08	3	W. Struve
1840.31	66.5	47.3	0.40 ±	0.35	+19.2	+0.05	3-1	O. Struve
1844.28	193.5	189.8	0.33	0.29	+ 3.7	+0.04	2-1	O. Struve
1846.29	207.6	205.2	0.55	0.48	+ 2.4	+0.07	1	O. Struve
1847.34	214.2	210.1	0.54	0.52	+ 4.1	+0.02	1	O. Struve
1848.25	213.0	214.1	0.53	0.52	- 1.1	+0.01	1	O. Struve
1849.32	223.3	218.8	0.48	0.50	+ 5.5	-0.02	1	O. Struve
1850.30	228.6	223.9	0.42	0.45	+ 4.7	-0.03	1	O. Struve
1851.26	239.7	230.1	0.33	0.39	+ 9.6	-0.06	1	O. Struve
1861.26	8.9	9.9	0.67	0.58	- 1.0	+0.09	1	O. Struve
1863.11	14.8	14.6	0.7	0.66	+ 0.2	+0.04	1	Dembowski
1864.30	13.0	18.1	0.71	0.73	- 5.1	-0.02	1	O. Struve
1865.77	26.8	21.3	0.80	0.78	+ 5.5	+0.02	2	Englemann
1867.65	21.3	24.8	0.70	0.79	- 3.5	-0.09	5	Dembowski
1868.30	27.6	26.2	0.81	0.79	+ 1.4	+0.02	2	O. Struve
1869.31	26.1	28.2	0.88	0.76	- 2.1	+0.12	1	O. Struve
1870.38	28.6	30.6	0.57	0.71	- 2.0	-0.14	3	Dunér, 2; Gledhill 1
1871.30	31.8	32.8	0.65	0.65	- 1.0	0.00	11	Gl. 1; Du. 3; OΣ. 2; Dem. 5
1872.20	36.4	35.6	0.68	0.58	+ 0.8	+0.10	1-2	OΣ. 1; Dunér 0-1
1873.70	34.3	42.8	0.5 ±	0.42	- 8.5	+0.08	9-1	Dembowski 8-0; Gledhill 1
1874.28	46.7	46.7	0.53	0.36	0.0	+0.17	2	O. Struve
1875.27	63.0	63.0	0.25	0.22	0.0	+0.03	8-5	Du. 1; OΣ. 1; Sch. 4; Dem. 2
1878.21	185.2	188.4	0.25	0.28	- 3.2	-0.03	1	Burnham
1879.57	200.4	200.2	0.43	0.41	+ 0.2	+0.02	5	Schiaparelli
1880.28	200.0	205.1	0.43	0.48	- 5.1	-0.05	4	Hall 3; Burnham 1
1881.34	205.3	210.1	0.46	0.52	- 4.8	-0.06	2	Schiaparelli
1882.28	205.8	214.1	0.45	0.52	- 8.3	-0.07	4	Schiaparelli
1883.27	221.2	218.3	0.45	0.50	+ 2.9	-0.05	6-12	En. 6; Sch. 0-3; Hall 0-3
1884.39	222.7	224.5	0.38	0.44	- 1.8	-0.06	4	Schiaparelli
1885.30	215.8	230.5	0.4 ±	0.39	-14.7	+0.01	3	Schiaparelli
1886.33	221.2	239.9	0.27	0.32	-17.7	-0.05	4	Englemann
1887.27	250.4	252.5	0.22	0.27	- 2.1	-0.05	9	Schiaparelli
1888.27	286.3	272.6	0.22 ±	0.22	+13.7	0.00	7	Schiaparelli
1889.30	312.3	299.7	0.23 ±	0.21	+12.6	+0.02	7	Schiaparelli
1890.29	332.9	323.5	0.27 ±	0.24	+ 8.4	+0.03	4	Schiaparelli
1891.29	343.3	340.8	0.34	0.30	+ 4.2	+0.04	6	Hall 4; Schiaparelli 2
1892.26	355.3	354.0	0.41 ±	0.37	+ 1.3	+0.04	7	Schiaparelli
1893.25	2.3	359.7	0.47	0.43	+ 2.6	+0.04	7-2	Schiaparelli
1894.22	5.2	5.0	0.48	0.50	+ 0.2	-0.02	9	Wilson 1; Comstock 3; Sch. 5
1895.29	10.7	9.8	0.53	0.58	+ 0.9	-0.05	5	See 2; Comstock 3



Some of the observations are vitiated by sensible systematic errors, so that occasionally our best observers differ by so much as 12° ; and in succeeding years the angles are made to retrograde where they ought to be steadily advancing. Under these circumstances the residuals may be considered small, and the elements very satisfactory for so close and difficult a star. In following this star, observers should take every precaution against systematic error, since the orbit is highly inclined, and a small error in angle greatly affects the distance. Good observations are essential for any further improvement of the elements:

EPHEMERIS.					
t	θ_c	ρ_c	t	θ_c	ρ_c
1896.30	13.5	0.64	1899.30	20.7	0.77
1897.30	16.2	0.69	1900.30	22.7	0.79
1898.30	18.5	0.74			

Since the companion is now approaching its maximum distance, the star will be relatively easy for a number of years.

ω LEONIS = Σ 1356.

$\alpha = 9^h 23^m.1$; $\delta = +9^\circ 30'$.
6, yellow ; 7, yellow.

Discovered by Sir William Herschel, February 8, 1782.

OBSERVATIONS.									
t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1782.86	110.9	—	1	Herschel	1841.18	354.5	—	1	Dawes
1803.09	130.9	—	2	Herschel	1841.35	194.0	0.3	1	Mädler
1825.21	153.9	0.97	5	Struve	1842.21	249.8	elong.?	1	Mädler
1830.24	146.5	wedge-shaped	1	Herschel	1842.31	302.3	0.3	4	O. Struve
1832.25	163.4	0.51	3	Struve	1842.33	einfach		1	Mädler
1833.29	172.8	0.45	3	Struve	1843.30	einfach, rund		3	Mädler
1835.33	178.3	0.3 $\pm$	3-1	Struve	1843.30	316.8	0.37	2	O. Struve
1836.28	358.7	0.35 $\pm$	3-2	Struve	1844.29	320.9	0.48	3	O. Struve
1836.28	359.8	—	3	O. Struve	1844.32	337.0	0.32	4	Mädler
1836.30	171.8	—	1	Mädler	1845.31	321.1	0.44	3	O. Struve
1840.29	247.5	0.3	2	O. Struve	1846.28	326.9	0.35	11	Mädler
1840.29	255.	—	—	Dölln	1846.30	322.9	0.45	2	O. Struve
1840.31	250.3	—	—	W. Struve	1847.28	337.0	0.37	3	Mädler
					1847.33	328.8	0.53	2	O. Struve

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1848.32	332.1	0.43	4	O. Struve	1870.24	44.4	0.25 $\pm$	5-1	Peirce
1848.35	346.8	0.38	1	Mädler	1870.28	53.6	0.58	2	O. Struve
1849.32	331.8	0.43	3	O. Struve	1870.30	37.9	0.27 $\pm$	2	Dunér
1850.63	335.8	0.49	3	O. Struve	1871.16	52.6	cuneo	3	Dembowski
1851.23	342.6	0.35	9	Mädler	1871.30	56.7	0.57	3	O. Struve
1852.30	350.0	0.47	4	Mädler	1871.31	42.7	0.3 $\pm$	1	Dunér
1852.66	339.1	0.46	3	O. Struve	1872.18	66.3	0.48	2	Wilson
1853.18	343.3	0.45 $\pm$	2	Jacob	1872.31	58.8	0.52	2	O. Struve
1853.27	346.3	0.35	7-6	Mädler	1873.23	56.2	—	2	W. & S.
1853.96	350.0	0.4 $\pm$	2	Jacob	1873.29	57.0	0.4 $\pm$	1	Gledhill
1854.23	346.2	0.55	2	Dawes	1873.58	62.0	contatto	5	Dembowski
1854.28	348.3	0.53	10	Mädler	1873.96	63.6	0.59	3	O. Struve
1855.27	obl. ?	—	2	Mädler	1875.25	64.6	0.46	5	Dembowski
1855.32	348.7	0.47	2	O. Struve	1875.26	62.7	0.49	7	Schiaparelli
1855.34	6.2	—	1	Winnecke	1875.31	66.8	0.43	5	Dunér
1856.20	obl. ?	—	1	Mädler	1875.32	66.4	0.59	3	O. Struve
1856.42	1.0	0.36	10-7	Secchi	1876.16	69.4	0.44	2	Dembowski
1857.28	358.1	0.52	1	O. Struve	1876.24	52.7	—	3	Doberek
1857.31	obl. ?	—	1	Mädler	1876.27	73.5	0.55 $\pm$	2	W. & S.
1857.54	4.3	0.43 $\pm$	3	Jacob	1876.29	65.6	0.57	2	O. Struve
1858.28	16.2 ?	—	1	Mädler	1877.21	77.2	0.88	1	Copeland
1859.25	16.7	0.35	4-3	Mädler	1877.21	71.2	0.54	5 1	Plummer
1859.30	6.7	0.60	2	O. Struve	1877.21	73.0	0.51	3 1	Doberek
1860.28	9.2	—	—	Winnecke	1877.27	70.7	0.47	7	Schiaparelli
1860.28	10.2	0.62	2	O. Struve	1877.28	71.6	0.54	2	O. Struve
1860.33	19.1	0.25	1	Mädler	1877.36	76.6	0.41	2	Dembowski
1861.28	11.9'	0.56	2	O. Struve	1878.11	70.3	0.63	2	Burnham
1862.32	18.6	elong.	2	Mädler	1878.26	80.3	0.50	1	Doberek
1864.30	29.2	0.52	1	O. Struve	1878.28	74.7	0.44	5	Dembowski
1864.89	24.	cuneo	4	Dembowski	1878.63	77.7	0.60	3	O. Struve
1865.67	23.0	0.50	8	Englemann	1878.95	74.4	0.41	6	Hall
1866.30	32.9	0.3	1	Secchi	1879.31	76.6	0.55	7	Schiaparelli
1867.08	109.4	elong.	1	Winlock	1879.78	79.8	0.51	4	Burnham
1867.08	125.7	elong.	1	Searle	1880.23	79.7	—	1	Bigourdan
1867.32	29.3	elong.	1	Winlock	1880.26	95.2	obl.	4	Jedrzejewicz
1867.87	Kreisrund		1	Vogel	1880.26	81.3	0.46	6	Hall
1868.21	15.6	elong.	1	Peirce	1881.10	81.0	0.61	2	Bigourdan
1868.63	44.3	0.55	3	O. Struve	1881.24	82.3	0.50	5-2	Doberek
1869.13	317.2	elong.	1	Peirce	1881.26	98.7	obl.	2	Jedrzejewicz
1869.26	36.7	elong.	1	Peirce	1881.28	83.7	0.68	2	O. Struve
					1881.31	84.3	0.48	4	Hall
					1881.33	84.4	0.58	5	Schiaparelli

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1882.12	77.3	—	1	Dobereck	1888.21	97.4	0.68	3	Tarrant
1882.12	80.5	—	1	Copeland	1888.26	91.6	—	3	Smith
1882.23	80.0	0.56	7	Englemann	1888.27	98.5	0.68	6	Schiaparelli
1882.27	83.3	0.66	3	Dobereck	1888.29	98.3	0.66	5	Hall
1882.30	84.1	0.49	4	Hall	1888.33	94.9	0.87	2	O. Struve
1882.34	86.7	0.61	2	O. Struve	1888.57	95.8	0.71	7	Lv.
1882.36	90.0	0.55	4	Schiaparelli	1889.19	94.1	0.70	1	Hodges
1883.24	85.8	0.62	6	Englemann	1889.29	99.8	0.67	5	Hall
1883.31	90.5	0.65	6	Schiaparelli	1889.32	100.2	0.65	9	Schiaparelli
1883.34	90.9	0.62	3	Hall	1890.27	101.8	0.68	2	Comstock
1884.18	90.6	0.55	2	Perrotin	1890.31	101.2	0.64	4	Hall
1884.23	91.4	0.66	4	Englemann	1890.31	101.6	0.68	4	Schiaparelli
1884.26	87.6	0.71	2	O. Struve	1891.21	102.1	0.76	2	Bigourdan
1884.30	91.3	0.58	5	Schiaparelli	1891.28	101.2	0.75	5	Hall
1884.32	93.3	0.55	4	Hall	1891.31	103.9	0.66	5	Schiaparelli
1884.34	90.6	—	10	Bigourdan	1892.25	102.4	0.77	3	Maw
1884.39	85.9	1.0 $\pm$	3-2	Sea. & Sm.	1892.26	104.9	0.72	7	Schiaparelli
1885.27	90.6	0.72	3	Englemann	1892.27	104.5	0.87	5	Lv. & Col.
1885.17	93.3	—	1	Dobereck	1893.25	101.5	0.61	1	Comstock
1885.31	93.7	0.58	4	Schiaparelli	1893.28	105.7	0.70	9	Schiaparelli
1885.31	93.9	0.69	2	Tarrant	1894.22	104.5	1.30	1	Bigourdan
1885.35	88.9	1.00 $\pm$	1	Smith	1894.23	106.5	0.67	3	Comstock
1885.72	90.9	0.70	2	Perrotin	1894.25	103.3	0.74	2	H. C. Wilson
1886.24	90.1	1.19	2-1	Sea. & Sm.	1894.25	106.7	0.75	8	Schiaparelli
1886.32	92.2	0.73	6	Englemann	1894.88	287.4	0.91	3	Barnard
1887.26	95.0	0.62	9	Schiaparelli	1895.24	106.1	0.67	3	Comstock
1887.30	95.6	0.53	4	Hall	1895.28	106.1	0.83	2	See
1887.37	94.0	—	1	Smith					

At the time of discovery SIR WILLIAM HERSCHEL estimated the position-angle* to be between 95° and 100° , but later in the year found by measurement that the angle was $110^\circ.9$. The pair was soon found to be in slow orbital motion, and in 1804 HERSCHEL concluded that since 1782 the change in angle had amounted to $+19^\circ 59'$, and that the distance had sensibly increased. When the star was thus recognized as binary, it naturally claimed the attention of the principal double-star observers, and accordingly since the time of STRUVE, a long list of measures has been secured. But while the closeness of the companion in most parts of the apparent ellipse has made the pair a classic test-object for the dividing power of small telescopes, it has, on the other hand, rendered micrometrical measurement extremely difficult, and some of the observations are therefore far from satisfactory. In spite of the fact that the measures

* *Astronomische Nachrichten*, 3311.

are sometimes difficult to reconcile, the angles and distances of the best observers, when properly combined, in conjunction with the important principle of the preservation of areas, enable us to fix the apparent ellipse with a relatively high degree of precision, and the resulting elements are found to be incapable of any large variation. The orbit is based chiefly upon the observations of HERSCHEL, STRUVE, O. STRUVE, DAWES, DEMBOWSKI, BURNHAM, HALL, SCHIAPARELLI, and the measures which the writer recently secured at the McCormick Observatory in Virginia. The elements of ω *Leonis* are:

$P = 116.20 \text{ years}$
 $T = 1842.10$
 $e = 0.537$
 $a = 0''.88241$

$\Omega = 146^\circ.70$
 $i = 63^\circ.47$
 $\lambda = 124^\circ.22$
 $n = +3^\circ.0981$

Apparent orbit:

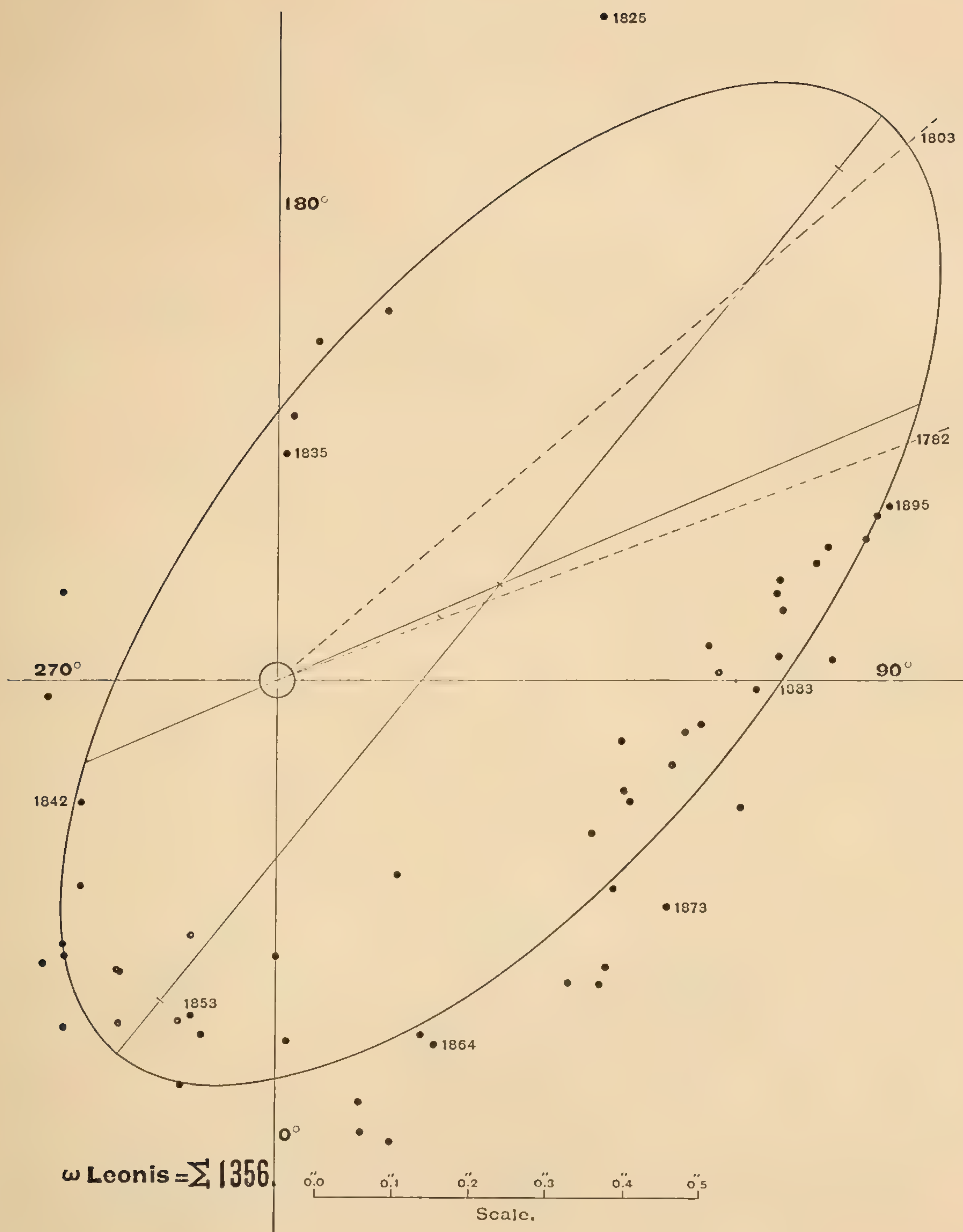
Length of major axis = $1''.576$
Length of minor axis = $0''.738$
Angle of major axis = $141^\circ.1$
Angle of periastron = $293^\circ.4$
Distance of star from centre = $0''.317$

Several astronomers have previously investigated the orbit of this star; the following table gives the elements hitherto published:

<i>P</i>	<i>T</i>	<i>e</i>	<i>a</i>	Ω	<i>i</i>	λ	Authority	Source
82.533	1849.76	0.6134	0.857	135.2	46.57	185.45	Mädler, 1841	Dorp. Obs. IX, 198
117.577	1843.408	0.6256	0.8505	159.83	50.61	120.45	Mädler, 1846	Fixt. Syst. I, p. 250
133.35	1846.44	0.3605	0.703	111.85	57.23	217.37	Klinkerf. 1856	A.N. 990
227.77	1841.40	0.7225	1.307	169.2	60.22	84.17	Klinkerf. 1856	A.N. 990
142.41	1843.39	0.6286	1.092	162.22	54.42	107.15	Klinkerf. 1858	A.N. 1127
136.4	1844.2	0.62	1.05	160.5	52.4	113.4	Klinkerfues	Theor. Astron. p. 395
107.62	1842.77	0.5028	—	151.57	65.37	122.9	Dobereck, 1876	A.N. 2078
110.82	1841.81	0.536	0.890	148.77	64.08	121.07	Dobereck, 1876	A.N. 2095
114.55	1841.57	0.5510	0.85	149.25	64.08	122.3	Dobereck	
115.30	1841.99	0.5379	0.864	147.1	64.15	122.9	Hall, 1892	A.J. 269
115.87	1842.16	0.533	0.8753	145.9	63.05	125.32	See, 1894	A.N. 3311

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

<i>t</i>	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	<i>n</i>	Observers
1782.86	110.9	112.1	—	0.89	— 1.2	—	1	Herschel
1803.09	130.9	130.3	—	1.08	+ 0.6	—	2	Herschel
1825.21	153.9	150.4	0.97	0.81	+ 3.5	+0.16	5	Struve
1832.25	163.4	164.9	0.51	0.52	— 1.5	—0.01	3	Struve
1833.29	172.8	168.8	0.45	0.47	+ 4.0	—0.02	3	Struve
1835.33	178.3	179.9	0.3±	0.35	— 1.6	—0.05	3-1	Struve
1836.28	176.8	187.8	0.35	0.30	—11.0	+0.05	7-2	Σ . 3-2; $O\Sigma$. 3-0; Mädler 1-0
1840.29	247.5	263.8	0.3	0.21	—16.3	+0.09	2	O. Struve
1841.26	274.2	281.6	0.3	0.24	— 7.4	+0.06	2-1	Dawes 1-0; Mädler 1
1842.31	302.3	295.8	0.3	0.28	+ 6.5	+0.02	4	O. Struve



t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1843.30	316.8	305.2	0.37	0.33	+11.6	+0.04	2	O. Struve
1844.31	320.9	312.3	0.48	0.38	+ 8.6	+0.10	3	O. Struve
1845.31	321.1	317.9	0.44	0.42	+ 3.2	+0.02	3	O. Struve
1846.30	322.9	322.6	0.45	0.45	+ 0.3	0.00	2	O. Struve
1847.31	328.8	326.8	0.53	0.48	+ 2.0	+0.05	2	O. Struve
1848.32	332.1	330.5	0.43	0.50	+ 1.6	-0.07	4	O. Struve
1849.32	331.8	334.0	0.43	0.52	- 2.2	-0.09	3	O. Struve
1850.63	335.8	338.2	0.49	0.53	- 2.4	-0.04	3	O. Struve
1851.23	342.6	340.1	0.35	0.53	+ 2.5	-0.18	9	Mädler
1852.48	344.5	344.1	0.46	0.54	+ 0.4	-0.08	7	Mädler 4; O. Struve 3
1853.47	346.5	347.0	0.45	0.54	- 0.5	-0.09	11-10	Jacob 2; Mädler 7-6; Jacob 2
1854.25	347.2	349.4	0.54	0.54	- 2.2	0.00	12	Dawes 2; Mädler 10
1855.32	348.7	353.1	0.47	0.53	- 4.4	-0.06	2	O. Struve
1856.42	1.0	356.3	0.36	0.53	+ 4.7	-0.17	10-7	Secchi
1857.41	2.4	359.5	0.47	0.52	+ 2.9	-0.05	4	O. Struve 1; Jacob 3
1859.27	11.7	5.6	0.60	0.51	+ 6.1	+0.09	6-5	Mädler 4-3; O. Struve 2
1860.30	14.6	9.2	0.62	0.50	+ 5.4	+0.12	3	O. Struve 2; Mädler 1
1861.28	11.9	12.8	0.56	0.50	- 0.9	+0.06	2	O. Struve
1864.59	24.0	25.0	0.52	0.48	- 1.0	+0.04	4-1	O. Struve 1; Dembowski 4-0
1865.67	23.0	28.1	0.50	0.48	- 5.1	+0.02	8	Englemann
1866.30	32.9	31.7	0.30	0.48	+ 1.2	-0.18	1	Secchi
1868.63	44.3	40.7	0.55	0.48	+ 3.6	+0.07	3	O. Struve
1870.28	47.3	47.1	0.68	0.49	+ 0.2	+0.18	9-5	Peirce 5-1; O. Struve 2; Dunér 2
1871.30	49.7	51.0	0.57	0.49	- 1.3	+0.08	7-4	Dembowski 3-0; O. Struve 3; Du. 1
1872.31	58.8	54.7	0.52	0.50	+ 4.1	+0.02	2	O. Struve
1873.62	60.3	59.2	0.52	0.51	+ 1.1	+0.01	11-4	W. & S. 2-0; Gl. 1; Dem. 5-0;
1875.27	64.7	64.9	0.46	0.52	- 0.2	-0.06	17	Dem. 5; Sch. 7; Du. 5 [O. Struve 3]
1876.21	71.4	67.7	0.49	0.53	+ 3.7	-0.04	4	Dem. 2; W. & S. 2 [Cop. 0-1]
1877.25	72.9	71.3	0.56	0.55	+ 1.6	+0.01	17-12	Pl. 5-1; Dk. 3-1; Sch. 7; Dem. 2;
1878.40	74.9	74.8	0.63	0.56	+ 0.1	+0.07	14	β . 2; Dk. 1; Dem. 5; Hall 6
1879.54	78.2	77.7	0.53	0.58	+ 0.5	-0.05	11	Schiaparelli 7; Burnham 4
1880.24	80.2	79.7	0.46	0.59	+ 0.5	-0.13	7-6	Bigourdan 1-0; Hall 6
1881.24	83.0	82.1	0.54	0.60	+ 0.9	-0.06	16-13	Big. 2; Dk. 5-2; Hl. 4; Sch. 5
1882.29	84.4	84.7	0.56	0.62	- 0.3	-0.06	18	En. 7; Dk. 3; Hl. 4; Sch. 4
1883.30	89.2	87.1	0.63	0.63	+ 2.1	0.00	15	En. 6; Sch. 6; Hl. 3 [Big. 10-0]
1884.27	91.4	89.2	0.58	0.65	+ 2.2	-0.07	25-15	Per. 2; En. 4; Sch. 5; Hall 4;
1885.37	92.9	90.9	0.66	0.66	+ 2.0	0.00	9-8	Dk. 1-0; Sch. 4; Tar. 2; Per. 2
1886.32	92.2	93.3	0.73	0.68	- 1.1	+0.05	6	Englemann
1887.31	94.9	95.2	0.57	0.70	- 0.3	-0.13	14-13	Sch. 9; Hall 4; Smith 1-0
1888.25	98.1	96.9	0.67	0.72	+ 1.2	-0.05	14	Tarrant 3; Sch. 6; Hall 5
1889.30	100.0	98.6	0.66	0.73	+ 1.4	-0.07	14	Hall 5; Schiaparelli 9
1890.30	101.5	100.3	0.67	0.75	+ 1.2	-0.08	10	Hall 4; Comstock 2; Sch. 4
1891.27	102.4	101.8	0.72	0.77	+ 0.6	-0.05	12	Hall 5; Bigourdan 2; Sch. 5
1892.26	103.9	103.3	0.79	0.79	+ 0.6	0.00	15	Maw 3; Sch. 7; Lv. & Col. 5
1893.26	103.6	104.8	0.74	0.80	- 1.2	-0.06	10	Comstock 1-0; Schiaparelli 9-5
1894.36	105.6	106.3	0.81	0.82	- 0.7	-0.01	17-13	Big. 1-0; Com. 3-0; H.C.W. 2;
1895.28	106.1	107.5	0.83	0.84	- 1.4	-0.01	2	See [Sch. 8; Bar. 3]

The elements given above confirm the substantial accuracy of the orbit found by HALL, and represent the observations as a whole remarkably well. The changes which future observations will introduce are likely to be very small.

The following is an ephemeris for the next five years:

EPHEMERIS.					
t	θ_c	ρ_c	t	θ_c	ρ_c
1896.28	108.7	0.85	1899.28	112.4	0.90
1897.28	110.0	0.87	1900.28	113.5	0.91
1898.28	111.2	0.88			

It is to be noted that the distance is steadily increasing, and that for many years the pair will be relatively easy. A number of observers of late years have sensibly underestimated the distance. Owing to the closeness of ω *Leonis* and its slow orbital motion, one would naturally think that this brilliant system probably has a small mass, and is comparatively near us in space; for if the mass be large, the slow motion of so close a system would indicate that it is very remote, and the resulting brightness of the components would be very great. The eccentricity of this orbit is so well determined that the value given above can hardly be in error by so much as 0.01, and a correction of half this amount does not seem probable.

η URSAE MAJORIS = σ^2208 .

$\alpha = 9^h 45^m.3$; $\delta = +54^\circ 33'$.
5.5, yellowish ; 5.5, yellowish.

Discovered by Otto Struve in 1842.

OBSERVATIONS.									
t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1842.30	4.2	0.42	1	Mädler	1852.39	16.1	0.32	2	O. Struve
1842.35	8.5	0.52	2	O. Struve	1852.40	209.8	0.25	4	Mädler
1843.37	5.6	0.48	3	Mädler	1853.40	16.7	0.34	3	O. Struve
1843.47	188.5	0.39	1	O. Struve	1854.28	25.9	0.4 $\pm$	1	Dawes
1844.26	186.6	0.51	1	O. Struve	1854.37	23.3	0.42	1	O. Struve
1846.01	193.8	0.45	3-2	Mädler	1857.34	30.6	0.3	1	Secchi
1846.37	9.2	0.42	1	O. Struve	1858.41	36.1	0.40	3	O. Struve
1847.41	196.8	0.30	2	Mädler	1859.37	43.9	0.33	1	Winnecke
1847.41	12.1	0.36	1	O. Struve	1859.39	37.6	0.35	2	O. Struve
1848.40	10.4	0.35	2	O. Struve	1861.40	55.0	0.44	1	Winnecke
1850.39	15.0	0.33	2	O. Struve	1861.41	48.5	0.37	2	O. Struve
1851.39	207.2	0.31	4	Mädler	1862.39	46.8	0.38	1	O. Struve
1851.40	13.7	0.33	2	O. Struve	1864.43	48.5	0.27	1	O. Struve

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1866.27	46.5	<0.4	1	Englemann	1882.19	139.0	<0.2	3	Englemann
1866.42	48.2	0.24	1	O. Struve	1882.34	342.0?	—	1	O. Struve
1869.40	45.0	oblong	2	Dunér	1887.43	218.9	0.23	4	Schiaparelli
1870.42	81.5	oblong	2	Dunér	1888.43	220.3	cuneiforme	1	O. Struve
1872.41	77.7	0.23	2	O. Struve	1889.39	214.0	cert. elong.	1	O. Struve
1873.44	87.5	—	1	Lindemann	1892.13	250.8	0.24	3	Burnham
1873.45	96.6	oblong	3	O. Struve	1892.31	60.4	0.29	1	Bigourdan
1873.47	95.4	—	1	H. Bruhns	1892.58	single	—	1	Comstock
1875.47	115.1	oblong	2	O. Struve	1893.36	339.55	0.30	1	Schiaparelli
1876.42	54.0	elongated?	1	O. Struve	1894.25	round	—	1	Comstock
1877.43	single	—	1	O. Struve	1894.40	82.7	—	3	Bigourdan
1879.44	single	—	1	O. Struve	1895.73	276.2	0.29	3	See

Although this close and rapid binary was discovered by OTTO STRUVE, the first observation was secured by MÄDLER, whose measures supplement STRUVE's work in a very happy manner, and enable us to fix the original position of the companion with much precision. For a long time these two astronomers alone followed the motion of the system, but in later years it has received occasional attention from several other observers. The stars are nearly equal in magnitude, and hence a few of the recorded angles require a correction of 180° . They already described amounts to about 270° , and as this covers the most critical parts of the orbit, most of the elements are defined with the desired precision. The chief difficulty encountered by observers lies in the closeness of the components, which places them beyond the reach of small, and even of moderate-sized, telescopes. The pair is, however, gradually widening out, and in a few years will be much more accessible to measurement.

The following elements of this star have been published by previous computers:

P	T	e	a	Ω	i	λ	Authority	Source
$\frac{\text{yrs.}}{115.4}$	1877.12	0.788	0.54	105.3	57.95	72.1	Casey, 1882	A.N. 2417
91.9	1885.4	0.45	0.29	165.7	34.7	19.0	Glas., 1892	A.N. 3119

Using all the available measures, we find the following elements:

$$\begin{aligned}
 P &= 97.0 \text{ years} & \Omega &= 160^\circ.3 \\
 T &= 1884.0 & i &= 30^\circ.5 \\
 e &= 0.440 & \lambda &= 15^\circ.9 \\
 a &= 0''.3443 & n &= +3^\circ.7114
 \end{aligned}$$

Apparent orbit:

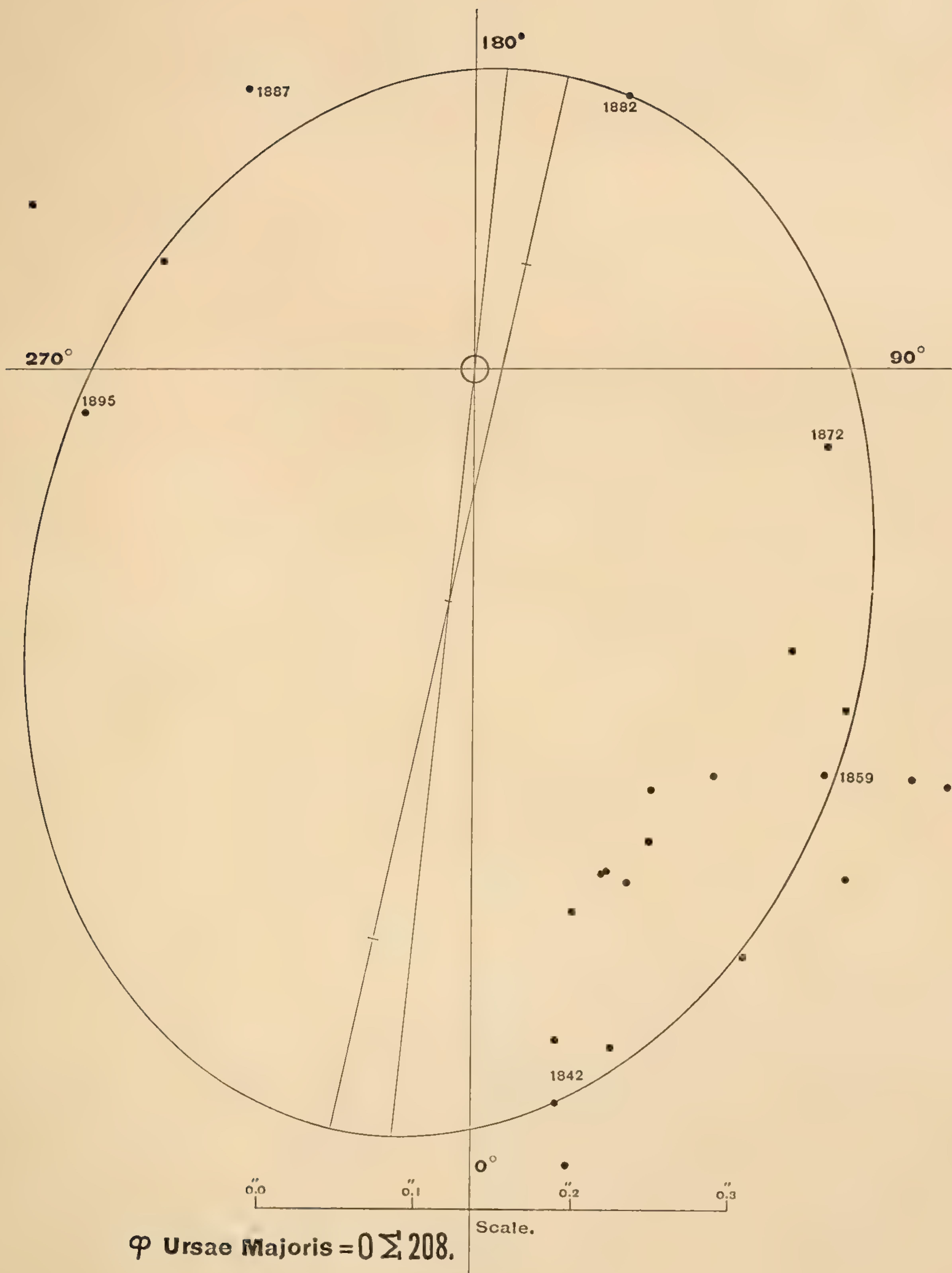
Length of major axis	= 0".69
Length of minor axis	= 0".53
Angle of major axis	= 167°.6
Angle of periastron	= 174°.1
Distance of star from centre	= 0".149

It will be seen that this orbit is essentially similar to that found by GLASENAPP. The table of computed and observed places shows so satisfactory an agreement for this close and difficult object that we may regard these elements as substantially correct, and confidently conclude that such alterations as future observations may render necessary will be of minor importance.

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
	°	°	"	"	°	"		
1842.32	6.3	4.0	0.47	0.48	+ 2.3	-0.01	3	Mädler 1; O. Struve 2
1843.42	7.0	5.7	0.43	0.47	+ 1.3	-0.04	4	Mädler 3; O. Struve 1
1844.26	6.6	7.0	0.51	0.47	- 0.4	+0.04	1	O. Struve
1846.19	11.5	10.1	0.44	0.46	+ 1.4	-0.02	4-3	Mädler 3-2; O. Struve 1
1847.41	14.4	12.0	0.33	0.45	+ 2.4	-0.12	3	Mädler 2; O. Struve 1
1848.40	10.4	13.8	0.35	0.45	- 3.4	-0.10	2	O. Struve
1850.39	15.0	17.2	0.33	0.43	- 2.2	-0.10	2	O. Struve
1851.40	20.4	19.1	0.32	0.43	+ 1.3	-0.11	6	Mädler 4; O. Struve 2
1852.40	22.9	20.9	0.29	0.42	+ 2.0	-0.13	6	O. Struve 2; Mädler 4
1853.40	16.7	22.9	0.34	0.41	- 6.2	-0.07	3	O. Struve
1854.32	24.6	24.7	0.41	0.41	- 0.1	± 0.00	2	Dawes 1; O. Struve 1
1857.34	30.6	31.3	0.30	0.38	- 0.7	-0.08	1	Secchi
1858.41	36.1	33.9	0.40	0.37	+ 2.2	+0.03	3	O. Struve
1859.38	40.8	36.2	0.34	0.36	+ 4.6	-0.02	3	Winnecke 1; O. Struve 2
1861.40	48.5	41.8	0.40	0.34	+ 6.7	+0.06	2-3	Winnecke 0-1; O. Struve 2
1862.39	46.8	44.6	0.38	0.33	+ 2.2	+0.05	1	O. Struve
1864.43	48.5	51.2	0.27	0.31	- 2.7	-0.04	1	O. Struve
1866.34	47.4	61.8	0.32	0.29	-14.4	+0.03	2	Englemann 1; O. Struve 1
1869.40	45.0	70.0	oblong	0.27	-25.0	—	2	Dunér
1870.42	81.5	75.6	oblong	0.26	+ 5.9	—	2	Dunér
1872.41	77.7	86.4	0.23	0.24	- 8.7	-0.01	2	O. Struve
1873.46	96.0	92.4	oblong	0.24	+ 3.6	—	4	O. Struve 3; H. Bruhns 1
1875.47	115.1	105.1	oblong	0.22	+10.0	—	2	O. Struve
1877.43	single	118.9	single	0.21	—	—	1	O. Struve
1879.44	single	134.7	single	0.21	—	—	1	O. Struve
1882.26	150.5	149.6	0.20	0.20	+ 0.9	± 0.00	4-3	Englemann 3; O. Struve 1-0
1887.43	218.9	206.6	0.23	0.19	+12.3	0.04	4	Schiaparelli
1888.43	220.3	216.1	eune.	0.19	+ 4.2	—	1	O. Struve
1889.39	214.0	225.2	elong.	0.19	-11.2	—	1	O. Struve
1892.13	250.8	248.3	0.21	0.21	+ 2.5	± 0.00	3-2	Burnham
1893.36	249.6	257.1	0.30	0.22	- 7.5	+0.08	1	Schiaparelli
1894.40	262.7	264.0	—	0.23	- 1.3	—	3-0	Bigourdan
1895.73	276.2	271.6	0.25	0.25	+ 4.6	± 0.00	3-1	See

Some changes will doubtless be required in all the elements, but the two elements of chief interest, the period and the eccentricity, will hardly be varied



by more than five years, and ± 0.03 respectively. It is desirable to have the theory of this system carefully confirmed, and observers with good telescopes will find it worthy of regular attention. The motion is still tolerably rapid, but is gradually slowing up, as will be seen in the following ephemeris:

t	θ_c	ρ_c	t	θ_c	ρ_c
1896.40	275.2	0.26	1899.40	288.8	0.29
1897.40	280.1	0.27	1900.40	292.7	0.30
1898.40	284.6	0.28			

ξ URSAE MAJORIS = Σ1523.

$\alpha = 11^h 12^m.9$; $\delta = +32^\circ 6'$.
4, yellow ; 5, yellowish.

Discovered by Sir William Herschel, May 2, 1780.

OBSERVATIONS.

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1781.97	143.8	4.±	—	Herschel	1833.14	189.9	2.06	8-2	Herschel
1802.09	97.5	—	—	Herschel	1833.23	189.8	1.98	4	Dawes
1804.09	92.6	—	—	Herschel	1833.38	188.2	1.69	5	Struve
1819.10	284.5	—	2	Struve	1834.44	184.1	1.87	2	Struve
1820.13	276.3	—	3	Struve	1834.50	182.5	2.17	4-1	Mädler
1821.78	264.7	1.92	3	Struve	1835.27	176.1	1.93	1	Mädler
1825.22	244.5	2.44	6-4	South	1835.41	180.2	1.76	5	Struve
1826.20	238.7	1.77	3	Struve	1835.56	175.8	—	4	Mädler
1827.27	228.3	1.71	4	Struve	1836.28	171.4	1.92	1	Dawes
1828.37	224.0	2.01	2	Herschel	1836.28	172.7	1.94	7-2	Mädler
1829.02	219.0	2.00	1	Herschel	1836.44	171.2	1.97	4	Struve
1829.35	213.6	1.67	7	Struve	1837.47	165.3	1.93	3	Struve
1830.18	211.4	—	10±	Herschel	1838.43	160.4	2.26	9	Struve
1830.98	200.9	2.23	10±	Herschel	1839.47	157.9	1.89	—	Galle
1831.08	201.5	1.86	5	Bessel	1840.25	152.2	2.08	40-31 obs.	Kaiser
1831.23	201.1	1.93	6-4	Herschel	1840.29	150.8	2.44	6-4	Dawes
1831.34	201.9	1.98	17-4	Dawes	1840.40	153.6	2.28	6	O. Struve
1831.44	203.8	1.71	5	Struve	1840.44	—	2.29	—	W. Struve
1832.16	198.2	—	5	Herschel	1841.21	148.0	2.40	4-3	Dawes
1832.27	196.7	1.76	10-8	Dawes	1841.29	150.2	2.44	7-6	Mädler
1832.41	195.9	1.75	5	Struve	1841.40	147.5	2.23	6	O. Struve
					1842.24	147.0	2.41	4	Mädler
					1842.27	144.8	2.44	4	Dawes
					1842.40	147.5	2.34	4	O. Struve

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1843.28	142.2	2.48	7	Dawes	1854.35	116.3	2.90	15	Mädler
1843.38	143.7	2.37	4	Mädler	1854.36	115.9	2.96	3	Dawes
1843.48	141.9	2.71	9	Schlüter	1854.37	115.6	3.46	1	Luther
1844.31	140.4	2.15	3	O. Struve	1854.38	115.9	2.90	4	O. Struve
1844.34	141.0	2.60	11-10	Mädler	1854.51	116.6	3.06	5	Dembowski
1844.36	141.0	2.47	—	Liapunow	1855.09	116.6	—	12	Powell
1844.36	141.5	2.65	—	Döllen	1855.15	115.6	3.23	7	Dembowski
1845.46	138.1	2.51	2	O. Struve	1855.29	114.3	2.96	1	Secchi
1845.82	135.8	3.11	2	Jacob	1855.33	114.1	2.98	1	Winnecke
1846.37	137.2	2.56	1	O. Struve	1855.44	115.7	2.87	2	Mädler
1847.30	131.6	2.58	1	Dawes	1855.44	115.2	2.85	3	O. Struve
1847.38	132.0	2.71	10	Mädler	1856.05	114.2	—	6	Powell
1847.41	133.2	2.61	3	O. Struve	1856.18	111.9	3.12	3	Jacob
1848.13	129.5	2.70	1	Dawes	1856.26	113.9	3.13	4	Secchi
1848.19	129.3	2.94	3	Dawes	1856.33	114.1	2.99	3	Winnecke
1848.31	129.7	2.71	4	Mädler	1856.34	112.3	3.15	7	Dembowski
1848.41	130.0	2.66	5	O. Struve	1856.42	112.7	2.98	13	Mädler
1848.45	129.1	2.90	2	W.C. Bond	1856.82	110.9	2.99	2	Jacob
1849.30	126.6	3.01	5	Dawes	1857.36	109.7	3.11	2	Secchi
1849.37	127.6	2.78	4	O. Struve	1857.43	109.6	2.74	8	Mädler
1850.01	127.0	2.65	1	Johnson	1857.46	110.2	2.97	3	O. Struve
1850.30	124.2	3.37	2	Jacob	1858.00	108.1	2.90	4	Jacob
1850.39	124.1	2.68	4	O. Struve	1858.20	108.1	2.85	2	Morton
1850.85	124.6	2.85	2	Mädler	1858.20	108.1	3.10	6	Dembowski
1851.19	123.1	2.83	6-5	Fletcher	1858.39	108.9	2.97	3	O. Struve
1851.27	123.3	2.93	6	Mädler	1858.43	108.8	2.96	5	Mädler
1851.31	122.9	2.98	2	Dawes	1859.39	106.1	2.91	6-3	Mädler
1851.41	123.0	2.80	5	O. Struve	1859.57	104.9	2.81	5	O. Struve
1851.79	122.1	2.91	9	Mädler	1860.08	105.2	2.84	2	Morton
1852.13	122.3	2.90	7	Miller	1860.16	104.1	2.99	6-5	Powell
1852.20	119.8	2.92	6	Fletcher	1860.32	105.2	2.88	2-1	Dawes
1852.29	120.9	3.01	1	Jacob	1860.36	102.8	—	—	Oblomievsky
1852.34	120.8	2.73	6	Mädler	1860.36	103.6	—	—	Schiaparelli
1852.36	118.2	2.85	2	Morton	1860.36	103.9	—	—	Wagner
1852.38	120.0	—	1	Dawes	1860.39	104.1	3.15	2	Mädler
1852.40	120.6	2.76	4	O. Struve	1861.14	100.6	3.09	6-2	Powell
1853.19	118.8	3.01	4	Miller	1861.40	101.1	2.70	4	O. Struve
1853.20	119.5	3.01	2	Jacob	1861.42	100.8	2.83	8	Mädler
1853.20	119.2	—	6	Powell	1861.76	100.4	3.01	5	Auwers
1853.23	118.9	2.98	6	Fletcher	1862.36	100.1	2.95	4	Mädler
1853.40	119.0	2.88	4	O. Struve	1862.39	99.3	2.62	4	O. Struve
1853.45	118.8	2.94	13	Mädler	1862.42	100.2	3.20	—	Oblomievsky
1854.12	117.2	3.1	10-1	Powell	1863.20	89.5	2.61	2	Main
					1863.23	96.6	2.55	19	Dembowski
					1863.46	95.7	2.55	2	O. Struve

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1864.31	94.0	2.29	9	Dembowski	1873.28	2.2	0.9	2-1	W. & S.
1864.38	92.9	2.40	3	Secchi	1873.33	358.9	0.98	10	Dembowski
1864.42	94.2	2.33	3	O. Struve	1873.42	358.4	0.88	1	Dunér
1864.46	92.8	2.44	1	Englemann	1873.43	358.4	0.96	5	O. Struve
1864.50	93.9	2.42	1	Dawes	1873.78	347.1	0.83	3	Gledhill
1865.12	91.4	2.44	19	Englemann	1874.13	338.4	1.00	3	Gledhill
1865.30	90.1	2.17	10	Dembowski	1874.20	336.2	0.92	2-1	W. & S.
1865.51	89.9	2.53	4	Secchi	1874.21	337.0	1.48	1	Ferrari
1866.25	92.8	2.72	4-3	Leyton Obs.	1874.26	335.5	—	2	Leyton Obs.
1866.30	86.5	2.26	3	Secchi	1874.35	333.6	1.02	6	Dembowski
1866.30	86.8	2.05	10	Dembowski	1874.41	338.1	1.03	3	O. Struve
1866.39	86.7	2.09	5	Kaiser	1874.45	335.1	0.96	4-5	Dunér
1866.40	85.4	2.12	3	O. Struve	1875.27	317.6	1.09	8	Dembowski
1866.45	87.8	2.08	5	Kaiser	1875.31	317.5	1.31	7	Schiaparelli
1866.49	81.1	—	2	Guldén	1875.34	317.2	1.28	4-3	W. & S.
1866.49	83.6	—	2	Abbe	1875.45	315.8	1.10	4	O. Struve
1866.49	87.0	—	2	Foss	1875.45	316.4	1.12	14	Dunér
1867.21	75.5	2.89	1	Winlock	1875.99	311.7	—	1	Doberck
1867.23	82.2	—	1	Leyton Obs.	1876.27	306.3	1.75	13-2	Doberck
1867.31	82.2	1.90	8	Dembowski	1876.30	304.8	1.24	7	Dembowski
1867.47	81.0	1.91	2	O. Struve	1876.34	334.5	1.65	1	Leyton
1868.14	80.8	1.76	1	Searle	1876.36	305.5	1.45	3	W. & S.
1868.23	79.1	2.49	2	Leyton Obs.	1876.42	303.5	1.35	3	O. Struve
1868.30	77.1	1.72	8	Dembowski	1876.46	301.2	1.52	5-4	Plummer
1868.39	77.1	1.77	1	Main	1877.20	297.0	1.57	7-6	Plummer
1868.42	72.6	1.63	4	O. Struve	1877.26	294.9	1.42	6	Dembowski
1869.40	68.6	1.34	11	Dunér	1877.26	294.2	1.76	10-9	Doberck
1869.42	69.9	—	—	Krüger	1877.34	293.0	1.52	8	Schiaparelli
1870.18	59.2	1.32	4	O. Struve	1877.40	294.6	1.52	3	W. & S.
1870.24	57.3	1.39	9	Dembowski	1877.43	291.6	1.45	2	O. Struve
1870.33	57.2	1.35	2	Gledhill	1877.	291.5	1.35	1	Pritchett
1870.35	70.8	—	—	Leyton Obs.	1877.41	294.5	2.10	2-1	Hall
1870.43	53.8	1.20	9	Dunér	1878.20	—	2.01	4	Doberck
1871.22	47.7	1.20	8	Dembowski	1878.32	286.8	1.66	6	Dembowski
1871.31	47.7	1.2	2	Gledhill	1878.36	286.3	1.50	3	O. Struve
1871.39	66.2	—	—	Leyton Obs.	1879.27	284.2	1.82	3	Hall
1871.40	45.7	1.12	2	O. Struve	1879.33	280.3	1.79	7	Schiaparelli
1871.47	40.0	1.02	11-10	Dunér	1879.41	278.5	1.74	2	O. Struve
1871.48	43.9	1.1	1	Wilson	1880.13	278.2	2.07	6	Franz
1872.05	30.7	1.05	2	Gledhill	1880.27	276.2	1.80	6	Hall
1872.26	23.2	1.09	7-6	W. & S.	1880.28	274.9	2.05	5	Doberck
1872.33	19.3	1.07	6	Knott	1880.39	273.0	1.90	2	Bigourdan
1872.35	68.0	1.28	1-2	Leyton Obs.	1880.48	272.0	1.82	3	Jedrzejewicz
1872.41	17.8	0.97	10	Dembowski	1881.23	270.3	1.84	4	Doberck
1872.46	16.6	0.94	14	Dunér	1881.31	268.0	1.80	2-1	Bigourdan
1872.48	15.4	0.98	8	Ferrari					

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1881.34	269.2	1.84	7	Hall	1889.37	216.9	1.81	3	Maw
1881.35	269.7	1.66	4-3	Burnham	1889.39	218.5	1.61	2	O. Struve
1881.36	268.9	1.92	6	Schiaparelli	1889.40	217.4	1.68	5	Tarrant
1882.25	263.5	1.99	6	Hall	1890.27	210.0	1.61	6	Hall
1882.25	259.4	2.00	4-3	Doberek	1890.36	209.7	1.61	7	Schiaparelli
1882.25	262.1	1.99	4	Englemann	1890.40	209.1	1.96	3	Maw
1882.39	261.1	1.93	9	Schiaparelli	1890.42	313.3	1.51	1	Hayn
1882.42	260.4	1.72	3	O. Struve	1890.45	209.4	1.87	2	Knorre
1883.32	257.8	2.00	6	Englemann	1891.13	202.6	1.78	1	Bigourdan
1883.38	257.1	1.88	11	Schiaparelli	1891.15	202.1	1.63	1	Flint
1883.40	258.2	1.95	6	Hall	1891.30	200.6	1.59	6	Hall
1883.41	258.1	1.88	3	Jedrzejewicz	1891.31	204.1	1.92	1	Knorre
1884.28	249.2	1.69	3-1	Perrotin	1891.41	199.8	1.60	10	Schiaparelli
1884.32	249.0	1.89	7	Hall	1891.47	199.9	1.71	3	Maw
1884.35	247.6	—	14	Bigourdan	1892.32	196.9	1.75	4	Maw
1884.38	249.3	1.82	11	Schiaparelli	1892.35	195.1	1.57	11-10	Schiaparelli
1884.41	249.6	1.92	1	Englemann	1892.36	194.1	1.78	1	Bigourdan
1884.44	249.2	1.56	1	O. Struve	1892.39	197.4	1.70	6	Knorre
1885.35	244.7	1.80	5	Hall	1892.45	196.6	1.60	2	Leavenworth
1885.36	245.2	2.12	4	Englemann	1892.46	197.5	1.57	4	Comstock
1885.39	245.4	1.72	10	Schiaparelli	1893.27	188.0	2.05	2	Knorre
1885.41	243.4	1.87	3	Tarrant	1893.33	187.3	1.72	1	Maw
1886.37	237.3	1.63	5	Hall	1893.36	186.4	1.65	7	Schiaparelli
1886.37	237.4	2.06	8	Englemann	1893.37	186.1	1.75	1	Dav. Photog.
1886.45	237.0	1.80	3	Jedrzejewicz	1894.22	183.2	1.79	3	Comstock
1887.01	226.9	—	1	Glasenapp	1894.30	181.1	2.00	1	Ebell
1887.35	230.3	1.61	5	Hall	1894.32	182.8	1.79	1	H. C. Wilson
1887.36	230.9	1.65	12	Schiaparelli	1894.34	183.6	1.84	2	Knorre
1888.28	222.2	1.68	6	Hall	1894.35	183.0	1.87	3	Maw
1888.29	222.7	1.63	4	Schiaparelli	1894.47	181.7	1.78	8	Bigourdan
1888.43	226.2	1.61	1	O. Struve	1894.56	184.6	1.77	1	Glasenapp
1888.51	222.7	2.20	1	Maw	1895.30	176.5	1.93	3	Comstock
1889.28	218.1	2.09	2-1	Glasenapp	1895.31	176.0	1.78	1	Dav. Photog.
1889.29	216.5	1.68	5	Hall	1895.32	176.0	1.98	1	Lewis
1889.36	215.9	1.61	9	Schiaparelli	1895.33	176.6	1.95	3	See
					1895.46	175.9	1.79	4	Schwarzschild

This celebrated system was first measured by HERSCHEL in 1781. A repetition of the measures in 1802 and 1804 showed* that the smaller star had a rapid relative motion (*Phil. Trans.* 1804, p. 363), and indeed gave indications for the first time that the motion of certain double stars is of an orbital nature. ξ *Ursae Majoris* thus enjoys the unique distinction of having first aroused interest in observational proof of the universality of the Newtonian law. This

* *Astronomische Nachrichten*, 3323.

star also led SAVARY in 1827 to derive a method for finding the orbit of a double star on gravitational principles, and the first orbit ever computed appeared in the *Connaissance des Temps* for 1830. When SAVARY's method for finding double-star orbits had been successfully applied to ξ *Ursae Majoris*, the subject was taken up by ENCKE and HERSCHEL, who published methods of superior elegance and of greater practical utility, with the result that numerous orbits were soon computed.

The rapid orbital motion of ξ *Ursae Majoris* insured it ample attention, and accordingly since the time of SIR JOHN HERSCHEL and STRUVE, measures have been secured annually by the best observers. The number of orbits computed for this star is very large; the following list is fairly complete:

<i>P</i>	<i>T</i>	<i>e</i>	<i>a</i>	Ω	<i>i</i>	λ	Authority	Source
yr.			"	°	°	°		
58.2625	1817.25	0.4164	3.857	95.37	59.67	131.63	Savary, 1828	Conn. des Temps, 1830
60.72	1816.73	0.3777	3.278	97.78	56.1	134.37	Herschel, 1832	Mem. R.A.S. V, p. 209
60.4596	1816.95	0.40368	2.290	95.0	52.27	129.68	Mädler, 1836	A.N. 319
61.464	1816.44	0.4135	2.417	98.87	54.93	130.8	Mädler, 1843	A.N. 486
61.30	1817.102	0.4037	2.295	96.35	50.92	132.47	Mädler, 1847	Fixt.-Syst. I, p. 233
61.175	1816.66	0.4116	2.82	96.1	53.87	129.47	Jacob, 1846	Mem. R.A.S. XVI, p. 322
61.576	1816.86	0.4315	2.439	95.83	52.82	128.57	Villargeau 1849	A.N. 680
63.14	1816.32	0.3929	2.454	97.3	52.27	132.88	Breen, 1862	M.N. XXII, p. 158
59.88	1816.405	0.3786	2.591	103.6	53.1	135.3	Ball, 1872	Proc. R.I.A., June, 1872
60.679	1815.008	0.3830	2.587	100.7	56.33	127.15	Knott, 1873	M.N. XXXIII, p. 101
60.63	1875.50	0.371	2.535	101.0	55.0	216.0	Flam., 1873	Cat. des Ét. Doub. p. 65
60.79	1875.29	0.3952	2.549	101.5	56.9	234.3	Dunér, 1876	Meas. Micr., p. 196
60.80	1875.26	0.4159	2.580	100.22	56.67	235.0	Pritchard, 1878	Oxford Obs., No. 1
60.50	1814.8	0.410	2.55	280.7	122.9	305.8	Birk., 1879	K. Akad. Wiss. Wien, Bd. 93

It will be seen that among the more recent orbits there is no wide range of values, and yet the elements are by no means identical. The different results depend upon the observations used and the method of computation employed.

From an investigation of all the observations, I am led to the following elements:

$$\begin{aligned}
 P &= 60.00 \text{ years} & \Omega &= 100^\circ.8 \\
 T &= 1875.22 & i &= 55^\circ.92 \\
 e &= 0.397 & \lambda &= 126^\circ.33 \\
 a &= 2''.508 & n &= -6^\circ.0000
 \end{aligned}$$

Apparent orbit:

$$\begin{aligned}
 \text{Length of major axis} &= 4''.76 \\
 \text{Length of minor axis} &= 2''.70 \\
 \text{Angle of major axis} &= 104^\circ.6 \\
 \text{Angle of periastron} &= 318^\circ.0 \\
 \text{Distance of star from centre} &= 0''.75
 \end{aligned}$$

The following table of computed and observed places shows that these elements are extremely satisfactory.

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1781.97	143.8	148.4	4 $\pm$	2.34	-4.6	+1.66 $\pm$	1	Herschel
1802.09	97.5	99.0	—	2.70	-1.5	—	1	Herschel
1804.09	92.6	93.3	—	2.47	-0.7	—	1	Herschel
1819.10	284.5	282.1	—	1.69	+2.4	—	2	Struve
1820.13	276.3	274.0	—	1.79	+2.3	—	3	Struve
1821.78	264.7	264.5	1.92	1.84	+0.2	+0.08	3	Struve
1823.29	258.4	255.8	2.81	1.83	+2.6	+0.98	58-20	Herschel and South
1825.22	244.5	244.5	2.44	1.78	± 0.0	+0.66	7-4	South
1826.20	238.7	238.4	1.77	1.75	+0.3	+0.02	3	Struve
1827.27	228.3	231.6	1.71	1.72	-3.3	-0.01	4	Struve
1828.37	224.0	224.3	2.01	1.69	-0.3	+0.32	2	Herschel
1829.35	213.6	217.7	1.67	1.67	-4.1	± 0.00	7	Struve
1830.58	206.1	209.3	2.23	1.67	-3.2	+0.56	10 $\pm$	Herschel
1831.28	202.4	204.5	1.85	1.68	-2.1	+0.17	27-14	Bessel 5; Dawes 17-4; W. Struve 5
1832.34	196.3	197.3	1.76	1.69	-1.0	+0.07	15-13	Dawes 10-8; W. Struve 5
1833.30	189.0	191.0	1.83	1.72	-2.0	+0.11	9	Dawes 4; W. Struve 5
1834.47	183.3	183.7	1.87	1.78	-0.4	+0.09	6-2	W. Struve 2; Mädler 4-0
1835.34	178.3	178.7	1.84	1.82	-0.4	+0.02	6	Mädler 1; W. Struve 5
1836.33	171.7	173.1	1.94	1.89	-1.4	+0.05	12-7	Dawes 1; Mädler 7-2; W. Struve 4
1837.47	165.3	167.2	1.93	1.97	-1.9	-0.04	3	Struve
1838.43	160.4	162.7	2.26	2.05	-2.3	+0.21	9	Struve
1839.47	157.9	157.4	1.89	2.14	+0.5	-0.25	—	Galle
1840.34	152.2	154.5	2.36	2.20	-2.3	+0.16	12-10	Dawes 6-4; O. Struve 6
1841.30	148.6	150.2	2.36	2.29	-1.6	+0.07	17-15	Dawes 4-3; Mädler 7-6; O. Struve 6
1842.30	146.4	147.3	2.40	2.37	-0.9	+0.03	12	Mädler 4; Dawes 4; O. Struve 4
1843.33	143.0	143.9	2.42	2.45	-0.9	-0.03	11	Dawes 7; Mädler 4
1844.34	140.7	140.7	2.52	2.54	± 0.0	-0.02	14-13	O. Struve 3; Mädler 11-10
1845.74	136.9	136.6	2.81	2.65	+0.3	+0.16	4	O. Struve 2; Jacob 2
1846.37	137.2	134.9	2.56	2.69	+2.3	-0.13	4	O. Struve
1847.36	132.3	132.3	2.63	2.76	± 0.0	-0.13	14	Dawes 1; Mädler 10; O. Struve 3
1848.30	129.5	130.0	2.78	2.82	-0.5	-0.04	15	Dawes 1; Dawes 3; Mädler 4; O. Struve 5; Bond 2
1849.33	127.1	127.3	2.89	2.87	-0.2	+0.02	9	Dawes 5; O. Struve 4
1850.51	124.3	121.8	2.96	2.94	-0.5	+0.02	8	Jacob 2; O. Struve 4; Mädler 2
1851.39	122.9	122.9	2.89	2.97	± 0.0	-0.08	28-27	Flt. 6-5; Mädler 6; Dawes 2; O. Struve 5; Mädler 9
1852.30	120.3	120.9	2.84	3.00	-0.6	-0.16	27-26	Miller 7; Flt. 6; Jacob 1; Mä. 6; Mo. 2; Da. 1-0; OΣ. 4
1853.24	119.0	118.9	2.96	3.02	+0.1	-0.06	35-29	Miller 4; Jacob 2; Powell 6-0; Fl. 6; OΣ. 4; Mä. 13
1854.34	116.4	116.5	2.98	3.03	-0.1	-0.05	37-28	Powell 10-1; Mädler 15; Dawes 3; O. Struve 4; Dem. 5
1855.33	115.2	114.5	2.98	3.03	+0.7	-0.05	13	Dembowski 7; Sec. 1; Mädler 2; O. Struve 3
1856.45	112.4	112.1	3.07	3.02	+0.3	+0.05	29	Jacob 3; Sec. 4; Dembowski 7; Mädler 13; Jacob 2
1857.42	109.8	110.0	2.94	3.00	-0.2	-0.06	13	Sec. 2; Mädler 8; O. Struve 3
1858.24	108.4	108.3	2.96	2.97	+0.1	-0.01	20	Jacob 4; Morton 2; Dembowski 6; O. Struve 3; Mä. 5
1859.48	105.5	105.4	2.87	2.91	+0.1	-0.04	11-8	Mädler 6-3; O. Struve 5
1860.24	104.6	103.6	2.96	2.86	+1.0	+0.10	12-10	Morton 2; Powell 6-5; Dawes 2-1; Mädler 2
1861.32	100.8	101.0	2.87	2.77	-0.2	+0.10	18-14	Powell 6-2; O. Struve 4; Mädler 8
1862.38	99.7	98.2	2.78	2.67	+1.5	+0.11	8	Mädler 4; O. Struve 4;
1863.34	96.7	95.6	2.55	2.56	+1.1	-0.01	21	Dembowski 19; O. Struve 2
1864.40	93.7	92.2	2.36	2.42	+1.5	-0.06	16	Dembowski 9; Sec. 3; O. Struve 3; Dawes 1
1865.31	90.5	89.0	2.37	2.27	+1.5	+0.10	33	Englemann 19; Dembowski 10; Sec. 4
1866.33	86.2	85.5	2.14	2.13	+0.7	+0.01	16	Sec. 3; Dembowski 10; O. Struve 3
1867.39	81.6	79.5	1.91	1.89	+2.1	+0.02	11	Dembowski 8; O. Struve 2
1868.28	76.8	75.0	1.70	1.73	+1.8	-0.03	13	Searle 1; Dembowski 8; O. Struve 4
1869.40	68.6	65.3	1.34	1.45	+3.3	-0.11	11	Dunér



ξ Ursae Majoris = Σ 1523.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
	$^{\circ}$	$^{\circ}$	$''$	$''$	$^{\circ}$	$''$		
1870.19	56.9	57.0	1.32	1.27	-0.1	+0.05	24	O. Struve 4; Dembowski 9; Gledhill 3; Dunér 9
1871.35	45.0	40.3	1.13	1.05	+4.7	+0.08	24-23	Dem. 8; Gl. 2; O. Struve 2; Dunér 11-10; Wilson 1
1872.36	20.4	19.5	1.01	0.92	+0.9	+0.09	47-46	Gl. 2; W. & S. 7-6; Kn. 6; Dem. 10; Du. 14; Fer. 8
1873.36	359.2	355.4	0.93	0.90	+3.8	+0.03	18-17	W. & S. 2-1; Dembowski 10; Dunér 1; O. Struve 5
1874.29	336.4	334.5	0.99	0.98	+1.9	+0.01	19-18	Gl. 3; W. & S. 2-1; Fer. 1-0; Dem. 6; OZ. 3; Du. 4-5
1875.47	316.1	314.4	1.20	1.18	+1.7	+0.02	34-32	Dem. 8; Sch. 7; W. & S. 4-3; Dunér 14; Doberck. 1-0
1876.35	304.3	303.5	1.34	1.33	+0.8	+0.01	28-10	Doberck 13-2; Dem. 7; W. & S. 3; Plummer 5-0
1877.31	294.7	294.0	1.52	1.39	+0.7	+0.13	36-33	Pl. 7-6; Dem. 6; Dk. 10-9; Sch. 8; W. & S. 3; Ill. 2-0
1878.32	286.8	286.5	1.66	1.62	+0.3	+0.04	6	Dembowski
1879.30	282.2	279.3	1.80	1.73	+2.9	+0.07	10	Hall 3; Schiaparelli 7
1880.31	274.8	273.1	1.83	1.80	+1.7	+0.03	22-11	Franz 6-0; Hall 6; Doberck 5-0; Bigourdan 2; Jed. 3
1881.32	269.2	267.1	1.82	1.83	+2.1	-0.01	23-21	Doberck 4; Bigourdan 2-1; Hall 7; β . 4-3; Sch. 6
1882.28	261.5	261.7	1.97	1.84	-0.2	+0.13	23-19	Hall 6; Doberck 4-0; Englemann 4; Schiaparelli 9
1883.38	257.8	255.3	1.90	1.82	+2.5	+0.08	26-20	Englemann 6-0; Schiaparelli 11; Hall 6; Jedrzejewicz 3
1884.35	248.9	248.8	1.83	1.80	+0.1	+0.03	39-26	Perrotin 3-4; Hall 7; Bigourdan 14-0; Sch. 11; En. 4
1885.38	244.5	243.5	1.80	1.77	+1.0	+0.03	18	Hall 5; Schiaparelli 10; Tarrant 3
1886.39	237.2	237.1	1.71	1.74	+0.1	-0.03	16-8	Hall 5; Englemann 8-0; Jedrzejewicz 3
1887.35	230.6	231.5	1.63	1.72	-0.9	-0.09	17	Hall 5; Schiaparelli 12
1888.36	222.5	224.4	1.65	1.69	-1.9	-0.04	14-10	Hall 6; Schiaparelli 4; Maw 4-0
1889.32	216.8	217.9	1.70	1.68	-1.1	+0.02	19-17	Glaserapp 2-0; Hall 5; Schiaparelli 9; Maw 3
1890.37	209.5	210.7	1.77	1.67	-1.2	+0.10	18	Hall 6; Schiaparelli 7; Maw 3; Knorre 2
1891.30	201.5	204.3	1.71	1.68	-2.8	+0.03	22	Big. 1; Flint 1; Hall 6; Knorre 1; Sch. 10; Maw 3
1892.39	196.3	197.3	1.66	1.69	-1.0	-0.03	28-17	Maw 4; Sch. 11-10; Big. 1; Knorre 6; Lv. 2; Com. 4
1893.33	188.0	191.0	1.71	1.72	-3.0	-0.01	14-12	Knorre 2-0; Maw 4; Schiaparelli 7; Davidson 1
1894.38	182.9	184.3	1.81	1.77	-1.4	+0.04	17	Com. 3; H.C.W. 1; Knorre 2; Maw 3; Big. 8; Glas. 1
1895.32	176.2	178.5	1.90	1.83	-2.3	+0.07	5	Davidson 1; Lewis 1; See 3

Future observations are likely to produce only very slight alterations in the above values. Thus the period is not likely to be in error by more than one-tenth of a year, and the error in the eccentricity can hardly surpass ± 0.005 . Indeed the orbit ξ *Ursae Majoris* is practically all that can be desired in the present state of double-star measurement. In order to effect any further improvement of the orbit, astronomers will need to take every precaution against systematic errors; and rough measures by inexperienced observers are unlikely to prove to be of any considerable value.

We remark, however, that continued observation of this star is desirable, because the micrometrical measures of skilled observers will be valuable in throwing light upon the question of the existence of dark bodies or other disturbing influences, and in proving with all possible experimental accuracy that the force which retains the companion in its orbit is directed exactly towards the central star.

ξ *Ursae Majoris*, like ζ *Herculis*, has a large proper motion in space, and this circumstance in connection with the brilliancy of the components, conduces to the belief that the system is comparatively near the earth. Measurement for parallax has never been attempted, but if suitable comparison stars could be found, effort in this direction would be likely to prove successful.

OY 234.

$\alpha = 11^h 25^m.4$; $\delta = +41^\circ 50'$.
7, yellowish ; 7.8, yellowish.

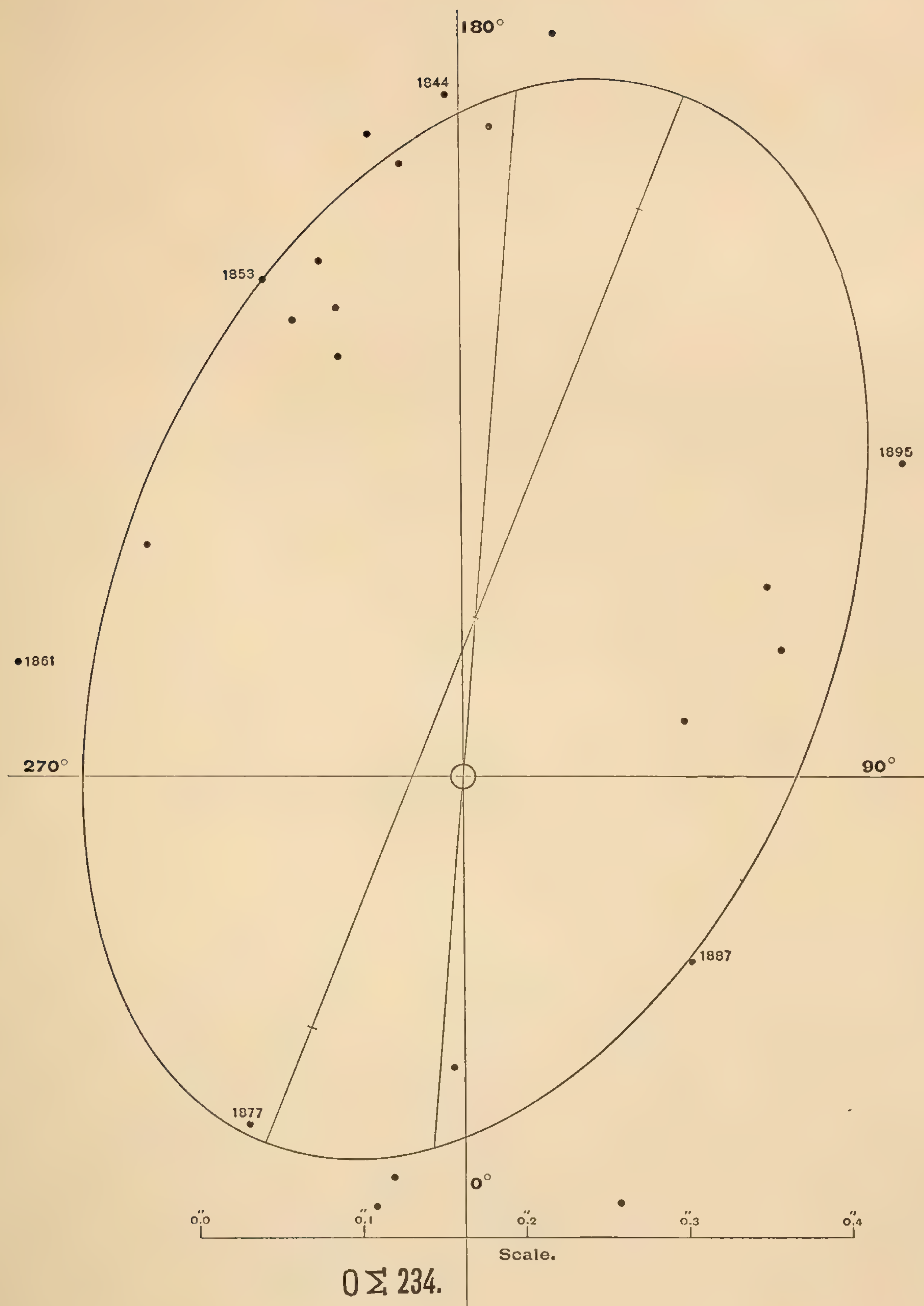
Discovered by Otto Struve in 1843.

OBSERVATIONS.

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1843.29	182.5	0.42	1	O. Struve	1870.46	281.8	cert. obl.	1	O. Struve
1843.33	179.6	0.25	—	Mädler	1877.26	127.3	0.25	2	Dembowski
1844.31	172.7	0.46	1	O. Struve	1877.32	cuneiforme sous 349°		1	O. Struve
1845.42	194.6	0.30	2	Mädler	1878.28	168.4	0.27	2-1	Burnham
1846.37	177.2	0.40	1	O. Struve	1880.37	178.4	0.18	1	Burnham
1847.40	187.2	0.25	1	Mädler	1882.	130.	<0.3	3	Englemann
1847.41	183.7	0.38	1	O. Struve	1883.	350.	<0.25	3	Englemann
1848.25	187.9	0.40	1	O. Struve	1884.10	20.	0.28	1	Englemann
1850.31	195.2	0.33	1	O. Struve	1887.42	231.2	0.18	6	Schiaparelli
1851.36	200.4	0.3	1	Mädler	1889.39	cuneiforme sous 98°		1	O. Struve
1851.42	199.3	0.30	2	O. Struve	1891.23	104.2	0.14	3	Burnham
1852.46	196.	0.27	1	O. Struve	1892.28	114.2	0.18	3	Burnham
1853.41	201.3	0.33	1	O. Struve	1892.39	107.0	0.24	2-1	Bigourdan
1858.36	cert. elong. in 244°		1	O. Struve	1892.40	293.6	0.22	1	Schiaparelli
1859.40	233.	0.24	1	O. Struve	1894.29	123.2	$0.22 \pm$	2	Comstock
1861.26	255.0	0.28	2-1	O. Struve	1894.84	121.7	0.21	3	Barnard
1862.39	260.	oblong	1	O. Struve	1895.20	122.2	$0.30 \pm$	1	Comstock
1866.20	single	—	1	Dembowski	1895.75	125.1	0.36	1	See
1866.49	oblong in 283°		1	O. Struve					

Since the discovery of this pair by OTTO STRUVE, the companion has described an arc of 305° . The object is always close and difficult, and hence the measures are by no means so good as could be desired; yet when account is taken of both angles and distances, there is reason to believe that elements based on the observations now available will never be greatly changed. MR. GORE is the only computer who has previously investigated the orbit of this pair; using the measures prior to 1886, he found the following elements:

$$\begin{aligned}
 P &= 63.45 \text{ years} & \Omega &= 124^\circ.2 \\
 T &= 1881.15 & i &= 47^\circ.35 \\
 e &= 0.3629 & \lambda &= 71^\circ.97 \\
 a &= 0''.339
 \end{aligned}$$



We find the following orbit of OY234:

$P = 77.0$ years	$\Omega = 157^\circ.5$
$T = 1880.10$	$i = 50^\circ.6$
$e = 0.302$	$\lambda = 206^\circ.6$
$a = 0''.3467$	$n = +4^\circ.6754$

Apparent orbit:

Length of major axis	$= 0''.695$
Length of minor axis	$= 0''.437$
Angle of major axis	$= 158^\circ.0$
Angle of periastron	$= 355^\circ.2$
Distance of star from center	$= 0''.098$

The accompanying table shows that these elements are very satisfactory; the period is perhaps uncertain by five years, and the eccentricity by perhaps ± 0.04 . Larger variations in these elements are not to be anticipated. It is probably worth noting that BURNHAM'S distance in 1891 is sensibly smaller than the computed distance, although the angle agrees perfectly. By this we are not to infer that he under-measured the distance with the great Refractor of the Lick Observatory, but that all small distances with a great Telescope appear diminished in comparison with their magnitude in a small instrument—a phenomenon due mainly to the diminution of the spurious discs under the superior separating power of great Telescopes. The computer must therefore take account of the inequality of the distances due to the different power of the Telescopes employed; but as most of the observations of OY234 were made with instruments of about 15-inch aperture, I preferred to make the scale of the major axis such, that on the whole the computed would agree with the observed distances.

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
	$^\circ$	$^\circ$	$''$	$''$	$^\circ$	$''$		
1843.31	181.0	178.1	0.42	0.41	+ 2.9	+0.01	2-1	OY.1; Mädler 1-0
1844.31	172.7	180.1	0.46	0.41	- 7.4	+0.05	1	O. Struve
1845.42	194.6	182.3	0.30	0.40	+12.3	-0.10	2	Mädler
1846.37	177.2	184.2	0.40	0.39	- 7.0	+0.01	1	O. Struve
1847.40	185.4	186.6	0.38	0.38	- 1.2	± 0.00	2-1	Mädler 1-0; OY.1
1848.25	187.9	188.5	0.40	0.38	- 0.6	+0.02	1	O. Struve
1850.31	195.2	193.7	0.33	0.36	+ 1.5	-0.03	1	O. Struve
1851.39	199.8	196.6	0.30	0.35	+ 3.2	-0.05	3	Mädler 1; OY.2
1852.46	196.	199.3	0.27	0.34	- 3.3	-0.07	1	O. Struve
1853.41	201.3	202.7	0.33	0.33	- 1.4	± 0.00	1	O. Struve
1858.36	244.	222.1	cert. elong.	0.27	+21.9	-	1	O. Struve
1859.40	233.	227.0	0.24	0.26	+ 6.0	-0.02	1	O. Struve
1861.26	255.0	237.0	0.28	0.25	+18.0	+0.03	2-1	O. Struve
1862.39	260.	243.8	oblong	0.24	+16.2	-	1	O. Struve
1866.49	283.	271.3	oblong	0.24	+11.7	-	1	O. Struve

<i>t</i>	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	<i>n</i>	Observers
1870.46	281.8	297.5	[°] cert. oblong 0.24	0.24	— 15.7	—	1	O. Struve
1877.29	328.1	337.3	0.25	0.25	— 9.2	±0.00	3	Dembowski 2; OΣ.1
1878.28	348.4	343.0	0.27	0.25	+ 5.4	+0.02	2-1	Burnham
1880.37	358.4	375.5	0.18	0.23	+ 0.9	—0.05	1	Burnham
1883.	350.	18.7	<0.25	0.20	—28.7	+0.05	3	Englemann
1884.10	20.	30.2	0.28	0.19	—10.2	+0.09	1	Englemann
1887.42	51.2	68.5	0.18	0.18	—17.3	±0.00	6	Schiaparelli
1889.39	98.	89.8	cune.	0.20	+ 8.2	—	1	O. Struve
1891.23	104.2	104.4	0.14	0.23	— 0.2	—0.09	3	Burnham
1892.36	111.6	111.5	0.21	0.25	+ 0.1	—0.04	6-5	Big. 2-1; β. 3; Sch.1
1894.56	121.7	122.6	0.22	0.29	— 0.9	—0.07	3-5	Comstock 2; Barnard 3
1895.20	125.1	125.2	0.33	0.30	— 0.1	+0.03	1-2	Comstock 0-1; See 1

The observation of this star which I made at Madison, is discordant in angle (*A.J.* 359), and hence I am led to think that an error of 30° occurred in reading the circle; the unreduced reading was 94°.3, whereas it doubtless should read 64°.3. As the angle was estimated at 130°, this correction is amply justified.

If good observations can be secured for the next decade, this orbit can be rendered very exact. The following ephemeris will be useful to observers:

<i>t</i>	θ_c	ρ_c	<i>t</i>	θ_c	ρ_c
1896.40	127.0	0.31	1899.40	136.8	0.36
1897.40	130.4	0.33	1900.40	139.5	0.37
1898.40	133.7	0.34			

OΣ235.

$\alpha = 11^h 26^m.7$; $\delta = +61^\circ 38'$.
6, yellowish ; 7.8, yellowish.

Discovered by Otto Struve in 1843.

OBSERVATIONS.

<i>t</i>	θ_o	ρ_o	<i>n</i>	Observers	<i>t</i>	θ_o	ρ_o	<i>n</i>	Observers
1844.33	289.3	0.67	1	O. Struve	1852.46	329.5	0.57	1	O. Struve
1845.47	296.7	0.54	1	O. Struve	1853.41	333.5	0.54	1	O. Struve
1846.42	306.8	0.57	1	O. Struve	1855.47	345.6	0.51	1	O. Struve
1847.45	315.8	0.53	1	O. Struve	1856.55	350.3	0.52	1	O. Struve
1849.47	320.8	0.49	1	O. Struve	1857.51	350.4	0.55	1	O. Struve
1850.31	316.5	0.56	1	O. Struve	1858.44	358.7	0.75	1	O. Struve
1851.42	328.0	0.54	2	O. Struve	1859.41	358.7	0.62	1	O. Struve

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1861.42	13.3	0.65	2	O. Struve	1879.44	55.5	1.07	3	Hall
1862.38	20.3	0.76	1	O. Struve	1882.59	64.8	1.26	6	Englemann
1864.43	25.3	0.80	1	O. Struve	1887.43	73.0	0.93	5-3	Schiaparelli
1866.49	33.3	0.83	1	O. Struve	1888.43	69.4	1.12	1	O. Struve
1867.45	40.1	separated	1	Dembowski	1888.69	72.6	1.32	4	Tarrant
1868.13	31.0	0.84	1	Dembowski	1889.35	76.9	1.07	5	Hall
1870.18	42.6	0.9	1	Dembowski	1889.39	67.3	0.90	1	O. Struve
1870.46	37.4	0.98	1	O. Struve	1891.29	81.7	1.04	1	Bigourdan
1872.40	42.0	0.8	1	Dembowski	1892.12	84.3	0.97	3	Burnham
1872.60	43.1	1.00	1	O. Struve	1892.44	88.1	1.03	1	Bigourdan
1876.63	51.0	0.95	1	O. Struve	1892.45	85.4	0.80	2	Lv.
1877.26	55.5	1.07	2	Dembowski	1892.54	84.2	0.94	3-2	Comstock
1877.32	54.7	1.04	1	O. Struve	1893.37	90.2	0.92	1	Comstock
1878.35	58.1	1.18	4	Dembowski	1893.41	86.6	0.85	6-9	Bigourdan
1879.44	58.2	0.76	1	O. Struve	1894.24	90.1	0.75	3	Comstock
					1895.27	93.9	0.79	3	Comstock
					1895.74	97.3	0.81	2	See

For a number of years after the discovery of this pair, OTTO STRUVE alone noted the position of the companion, but as his measures soon established the rapid motion of the system, DEMBOWSKI, HALL, SCHIAPARELLI, and other subsequent observers have contributed to the material now available for the investigation of the orbit.

The observations are not very numerous, but for an object of this difficulty, they are comparatively good.

The are described by the companion since 1844 is only 166° , and yet the motion around the apastron of the apparent orbit defines the elements with considerable precision. DOBERCK is the only astronomer who has previously investigated the motion of this pair; his elements are as follows:—

P	T	e	a	Ω	i	λ	Authority	Source
94.4	1839.1	0.500	0.98	99.6	54.5	134.9	Doberck, 1879	A.N. 2294
94.406	1839.10	0.5870	1.066	96.28	60.22	129.92	Doberck, 1879	

A careful study of all the observations leads to the following elements:

$$\begin{aligned}
 P &= 80.0 \text{ years} & \Omega &= 81^\circ.7 \\
 T &= 1834.30 & i &= 49^\circ.32 \\
 e &= 0.324 & \lambda &= 137^\circ.78 \\
 a &= 0''.8690 & n &= +4^\circ.5
 \end{aligned}$$

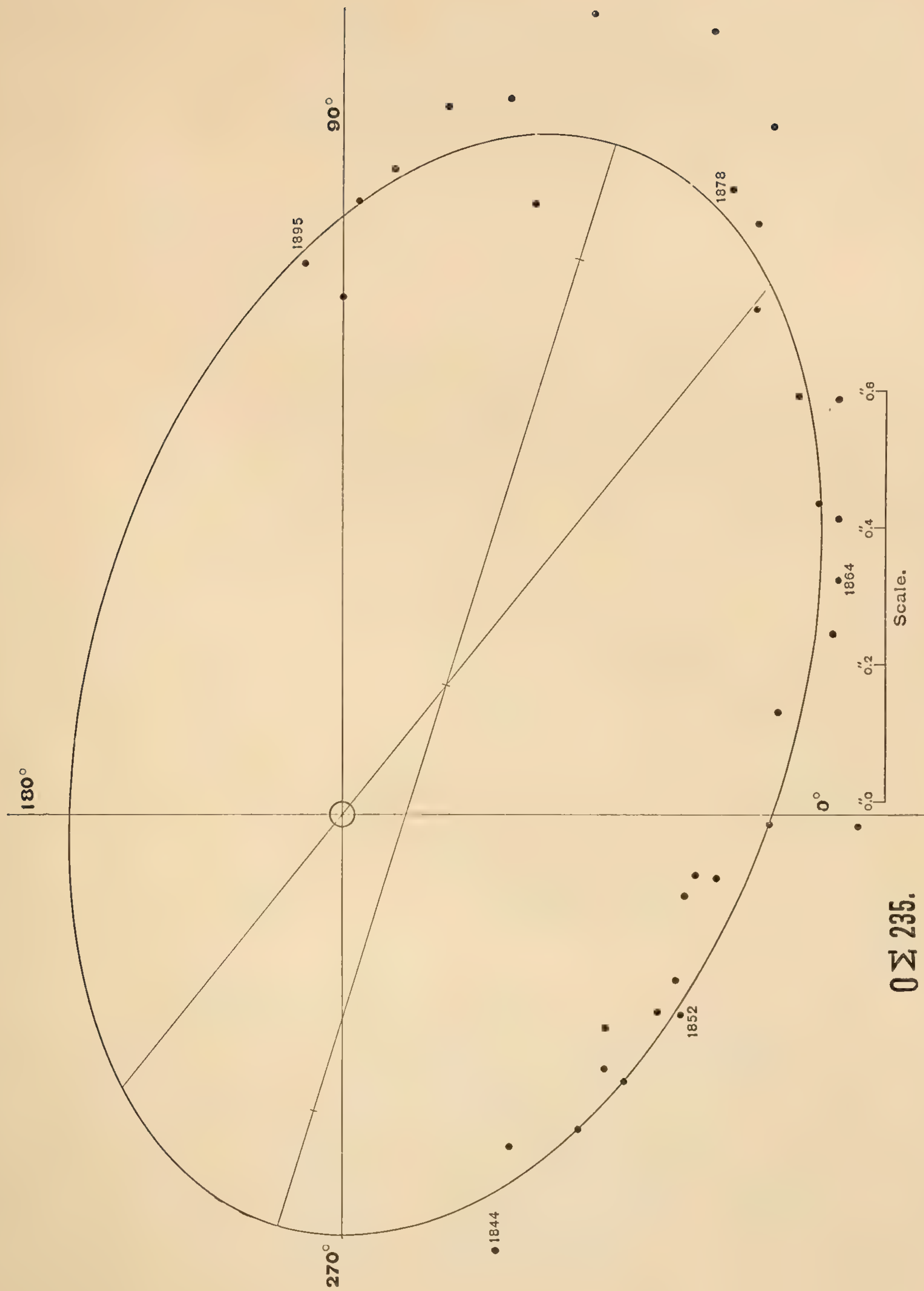
Apparent orbit:

Length of major axis	= 1".682
Length of minor axis	= 1".02
Angle of major axis	= 72°.8
Angle of periastron	= 231°.1
Distance of star from centre	= 0".242

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1844.33	289.3	288.6	0.67	0.60	+ 0.7	+0.07	1	O. Struve
1845.47	296.7	293.5	0.54	0.59	+ 3.2	-0.05	1	O. Struve
1846.42	306.8	298.1	0.57	0.58	+ 8.7	-0.01	1	O. Struve
1847.45	315.8	303.7	0.53	0.57	+12.1	-0.04	1	O. Struve
1849.47	320.8	314.9	0.49	0.56	+ 5.9	-0.07	1	O. Struve
1850.31	316.5	318.7	0.56	0.56	- 2.2	± 0.00	1	O. Struve
1851.42	328.0	324.7	0.54	0.56	+ 3.3	-0.02	2	O. Struve
1852.46	329.5	330.2	0.57	0.56	- 0.7	+0.01	1	O. Struve
1853.41	333.5	335.5	0.54	0.57	- 2.0	-0.03	1	O. Struve
1855.47	346.6	346.3	0.51	0.59	+ 0.3	-0.08	1	O. Struve
1856.55	350.3	351.8	0.52	0.60	- 1.5	-0.08	1	O. Struve
1857.51	350.4	356.6	0.55	0.61	- 6.2	-0.06	1	O. Struve
1858.44	358.7	1.0	0.75	0.63	- 2.3	+0.12	1	O. Struve
1859.41	358.7	5.5	0.62	0.65	- 6.8	-0.03	1	O. Struve
1861.42	13.3	13.7	0.65	0.69	- 0.4	-0.04	2	O. Struve
1862.38	20.3	17.5	0.76	0.71	+ 2.8	+0.05	1	O. Struve
1864.43	25.3	24.8	0.80	0.76	+ 0.5	+0.04	1	O. Struve
1866.49	33.3	30.8	0.83	0.81	+ 2.5	+0.02	1	O. Struve
1867.45	40.1	34.2	separated	0.84	+ 5.9	-	1	Dembowski
1868.13	31.0	36.0	0.84	0.86	- 5.0	-0.02	1	Dembowski
1870.32	40.0	40.4	0.94	0.90	- 0.4	+0.04	2	Dembowski 1; O. Struve 1
1872.50	42.6	47.1	0.90	0.96	- 4.5	-0.06	2	Dembowski 1; O. Struve 1
1876.63	51.0	55.9	0.95	1.02	- 4.9	-0.07	1	O. Struve
1877.29	55.1	57.3	1.05	1.03	- 2.2	+0.02	3	Dembowski 2; O. Struve 1
1878.35	58.1	59.3	1.18	1.04	- 1.2	+0.14	4	Dembowski
1879.44	58.2	61.5	1.07	1.05	- 3.3	+0.02	1-3	O. Struve 1; Hall 0-3
1882.59	64.8	67.3	1.26	1.05	- 2.5	+0.21	6	Englemann
1887.43	72.5	76.1	0.93	1.02	- 3.6	-0.09	4	Schiaparelli
1888.56	72.6	78.4	1.22	1.00	- 5.8	+0.22	4-5	OY. 0-1; Tarrant 4
1889.37	76.9	79.8	1.07	0.98	- 2.9	+0.09	5	Hall
1891.29	81.7	83.6	1.04	0.94	- 1.9	+0.10	1	Bigourdan
1892.39	85.5	85.9	0.94	0.92	- 0.4	+0.02	9-8	β . 3; Big. 1; Lv. 2; Com. 3-2
1893.39	88.4	88.2	0.89	0.89	+ 0.2	± 0.00	7-10	Comstock 1; Bigourdan 6-9
1894.24	90.1	90.1	0.75	0.87	± 0.0	-0.12	3	Comstock
1895.50	93.9	93.3	0.80	0.83	+ 0.6	-0.03	3	Comstock

A comparison of the computed with the observed places shows a very satisfactory agreement, and we cannot doubt that the elements given above will be found to approximate the truth. The period remains uncertain by perhaps five years, and the eccentricity may be varied by ± 0.05 ; but larger alterations in these elements are not to be expected. The motion of this pair will be accelerated in approaching periastron, and hence for a good many years will



deserve the regular attention of observers. If good measures can be secured during the next twenty years, the elements can be determined with great accuracy. The following is a short ephemeris:—

t	θ_c	ρ_c	t	θ_c	ρ_c
1896.50	95.9	0.80	1899.50	105.3	0.69
1897.50	98.9	0.76	1900.50	109.0	0.66
1898.50	102.0	0.73			

γ CENTAURI = H₂ 5370.

$\alpha = 12^h 36^m$; $\delta = -48^\circ 25'$.
4, yellowish ; 4, yellowish.

Discovered by Sir John Herschel, March 1, 1835.

OBSERVATIONS.

I. By SIR JOHN HERSCHEL:

MEASURES WITH THE EQUATORIAL.*

t	θ_c	ρ_c	n	Observers	Remarks
1835.257	354.8	<1	1	Herschel	Extremely close and very difficult, at least as close as γ Virginis; 273 barely elongates it.
1835.260	360.3	—	1	Herschel	Certainly double, but far too difficult for this telescope. Distinctly elongated, but the measures of no dependence.
1835.320	351.3	0.67	1	Herschel	Far too difficult for satisfactory measures; yet I must believe these to be somewhere about the truth.
1835.353	346.8	—	1	Herschel	A better set of measures than hitherto got with the equatorial, but it is too difficult for this object-glass.
1835.367	349.6	—	1	Herschel	Certainly seen double, i. e. elongated with parallel fringes.
1836.145	355.3	—	1	Herschel	Excessively close and difficult, but the power No. 4 will act to-night, though not quite so well as I could wish. Field strongly illuminated.
1836.156	362.0	—	1	Herschel	
1836.192	355.4	—	1	Herschel	Tolerably elongated with No. 4. Brandishes, dances, and spreads, yet occasionally an elongated centre caught.
1836.493	347.4	—	1	Herschel	
1837.140	361.9	1	1	Herschel	

OBSERVATIONS WITH THE REFLECTOR.

1835.166	—	—	—		γ Centauri, a star 4 ^m , which I am very much inclined to believe close double, but could not verify it owing to bad definition. Tried 320, but it will not bear that power.
1835.250	340.8	0.67	1	Herschel	180 with triangular aperture shows it elongated; 320 fairly double and almost divided. Pos. with 320 = 338°.3, with 480 (which shows a black division) = 343°.3. Both stars of 4th magnitude.
1836.382	340 ±	—	1	Herschel	Seen decidedly elongated with 320 and diminished aperture, but so violently agitated and ill defined that no measure could be got. That set down may err 20°.
1837.074	340 ±	—	1	Herschel	(γ Centauri). [Pos. estim. from diag]. Seen decidedly elongated in a position as per diagram, with 320 and triangular aperture, but all attempt at a measure confounded by constant boiling and working of the star.

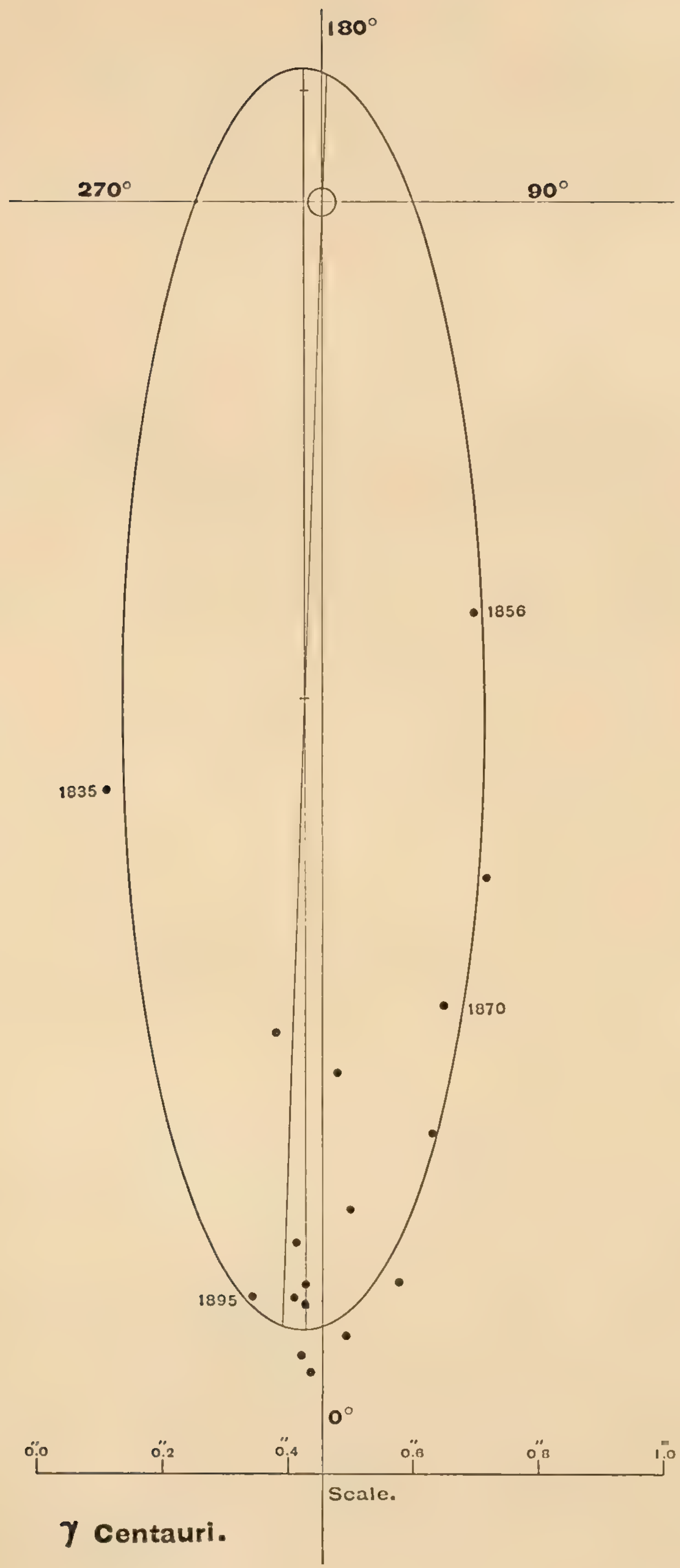
*Astronomische Nachrichten, 3339.

II. By other observers:

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1856.20	20.6	0.7 $\pm$	3	Jacob	1887.58	359.1	1.76	2-1	Tebbutt
1857.97	13.7	1.11	5	Jacob	1887.53	358.5	1.75	6	Pollock
1860.68	12.8	—	10obs.	Powell	1888.47	359.5	1.87	4-6	Tebbutt
1870.23	6.9	1.5 $\pm$	6	Powell	1889.32	359.1	1.73	4	Pollock
1871.38	3.8	1.18	1	Russell	1890.36	1.2	1.81	1	Sellers
1873.36	4.2	2.29	1	Russell	1890.36	359.0	1.81	2-1	Tebbutt
1874.26	1.6	1.61	1	Russell	1891.40	357.0	1.33	1	Sellers
1876.63	8.5	1.30	—	Ellery	1892.32	357.3	1.21	5	Sellers
1880.44	1.3	1.39	1	Russell	1892.48	358.7	1.66	7-8	Tebbutt
1882.22	2.1	—	1	Tebbutt	1893.36	356.7	1.40	3	Sellers
					1894.40	356.6	1.24	3	Sellers
					1895.33	356.4	1.75	11-7	Tebbutt

In the course of the three years following the discovery, HERSCHTEL secured several micrometrical measures with his seven-inch equatorial, but it appears that the records he has left us in his sweeps with the 20-foot reflector are much nearer the truth as regards the position-angle of the stars at that epoch. It is singular that his measures with the equatorial give angles almost identical with that of the pair at the present time ($356^\circ.4$), while his estimates made under the superior power of the reflector give the angle as $340^\circ \pm$. A careful study of all of his observations of γ Centauri (*Results of Observations at the Cape of Good Hope*, pp. 211, 256, 269), and of the other measures by subsequent astronomers leaves no doubt that his estimates with the reflector are essentially correct, while for some reason the measures taken with the equatorial are vitiated by systematic errors which render them worthless. In the above list of measures I have inserted HERSCHTEL's notes, with a view of throwing light upon this interpretation of his observations.

Contrary to the opinion of HERSCHTEL, it is now evident that the motion of γ Centauri is retrograde; and hence we perceive that the radius vector has swept over nearly an entire revolution since 1835. The recent measures of TEBBUTT, to whom we are so much indebted for observations of this star, prove beyond doubt that the distance of the components in angle 350° must be at least $1''.48$; and hence it could easily have been divided by HERSCHTEL with his seven-inch equatorial. He says, however, that the object was "extremely close and very difficult, at least as close as γ Virginis," and since it is known that γ Virginis, to which HERSCHTEL gave regular attention, was less than $0''.7$,



γ Centauri.

we may conclude that the distance of γ *Centauri* did not surpass $1''.0$. If this be the approximate distance at the epoch 1835.25 we see that the angle must have been substantially what HERSCHEL estimated with the reflector, and we are thus enabled to reconcile his measures with those of later observers. His estimate of $340^\circ \pm$ for the angle is based on three nights' work and can hardly be in error by more than two degrees. If we adopt the position thus indicated

$$1835.25 \quad 340^\circ \pm \quad 1'' \pm$$

and make use of the measures secured since 1856, we shall obtain an orbit which is near the truth, and the resulting elements will never be greatly changed. Mr. GORE is the only computer who has previously investigated the orbit of this binary; using HERSCHEL'S equatorial measures, and relying mainly on the angles, he found:

$$\begin{array}{ll} P = 61.88 \text{ years} & \Omega = 177^\circ.95 \\ T = 1840.84 & i = 84^\circ.1 \\ e = 0.6316 & \lambda = 46^\circ.81 \\ a = 1''.50 \end{array}$$

Making use of the mean places given in the following table, and basing our work on both angles and distances, we are led to the following elements of γ *Centauri*:

$$\begin{array}{ll} P = 88.0 \text{ years} & \Omega = 4^\circ.6 \\ T = 1848.0 & i = 62^\circ.15 \\ e = 0.800 & \lambda = 194^\circ.3 \\ a = 1''.0232 & n = -4^\circ.0911 \end{array}$$

Apparent orbit:

$$\begin{array}{ll} \text{Length of major axis} & = 2''.10 \\ \text{Length of minor axis} & = 0''.58 \\ \text{Angle of major axis} & = 0^\circ.1 \\ \text{Angle of periastron} & = 177^\circ.8 \\ \text{Distance of star from centre} & = 0''.794 \end{array}$$

The period here found may be uncertain by perhaps three years, and the eccentricity by ± 0.03 , but larger variations in these important elements are not to be expected. The orbit of γ *Centauri* is remarkable for its considerable inclination and high eccentricity, which renders the pair very difficult in the periastron part of the apparent ellipse. Binaries with equal components are very frequent among double stars, and are types of systems which possess a peculiar interest when studied in respect to their evolution.

It is clear that γ *Centauri* will move rather slowly for a good many years, but it deserves the regular attention of southern observers. The following is a short ephemeris:

<i>t</i>	θ_c	ρ_c	<i>t</i>	θ_c	ρ_c
1896.40	356.0	1.75	1899.40	354.8	0.71
1897.40	355.6	1.74	1900.40	354.4	1.70
1898.40	355.2	1.72			

COMPARISON OF THE COMPUTED WITH OBSERVED PLACES.

<i>t</i>	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	<i>n</i>	Observers
1835.25	340.±	338.2	1.00	0.88	+1.8	+0.12	3-1	Herschel
1856.20	20.6	19.7	0.7±	0.77	+0.9	-0.07	3	Jacob
1857.97	13.7	16.7	1.11	0.91	-3.0	+0.20	5	Jacob
1860.68	12.8	13.4	-	1.10	-0.6	-	10	Powell
1870.23	6.9	6.5	1.5±	1.54	+0.4	-0.04	6	Powell
1872.37	4.0	5.6	1.73	1.59	-1.6	+0.14	2	Russell
1874.26	1.6	4.7	1.61	1.64	-3.1	-0.03	1	Russell
1876.63	8.5	3.7	1.30	1.69	+4.8	-0.39	-	Ellery
1880.44	1.3	2.2	1.39	1.75	-0.9	-0.36	1	Russell
1882.22	2.1	1.4	-	1.77	+0.7	-	1	Tebbutt
1887.55	358.8	359.5	1.76	1.80	-0.7	-0.04	8-7	Tebbutt 2-1; Pollock 6
1888.47	359.5	359.1	1.87	1.80	+0.4	+0.07	4-6	Tebbutt
1889.32	359.1	358.8	1.73	1.80	+0.3	-0.07	4	Pollock
1890.36	360.1	358.4	1.82	1.80	+1.7	+0.02	2	Sellors 1; Tebbutt 1
1891.40	357.0	358.0	1.33	1.79	-1.0	-0.46	1	Sellors
1892.48	358.7	357.6	1.66	1.79	+1.1	-0.13	7-8	Tebbutt
1895.33	356.4	356.4	1.75	1.77	0.0	-0.02	11-7	Tebbutt

γ VIRGINIS = Σ1670.

$\alpha = 12^h 36^m.6$; $\delta = -0^\circ 54'$.
3, yellow ; 3.2, yellow.

Discovered by Bradley and Pound, March 15, 1718.

OBSERVATIONS.

<i>t</i>	θ_o	ρ_o	<i>n</i>	Observers	<i>t</i>	θ_o	ρ_o	<i>n</i>	Observers
1718.20	330.8	—	2	B. & P.	1819.40	—	3.56	—	Struve
1720.31	319.0	7.49*	1	Cassini	1820.28	284.9	2.76	5	Struve
1756.20	324.4	6.50	—	T. Mayer	1822.02	282.8	—	2	Struve
1777.±	310.±	9.8	—	C. Mayer	1822.25	283.4	3.79	2	H. & S.
1780.0	—	5.70±	—	Herschel	1823.19	—	3.30	—	Amici
1781.89	310.7	—	—	Herschel	1823.32	281.6	2.95	1-3	Struve
1803.37	300.2	—	Sobs.	Herschel	1825.32	276.9	3.26	4	South
					1825.32	277.9	2.37	6	Struve

* Computed from Lunar occultation — of no value.

t	θ_o °	ρ_o "	n	Observers	t	θ_o °	ρ_o "	n	Observers
1828.35	270.5	—	1	Herschel	1839.31	34.6	1.26	2-1	Dawes
1828.38	271.5	2.07	1	Struve	1839.35	35.5	1.30	5	Galle
1829.22	267.7	1.79	2	Herschel	1840.26	27.9	1.30	37-24	Kaiser
1829.39	268.3	1.78	5	Struve	1840.38	25.5	1.24	11-7	Dawes
1830.31	262.1	2.22	6-4	Herschel	1840.45	26.4	1.31	5	O. Struve
1830.59	262.2	1.59	7	Bessel	1841.19	20.9	1.42	2	Challis
1831.30	258.4	1.99	6-2	Dawes	1841.34	20.0	1.58	7-5	Dawes
1831.32	257.2	1.74	10-6	Herschel	1841.35	20.1	1.73	12-11	Mädler
1831.36	260.9	1.49	5	Struve	1841.41	22.4	1.63	4	O. Struve
1832.27	250.2	1.21	18-1	Herschel	1842.21	16.6	1.58	7-5	Mädler
1832.30	249.9	1.33	9-4	Dawes	1842.34	17.4	1.67	—	Main
1832.33	—	1.94	—	Cooper	1842.35	17.6	1.83	—	Airy
1832.52	253.5	1.26	4	Struve	1842.35	12.2	1.85	2	Challis
1833.20	241.8	1.41	12-3	Herschel	1842.38	14.9	1.73	9-5	Dawes
1833.24	64.9	1.14	1	Bessel	1842.41	17.1	1.86	4	O. Struve
1833.35	236.4	—	1	Mädler	1842.82	14.5	1.76	—	Kaiser
1833.36	240.1	1.14	8-2	Dawes	1842.88	14.7	1.84	6-1	Mädler
1833.37	245.5	1.05	7	Struve	1843.30	0.7	2.05	1	Challis
1834.29	227.3	—	8	Dawes	1843.35	12.0	1.77	7	Mädler
1834.34	214.8	—	1	Mädler	1843.39	13.6	2.08	—	Main
1834.37	223.1	1.51	8-1	Herschel	1843.40	12.2	1.83	10-5	Dawes
1834.38	231.6	0.91	5	Struve	1843.48	11.4	2.45	—	Eneke
1834.54	214.9	—	6	Herschel	1844.33	9.0	2.63	1	Challis
1834.84	213.6	—	1	Struve	1844.34	2.9	2.20	—	Richardson
1835.11	201.5	—	8	Herschel	1844.36	8.9	2.06	8-7	Mädler
1835.38	195.5	0.51	9	Struve	1844.38	8.6	2.27	—	Eneke
1835.39	195.2	0.57	1	Senff	1845.28	8.9	2.41	—	Eneke
1835.42	197.1	—	1	O. Struve	1845.37	7.0	—	—	Mädler
1836.28	169.5	—	2	Dawes	1845.46	4.5	2.23	2	O. Struve
1836.41	151.6	0.26	3	Struve	1846.28	5.0	—	—	Hind
1836.41	158.7	—	2	O. Struve	1846.32	2.2	2.91	2	Jacob
1836.41	153.8	—	1	Sabler	1846.39	6.3	2.25	—	Main
1836.59	113.9	—	—	Eneke	1846.39	2.9	2.35	2	O. Struve
1836.59	117.5	—	—	Mädler	1846.49	4.1	1.83	1	Mitchell
1837.41	78.3	0.58	1	Mädler	1846.90	3.8	2.45	2	Dawes
1837.41	77.9	0.58	6	O. Struve	1847.07	1.9	2.62	—	Hind
1837.41	78.5	0.67	1	Eneke	1847.35	2.5	2.40	8	Dawes
1837.41	77.9	—	1	Argelander	1847.41	13.0	2.37	—	Main
1838.08	57.5	0.67	1	Herschel	1847.42	2.5	2.40	3	O. Struve
1838.32	53.4	—	1	Dawes	1847.56	2.5	3.09	1	Mitchell
1838.36	—	1.24	—	Lamont	1847.94	359.9	2.88	2-1	Jacob
1838.40	51.9	0.86	—	Struve	1848.34	360.8	2.71	7-6	Mädler
1838.43	51.1	0.80	—	O. Struve	1848.37	360.6	2.62	9	Dawes
1838.43	49.2	0.83	3±	Ga. & Mä.	1848.43	359.1	2.55	3	O. Struve

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1848.45	360.4	2.60	2	W.C.&G.P.B.	1855.18	351.6	3.30	4	O. Struve
1848.45	360.6	2.80	1	Mitchell	1855.19	351.3	3.51	4	Dembowski
1848.48	360.5	2.60	2-3	Main	1855.30	353.4	—	4	Powell
					1855.39	353.5	3.45	—	Main
1849.37	359.0	2.85	5-1	Dawes	1855.40	352.6	3.37	1	Secchi
1849.41	352.9	2.64	2	O. Struve	1855.45	354.1	3.42	2	Mädler
1849.45	359.8	3.0	2	W.C.&G.P.B.	1855.46	351.2	3.31	4-3	Dawes
1849.50	357.0	2.92	3	Main	1855.53	353.3	3.51	3	Morton
1850.23	359.7	2.85	8	Johnson	1856.10	350.5	3.45	4	Jacob
1850.30	358.0	2.90	2	Jacob	1856.29	349.0	3.54	—	Main
1850.30	357.5	2.90	3	Hartnup	1856.38	351.7	3.55	6	Secchi
1850.36	356.7	2.95	6-3	Fletcher	1856.39	350.5	3.56	5	Dembowski
1850.39	355.2	2.74	4	O. Struve	1856.39	351.7	3.59	6	Mädler
1850.42	359.1	—	1	Mädler	1856.43	172.1	3.34	4	Winnecke
1850.48	359.7	2.94	4	Main	1856.96	353.0	3.64	—	Carpenter
					1856.97	351.6	3.66	3	Morton
1851.17	356.8	2.92	4	Philpot	1857.07	—	4.50	—	Schmidt
1851.19	357.7	3.12	2	Jacob	1857.09	348.4	3.76	6	Dembowski
1851.28	357.9	2.99	4	Mädler	1857.35	350.1	3.59	7	Dawes
1851.36	356.3	3.04	3	Main	1857.39	350.8	3.74	7	Secchi
1851.40	356.0	3.05	6	Fletcher	1857.40	352.9	3.58	6±	Baxendell
1851.40	356.5	2.99	5	Dawes	1857.41	351.6	3.54	—	Fletcher
1851.42	353.0	2.88	3	O. Struve	1857.42	350.2	3.59	9-8	Mädler
1851.47	355.9	3.04	3-1	Miller	1857.42	349.9	3.56	6	Dawes
1851.98	356.4	3.30	4-3	Mädler	1857.44	350.2	3.63	2	O. Struve
					1857.96	350.7	3.50	5	Jacob
1852.24	355.5	3.12	3	Jacob	1858.34	348.5	3.80	6	Dembowski
1852.26	355.5	3.12	6-3	Miller	1858.37	349.9	4.01	2	Mädler
1852.32	355.3	3.02	2	Dawes	1858.39	350.0	3.57	—	Fletcher
1852.42	355.4	3.15	5	Fletcher	1858.40	352.0	3.62	3	Secchi
1852.43	354.6	3.17	2	Mädler	1858.44	349.3	3.67	2	O. Struve
1852.43	353.0	3.00	3	O. Struve	1858.45	348.8	3.68	8	Dawes
1852.45	356.9	3.05	—	Fearnley	1858.47	348.0	3.85	—	Carpenter
1852.47	359.7	3.20	3	Main	1858.48	350.7	3.40	3	Morton
1853.24	353.2	3.12	2	Jacob	1859.15	350.7	3.95	4	Morton
1853.24	354.4	—	7	Powell	1859.37	349.2	3.88	9-8	Mädler
1853.27	354.9	3.10	7-5	Miller	1859.38	347.9	3.76	3	O. Struve
1853.32	354.6	3.18	6	Fletcher	1859.39	350.0	4.18	—	Wakelin
1853.36	354.1	3.06	3-2	Dawes	1859.44	349.5	3.91	3	Secchi
1853.38	357.4	3.30	2	Main	1859.46	348.2	3.77	5	Dawes
1853.39	354.2	3.25	6	Mädler					
1853.40	352.0	3.13	4	O. Struve	1860.24	347.9	3.95	1	Auwers
1853.91	353.0	3.06	2	Jacob	1860.30	358.0	2.90	—	Jacob
					1860.35	345.9	3.90	1	Mädler
1854.39	352.0	3.45	8	Mädler	1860.36	350.2	—	1	Schiaparelli
1854.39	352.7	3.21	8	Dawes	1860.36	347.1	—	1	Wagner
1854.40	352.1	3.40	3	Morton	1860.36	347.3	—	1	Oblomievsky
1854.47	353.6	3.23	7	Dembowski	1860.44	349.3	4.05	2	Knott

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1861.15	347.0	3.93	4	O. Struve	1869.22	344.9	4.77	—	Brünnnow
1861.19	357.7	3.12	—	Jacob	1869.22	340.9	5.27	2	Leyton Obs.
1861.28	347.8	3.99	4	Main	1869.49	339.8	4.74	3	Main
1861.31	346.1	3.93	5	Powell	1869.98	341.8	4.43	17	Dunér
1861.36	348.5	4.12	7	Auwers	1870.33	342.6	4.65	2	Gledhill
1861.41	347.8	4.11	3	Mädler	1870.38	340.6	4.76	6	Main
1862.03	346.5	3.95	5-3	Dawes	1870.39	338.6	—	—	Leyton Obs.
1862.33	345.3	3.90	3-2	Powell	1870.72	342.0	4.63	11	Dembowski
1862.38	345.5	4.39	3	Mädler	1870.77	343.4	4.45	3	O. Struve
1862.38	349.3	4.31	1	Auwers	1871.21	339.8	5.31	1	Peirce
1862.38	346.6	4.00	—	Main	1871.35	340.9	4.54	5	Main
1862.40	346.9	3.97	2	O. Struve	1871.38	343.1	4.76	—	Leyton Obs.
1862.42	347.6	3.62	1	Oblomievsky	1871.38	339.8	4.49	3	Knott
1863.25	346.7	4.06	3	Main	1871.38	339.7	5.35	2	W. & S.
1863.27	345.1	4.34	—	Bamberg	1871.53	341.8	4.77	3	Gledhill
1863.46	347.3	3.90	2	O. Struve	1872.12	341.1	4.59	17	Dunér
1863.63	345.6	4.08	2-6	Dembowski	1872.30	339.7	4.4	1	Gledhill
1864.40	345.7	4.27	2	Main	1872.34	342.2	5.59	3	W. & S.
1864.41	345.5	4.28	2	Secchi	1872.37	338.6	4.80	—	Leyton Obs.
1864.42	345.1	4.06	3	O. Struve	1872.40	341.5	4.82	1	Knott
1864.44	345.4	4.10	4	Dawes	1872.41	340.0	4.61	3	O. Struve
1864.44	345.4	4.27	2	Knott	1872.41	340.3	4.78	3	Main
1864.48	348.3	4.03	3	Englemann	1872.86	340.8	4.59	10	Dembowski
1865.45	345.4	4.02	5	Englemann	1873.40	340.2	4.83	5	Main
1865.36	345.2	4.28	4	Main	1873.41	339.7	4.65	2	Gledhill
1865.37	—	4.18	4	Kaiser	1873.43	340.8	4.55	3	O. Struve
1865.42	344.0	4.37	7-6	Dawes	1873.46	340.5	4.96	3	Lindstedt
1865.45	344.3	4.34	3	Knott	1874.27	340.5	5.08	2	Gledhill
1865.74	344.3	4.18	26	Dembowski	1874.30	341.8	5.00	1	W. & S.
1866.31	344.3	4.39	3	Secchi	1874.32	339.3	5.39	1	Leyton Obs.
1866.33	342.8	4.52	3-4	Leyton Obs.	1874.33	338.5	5.23	6	Main
1866.37	—	5.00	1	Winlock	1874.41	340.4	4.87	3	O. Struve
1866.38	344.6	4.21	6	Kaiser	1875.14	339.1	4.66	14	Dunér
1866.42	344.0	4.29	2	O. Struve	1875.22	338.5	4.86	4	Gledhill
1866.45	345.2	4.35	2	Main	1875.29	339.8	5.09	6	Main
1866.46	345.9	4.01	—	Kaiser	1875.30	340.0	4.97	1	Seabroke
1867.24	342.9	5.28	1	Leyton Obs.	1875.32	339.2	4.80	11	Dembowski
1867.29	344.3	4.50	5	Harvard	1875.41	339.6	4.86	13	Schiaparelli
1867.38	341.4	4.40	6	Main	1875.44	339.9	4.87	2	O. Struve
1867.80	343.2	4.30	12	Dembowski	1876.24	338.7	5.34	5	Doberck
1868.17	344.3	4.58	2	Searle	1876.27	338.7	4.78	13	Gledhill
1868.23	341.0	5.21	2	Leyton Obs.	1876.36	340.0	—	1	Leyton Obs.
1868.42	341.0	4.63	7-6	Main	1876.38	339.8	5.30	4	Cincinnati
1868.44	343.2	4.30	2	O. Struve	1876.40	339.7	4.64	1	Waldo
					1876.41	340.2	5.14	4	Hall

t	θ_o °	ρ_o "	n	Observers	t	θ_o °	ρ_o "	n	Observers
1876.42	339.7	4.95	3	O. Struve	1883.07	335.6	5.22	7-5	Englemann
1876.45	339.0	4.81	4	Schiaparelli	1883.36	336.8	5.15	5	Hall
1876.48	338.2	5.18	5	Main	1883.41	335.6	5.23	8	Schiaparelli
1877.07	338.5	—	2	Gledhill	1884.33	335.2	5.65	5-3	H. C. Wilson
1877.24	340.0	4.65	5-4	Plummer	1884.37	336.1	5.42	5	Hall
1877.28	335.8	5.04	—	Knott	1884.38	335.7	5.43	3	Perrotin
1877.30	338.1	5.19	8-7	Cincinnati	1884.40	337.0	5.53	2	Seabroke
1877.40	339.5	4.91	6	Jedrzejewicz	1884.89	336.1	5.32	4	Englemann
1877.41	337.9	4.91	14	Schiaparelli	1884.40	335.6	5.19	9	Schiaparelli
1877.43	338.4	4.96	—	Flammarion	1884.44	336.5	5.32	1	O. Struve
1877.43	338.9	4.97	2	O. Struve	1885.25	334.4	5.30	1	Cop. & Lohse
1877.83	338.1	4.97	8	Dembowski	1885.32	333.7	5.35	2	H. C. Wilson
1878.26	340.1	5.01	2	W. & S.	1885.38	336.8	5.35	3	Tarrant
1878.37	337.1	5.06	3-5	Goldney	1885.44	335.2	5.30	16	Schiaparelli
1878.37	337.5	5.03	1	O. Struve	1886.28	335.0	5.08	2	Glasenapp
1879.0	336.3	5.07	1	Pritchett	1886.30	336.4	5.38	2	H. C. Wilson
1879.12	337.3	5.20	20	Cincinnati	1886.36	334.9	5.57	4	Hall
1879.13	337.5	4.97	10	Schiaparelli	1887.26	335.7	5.63	2	Glasenapp
1879.35	338.6	5.00	1	Gledhill	1887.35	334.8	5.58	4	Hall
1879.37	338.3	5.20	3	Hall	1887.38	335.5	5.65	2	Tebbutt
1879.38	338.3	5.01	2	Sea. & Smith	1887.41	334.2	5.42	7	Schiaparelli
1879.44	340.0	5.09	1	O. Struve	1888.27	333.5	5.93	2	Glasenapp
1880.19	336.7	5.30	1	Burton	1888.33	334.6	5.50	5	Hall
1880.25	337.4	5.35	6	Radeliffe Obs	1888.35	334.2	5.33	2	Schiaparelli
1880.26	336.5	5.67	3-2	Tiss. & Big.	1888.40	335.1	5.29	2	Maw
1880.30	338.2	5.27	5	Hall	1888.43	333.3	5.53	1	O. Struve
1880.30	337.5	5.36	2	Burnham	1888.48	334.8	5.74	2	Tebbutt
1880.31	337.3	4.90	—	Gledhill	1888.91	333.8	5.50	9	Leavenworth
1880.32	336.9	5.13	6	Cincinnati	1889.27	333.5	5.93	2	Glasenapp
1880.37	338.1	4.95	3	Doberek	1889.31	333.4	5.72	3	Burnham
1880.40	337.5	4.89	2	Seabroke	1889.39	333.1	5.51	2	O. Struve
1880.40	337.1	5.74	2	Tebbutt	1889.43	333.0	5.54	5	Hall
1880.45	337.9	5.24	3	Jedrzejewicz	1889.44	333.8	5.41	3	Schiaparelli
1880.66	337.9	5.22	6	Franz	1890.36	333.3	5.10	4	Glasenapp
1880.70	338.4	5.32	2	Pritchett	1890.43	332.8	5.59	3	Hall
1881.24	336.3	5.40	—	Gledhill	1890.43	333.2	5.53	8	Schiaparelli
1881.24	337.1	5.02	4	Doberek	1890.44	336.0	6.13	1	Hayes
1881.30	336.1	5.57	3	E. J. Stone	1891.15	330.4	5.75	1	Flint
1881.35	337.7	5.33	4	Hall	1891.32	332.0	5.78	2	Wellmann
1881.39	336.8	5.20	9	Schiaparelli	1891.32	332.9	5.69	11	Knorre
1881.42	338.7	5.28	2	Hough	1891.39	333.1	5.64	3	Hall
1881.44	336.2	5.23	14-13	Bigourdan	1891.42	332.6	5.51	7-6	Schiaparelli
1882.28	335.0	5.13	3	H. C. Wilson					
1882.28	337.4	5.36	5-4	Doberek					
1882.34	335.8	5.50	2	Sea. & Hodges					
1882.41	336.6	5.23	10	Schiaparelli					

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1891.44	331.0	5.64	1	Bigourdan	1893.42	331.9	5.47	6	Schiaparelli
1891.44	332.5	5.70	3	See	1893.43	333.1	5.66	1	Comstock
1892.40	332.6	5.55	6	Schiaparelli	1893.46	331.7	5.64	4	Bigourdan
1892.43	332.2	5.67	2	Leavenworth	1894.40	332.1	5.50	2	Comstock
1892.49	333.6	5.55	3	Comstock	1894.42	332.2	5.62	2	Schiaparelli
1892.51	332.3	5.56	2	Tebbutt	1894.47	328.9	5.71	6	Bigourdan
1892.52	331.8	5.61	3	Bigourdan	1895.30	331.1	5.84	5-4	See
1892.96	332.1	5.83	2	Jones	1895.43	332.0	5.65	3	Comstock

The observations of this celebrated system date back almost to the beginning of double-star Astronomy. The only double star previously recognized which has proved to be binary is α *Centauri*.† It was resolved into its components in December, 1689, by FATHER RICHAUD, at Pondicherry, India. On putting one eye to the telescope, and looking at the heavens with the other, BRADLEY found the two components of γ *Virginis* to be approximately in line with the naked-eye stars α and δ *Virginis*; this allineation gives a position-angle of $330^\circ.8$ at the epoch 1718.20. Such an observation has of course some historical interest, but is worthy of little consideration in the discussion of a modern double-star orbit. Neither can any confidence be placed in the position for 1720, which was calculated from a lunar occultation observed by CASSINI while searching for evidence of an atmosphere surrounding the Moon.

The observation which results from the Catalogue of TOBIAS MAYER would be entitled to more weight were it not for the uncertainty of double-star positions deduced from differences of right ascension and declination.

Therefore in the present discussion of the orbit I have relied principally upon observations since the time of WILLIAM STRUVE, but have not entirely ignored the measures of SIR WILLIAM HERSCHEL, which appear to be as good as could be expected from the means at his disposal. After an examination of all the observations, it appeared advisable to base the orbit mainly upon the work of the great standard observers. This sifting of the observational material is rendered the more necessary by virtue of the great number and miscellaneous character of the observers who have occupied themselves with an easy‡ and celebrated star like γ *Virginis*. It is probable that more orbits have been computed for this star than for any other binary in the heavens, but as all of these are defective, according to trustworthy recent observations, a new determination of the elements based upon the best measures now available, would seem to be desirable. In dealing with an orbit which has long occupied the

† *Astronomical Journal*, 352.

‡ Some of the observations here omitted are good, but in working with the graphical method I have not thought it necessary to use all of the super-abundant material.

attention of eminent men, including SIR JOHN HERSCHEL and the illustrious ADAMS, we could hardly hope for material improvement over the results already obtained, were not the investigation rendered more complete by recent observations, and by the use of the observed distances, which have generally been rejected, but which here acquire a high importance owing to the slow angular motion. The nature of the motion of γ *Virginis* is such that some of the elements, especially the periastron passage and the eccentricity, are determined with great precision; but the period has been underestimated by nearly all recent investigators, and will still remain slightly uncertain, perhaps to the extent of one year.

ELEMENTS DERIVED FROM PREVIOUS INVESTIGATIONS.

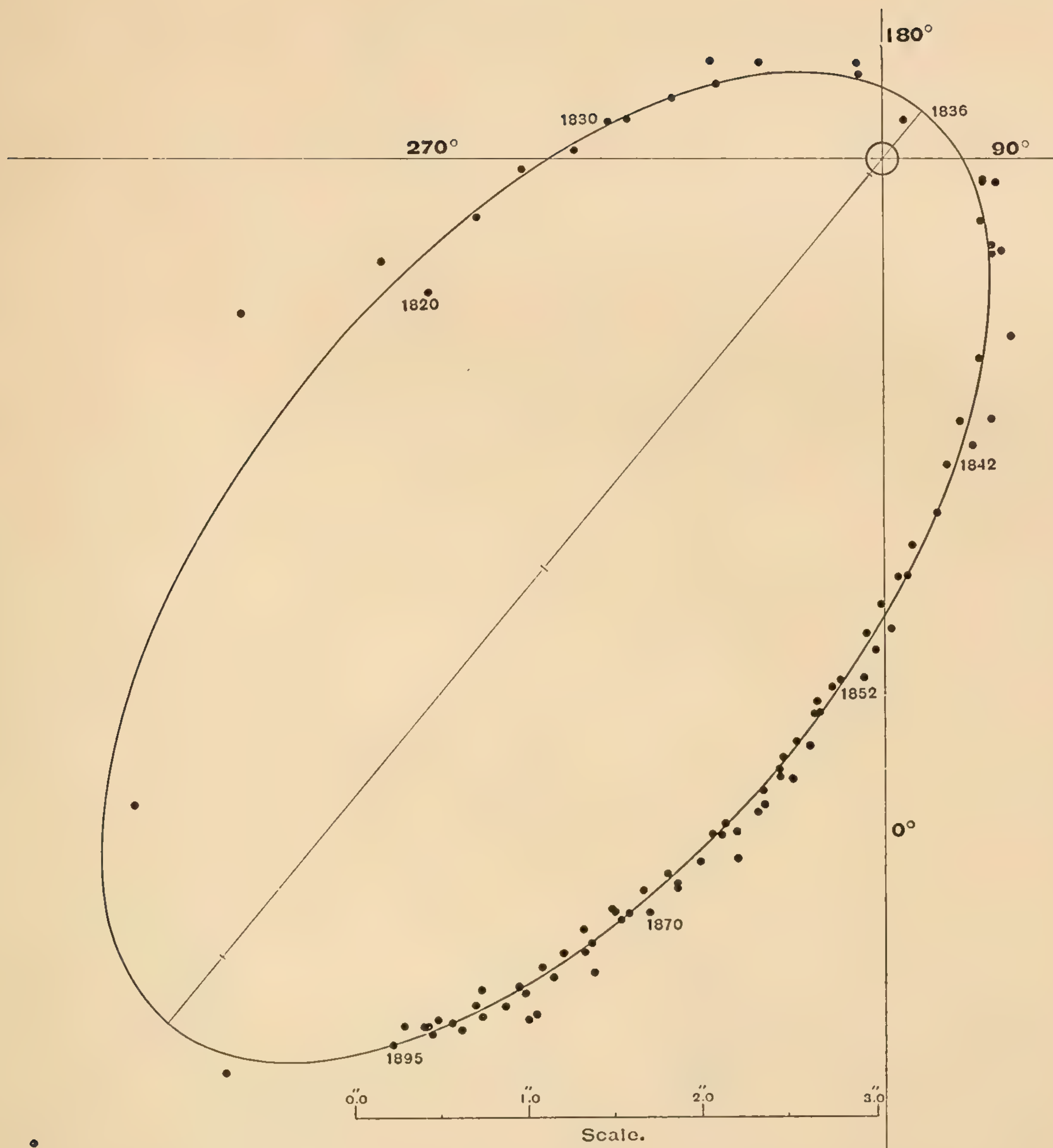
P	T	e	a	Ω	i	λ	Authority	Source
513.28 ^{vis.}	1834.01	0.8872	11.830	87.83	68.0	290.0	Herschel, 1831	Mem. R.A.S. vol. V. p. 193
628.90	1834.63	0.8335	12.09	97.4	67.03	282.35	Herschel, 1833	Mem. R.A.S., vol. VI. p. 152
145.409	1836.313	0.8681	3.402	60.63	24.65	78.37	Mädler, 1841	Dorpat Obs., 1841 p. 174
157.562	1836.103	0.8680	3.638	58.38	35.6	94.0	Mädler, 1841	A.N. 363
143.44	1836.29	0.8590	—	70.6	23.1	319.38	Hend'n, 1843	'Spec. Hartw.,' p. 345
141.297	1836.228	0.8566	—	78.47	25.23	319.77	Hind, 1845	Mem. R.A.S., vol. XVI,
133.5	1836.30	0.8525	3.499	69.67	24.6	249.3	Jacob, 1846	[p. 461
169.445	1836.279	0.8806	—	62.15	25.42	79.07	Mädler, 1847	Die Fixs.-Syst. II. p. 240
182.12	1836.43	0.8795	—	5.55	23.6	313.75	Herschel, 1847	'Results,' p. 297 [p. 67
183.137	1836.385	0.8860	4.336	28.7	30.65	290.5	Herschel, 1850	Mem. R.A.S., vol. XVIII,
171.54	1836.40	0.8804	—	20.57	27.38	300.2	Hind, 1851	M.N., vol. XI., p. 136
174.137	1836.34	0.8796	—	34.75	25.45	284.9	Adams, 1851	
184.53	1836.40	0.8794	—	19.12	27.6	295.2	Fletcher, 1853	M.N., vol. XIII, p. 258
148.2	1836.2	0.8725	3.617	41.67	31.95	269.3	Smyth, 1860	'Cycle,' p. 356
177.7	1836.50	0.8878	4.226	35.62	37.33	281.7	Smyth, 1860	'Cycle' cont., p. 451
185.0	1836.68	0.896	3.97	35.6	35.1	283.7	Thiele 1866	A.N., vol. XVIII
175.0	1836.45	0.8715	3.385	—	0.0	long. per. = 320.0	Fl., 1874	'Catalogue,' p. 72
180.54	1836.47	0.8978	4.09	45.82	37.0	93.98	Dobereck, 1881	Copernicus, vol. I, p. 143
179.65	1836.45	0.8904	3.94	46.0	33.95	93.92	Dobereck, 1881	Copern., vol. I, p. 143 [193
192.07	1836.51	0.895	4.144	54.9	34.12	274.23	See, 1893	Astron. & Astro.-Phys., Dec.

From an investigation of the long list of observations, including the very careful measures recently secured with the 26-inch refractor of the Leander McCormick Observatory of the University of Virginia, we find the following elements of γ *Virginis*:

$$\begin{aligned}
 P &= 194.0 \text{ years} & \Omega &= 50^\circ.4 \\
 T &= 1836.53 & i &= 31^\circ.0 \\
 e &= 0.8974 & \lambda &= 270^\circ.0 \\
 a &= 3''.989 & \alpha &= -1^\circ.8557
 \end{aligned}$$

Apparent orbit:

$$\begin{aligned}
 \text{Length of major axis} &= 6''.824 \\
 \text{Length of minor axis} &= 3''.530 \\
 \text{Angle of major axis} &= 140^\circ.4 \\
 \text{Angle of periastron} &= 140^\circ.4 \\
 \text{Distance of star from centre} &= 3''.062
 \end{aligned}$$



γ Virginis = Σ 1670.

The accompanying table of computed and observed places shows that these are perhaps the most exact elements yet determined for any star. For although all the measures have not been used in forming the mean observations on which the orbit is based, yet those measures which have been employed have been so combined as fairly to represent the best material for each year. Accordingly, the residuals are uniformly small, except just before periastron passage, when the object was extremely difficult; and, as no variation of the elements will materially improve the representation of the observations in this part of the orbit without a corresponding damage elsewhere, we infer that the differences are due mainly to systematic errors in STRUVE'S measures.

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers.
1718.20	330.8	326.2	—	6.27	+ 4.6	—	2	Bradley and Pound
1720.31	319.0	325.0	7.49	6.34	— 6.0	+1.15	1	Cassini
1756.20	324.4	318.7	6.50	6.46	+ 5.7	+0.04	—	Tobias Mayer
1781.89	310.7	308.1	5.70	5.67	+ 2.6	+0.03	1	Herschel
1803.37	300.2	299.6	—	4.60	+ 0.6	—	8 obs.	Herschel
1819.40	—	286.9	3.56	3.16	—	+0.40	1+	Struve
1820.28	284.9	284.9	2.76	2.97	0.0	—0.21	5	Struve
1822.25	283.4	283.4	3.79	2.85	0.0	—0.06	2	Herschel and South
1823.32	281.6	281.8	2.95	2.70	— 0.2	+0.25	1, 3	Struve
1825.32	277.9	278.2	2.37	2.43	— 0.3	—0.06	6	Struve
1828.38	271.5	271.4	2.07	2.01	+ 0.1	+0.06	1	Struve
1829.30	268.0	268.8	1.78	1.86	— 0.8	—0.08	7	H. 2; Σ . 5
1830.59	262.2	264.1	1.59	1.63	— 1.9	—0.04	7	Bessel
1831.36	260.9	260.8	1.49	1.50	+ 0.1	—0.01	5	Struve
1832.52	253.5	253.8	1.26	1.26	— 0.3	0.00	4	Struve
1833.36	240.1	247.2	1.14	1.09	— 7.1	+0.05	8, 2	Dawes
1833.37	245.5	247.1	1.05	1.08	— 1.6	—0.03	7	Struve
1834.38	231.6	235.0	0.91	0.84	— 3.4	+0.07	5	Struve
1834.84	213.6	226.5	—	0.72	—12.9	—	1	Struve
1835.38	195.5	212.2	0.51	0.58	—16.7	—0.07	9	Struve
1835.39	195.2	212.0	0.57	0.57	—16.8	0.00	1	Senff
1835.42	197.1	211.3	—	0.56	—14.2	—	1	O. Struve
1836.41	151.6	150.2	0.26	0.36	+ 1.4	—0.10	3	Struve
1836.41	158.7	150.2	—	0.36	+ 8.5	—	2	O. Struve
1836.41	153.8	150.2	—	0.36	+ 3.6	—	1	Sabler
1837.41	77.9	78.2	0.58	0.52	— 0.3	+0.06	6	O. Struve
1837.41	78.5	78.2	0.67	0.52	+ 0.3	+0.15	1	Encke
1838.08	57.5	58.0	0.67	0.70	— 0.5	—0.03	1	Herschel
1838.40	51.9	50.8	0.86	0.78	+ 1.1	+0.08	—	Struve
1838.43	51.1	50.0	0.80	0.79	+ 1.1	+0.01	—	O. Struve
1838.43	49.2	50.0	0.83	0.79	— 0.8	+0.04	3 $\pm$	Galle and Mädler
1839.33	35.5	37.3	1.26	1.01	— 1.8	+0.25	5, 1	Galle 5-0; Dawes 0-1
1840.36	26.3	28.1	1.28	1.23	— 1.8	+0.05	16, 12 $\pm$	Kaiser 1 $\pm$; Dawes 11-7; $O\Sigma$. 5
1841.41	22.4	22.0	1.63	1.44	+ 0.4	+0.19	4	O. Struve
1842.21	16.6	17.7	1.58	1.60	— 1.1	—0.02	7, 5	Mädler
1842.41	17.1	16.1	1.73	1.67	+ 1.0	+0.06	4, 5	$O\Sigma$. 4-0; Dawes 0-5
1843.37	12.1	13.7	1.80	1.78	— 1.6	+0.02	17, 12	Mädler 7; Dawes 10-5
1844.36	8.9	10.1	2.06	1.97	— 1.2	+0.09	8, 7	Mädler
1845.46	4.5	7.2	2.23	2.15	— 2.7	+0.08	2	O. Struve
1846.59	3.6	4.6	2.21	2.31	— 1.0	—0.10	5	$O\Sigma$. 2; Dawes 2; Mitchell 1

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1847.38	2.5	3.0	2.40	2.42	- 0.5	-0.02	11	Dawes 8; $O\Sigma$ 3
1848.34	0.8	1.3	2.71	2.55	- 0.5	+0.16	7, 6	Mädler
1848.40	359.8	1.1	2.57	2.56	- 1.3	+0.01	12	Dawes 9; $O\Sigma$ 3
1849.37	359.0	359.5	2.84	2.67	- 0.5	+0.17	5, 4	Dawes
1850.40	358.0	357.9	2.74	2.80	+ 0.1	-0.06	11, 4	Jacob 2-0; $O\Sigma$ 4; Mädler 1-0;
1851.28	357.9	356.8	2.99	2.90	+ 1.1	+0.09	4	Mädler [Mädler 4-0]
1851.40	356.5	356.4	2.99	2.95	+ 0.2	+0.04	5	Dawes
1852.38	354.6	355.4	3.06	3.01	- 0.8	+0.05	7	Dawes 2; Mädler 2; $O\Sigma$ 3
1853.30	353.6	354.3	3.21	3.13	- 0.7	+0.08	5, 4	Jacob 2; Dawes 3-2
1853.56	353.1	354.0	3.15	3.16	- 0.9	-0.01	12	Mädler 6; $O\Sigma$ 4; Jacob 2
1854.43	353.2	353.0	3.22	3.26	+ 0.2	-0.04	15	Dawes 8; Dembowski 7
1855.18	351.4	352.3	3.40	3.33	- 0.9	+0.07	8	$O\Sigma$ 3; Dembowski 4
1855.67	352.8	351.8	3.40	3.40	+ 1.0	0.00	10, 9	Senff 1; Mädler 3; Dawes 4-3;
1856.39	350.5	351.3	3.56	3.44	- 0.8	+0.12	5	Dembowski [Morton 3]
1857.28	349.1	350.2	3.70	3.56	- 1.1	+0.14	20	Dembowski 6; Dawes 7; Senff 7
1857.56	350.2	350.1	3.57	3.57	+ 0.1	0.00	22, 21	Mä. 9-8; Da. 6; $O\Sigma$; Ja. 5
1858.36	349.2	349.3	3.80	3.65	- 0.1	+0.15	8, 6	Dembowski 6; Mädler 2-0
1858.44	350.2	349.3	3.59	3.66	+ 0.9	-0.07	16	Senff 3; $O\Sigma$ 2; Da. 8; Mo. 3
1859.36	349.1	348.6	3.83	3.72	+ 0.5	+0.11	24, 23	Mo. 4; Mä. 9-8; $O\Sigma$ 3; Senff 3
1860.40	347.6	347.6	3.97	3.84	0.0	+0.13	3	Mädler 1; Knott 2 [Dawes 5]
1861.23	346.6	347.1	3.93	3.90	- 0.5	+0.03	9	$O\Sigma$ 4; Powell 5
1861.38	348.1	347.0	4.11	3.91	+ 1.1	+0.20	3+	Mädler 3; Auwers —
1862.28	346.0	346.3	4.01	3.99	- 0.3	+0.02	13, 10	Da. 5-3; Po. 3-2; Mä. 3; $O\Sigma$ 2
1863.54	346.4	345.5	3.99	4.06	+ 0.9	-0.07	28	$O\Sigma$ 2; Dembowski 26
1864.43	345.3	344.9	4.18	4.14	+ 0.4	+0.04	11	Senff 2; $O\Sigma$ 3; Da. 4; Kn. 2
1865.54	344.2	344.2	4.36	4.22	0.0	+0.14	36, 35	Da. 7-6; Kn. 3; Dem. 26
1866.36	344.1	343.7	4.34	4.28	+ 0.4	+0.06	5	Senff 3; $O\Sigma$ 2
1867.80	343.2	342.8	4.30	4.40	+ 0.4	-0.10	12	Dembowski
1868.43	342.2	342.4	4.47	4.45	- 0.2	+0.02	9	O. Struve 2; Main 7
1869.98	341.8	341.6	4.43	4.53	+ 0.2	-0.10	17	Dunér
1870.74	342.7	341.2	4.54	4.60	+ 0.5	-0.06	14	Dembowski 11; $O\Sigma$ 3
1871.43	340.5	340.9	4.87	4.65	- 0.4	+0.22	8	Kn. 3; Gled. 3; W. & S. 2
1872.12	341.1	340.5	4.59	4.68	+ 0.6	-0.09	17	Dunér
1872.63	340.4	340.0	4.61	4.74	+ 0.4	-0.13	13	$O\Sigma$ 3; Dembowski 10
1873.43	340.3	339.9	4.77	4.76	+ 0.4	+0.01	13	Gled. 2; $O\Sigma$ 3; Ma. 5; Lin. 3
1874.64	340.4	339.3	4.97	4.84	+ 1.1	+0.13	5	Gledhill 2; $O\Sigma$ 3
1875.18	338.8	339.0	4.76	4.88	- 0.2	-0.12	18	Dunér 14; Gledhill 4
1875.36	339.4	338.9	4.83	4.89	+ 0.5	-0.06	25	Dembowski 11; Schiaparelli 13
1876.34	339.1	338.5	5.02	4.95	+ 0.6	+0.07	26	Gled. 13; Ill. 4; Sch. 4; Dk. 5
1877.62	338.0	337.9	4.94	5.01	+ 0.1	-0.07	22	Schiaparelli 14; Dembowski 8
1878.37	337.1	337.6	5.06	5.06	- 0.5	0.00	3, 5	Goldney
1879.25	337.9	337.2	5.08	5.12	+ 0.7	-0.04	13	Schiaparelli 10; Hall 3
1880.30	337.5	336.8	5.36	5.17	+ 0.7	+0.19	2	Burnham
1881.44	336.2	336.3	5.28	5.22	- 0.1	+0.06	14, 17	Hall 0-4; Bigourdan 14-13
1882.41	336.6	335.9	5.23	5.28	+ 0.1	-0.05	10	Schiaparelli
1883.28	335.6	335.6	5.30	5.31	0.0	-0.01	20, 18	En. 7-5; Hall 0-5; Sch. 8
1884.38	335.8	335.1	5.34	5.38	+ 0.7	-0.04	17	Hall 5; Per. 3; Sch. 9
1885.35	334.1	334.8	5.32	5.40	- 0.7	-0.08	19	Cop. 1; H.C.W. 2; Sch. 16
1886.36	334.9	334.4	5.45	5.45	+ 0.5	0.00	4, 6	Hall 4; H.C.W. 0-2
1887.38	334.5	334.0	5.50	5.50	+ 0.5	0.00	11	Schiaparelli 7; Hall 4
1888.32	334.1	333.6	5.58	5.55	+ 0.5	+0.03	9	Glas. 2; Hall 5; Sch. 2
1889.40	333.4	333.3	5.56	5.60	+ 0.1	-0.04	11	Burnham 3; Hall 5; Sch. 3
1890.43	332.8	332.9	5.59	5.64	- 0.1	-0.05	3	Hall
1891.44	332.5	332.6	5.70	5.67	- 0.1	+0.03	3	See [Jones 2]
1892.56	332.3	332.2	5.64	5.71	+ 0.1	-0.07	16	Sch. 6; Iv. 2; Com. 3; Big. 3;
1893.44	332.2	331.9	5.65	5.75	+ 0.3	-0.10	11, 5	Sch. 6; Com. 1; Big. 4
1894.33	331.1	331.6	5.71	5.79	- 0.5	-0.08	10, 6	Com. 2-0; Sch. 2-0; Big. 6
1895.30	331.1	331.3	5.84	5.83	- 0.2	+0.01	5, 4	See

It will be seen that in this orbit the line of nodes coincides with the minor axis of the real ellipse, which is also the minor axis of its projection; and owing to the small inclination the apparent ellipse is only slightly less eccentric than the real ellipse, so that the foci of the two ellipses very nearly coincide. This renders the motion of the radius vector in the apparent orbit very nearly the same as in the real orbit, and makes γ *Virginis* an object of peculiar interest from the point of view of the study of the law of attraction in the stellar systems. From direct observation we are enabled to say that if there is any deviation from the Keplerian law of areas, it must be extremely slight. Therefore the force is certainly central, and the probabilities are overwhelming that the principal star, which is so near the focus of the apparent orbit, occupies the focus of the real orbit, or that the law of attraction is Newtonian gravitation. Other researches in double-star Astronomy increase the probability of the law of gravitation, and leave no adequate ground for doubt as to its absolute universality. Yet a prolonged study of the motion of γ *Virginis* will eventually give a very precise criterion for the rigor of this law, as well as throw light upon the question of the existence of disturbing bodies in binary systems.

The orbit of γ *Virginis* is very remarkable for its high eccentricity, which surpasses that of any other known stellar orbit. This characteristic of γ *Virginis*, which SIR JOHN HERSCHEL recognized when he declared the eccentricity to be "physically speaking, the most important of all the elements" (*Results at Cape of Good Hope*, p. 294), seems to preclude the permanent existence of a third body in the system; for if a companion to either of the components existed, its motion would be affected by an equation of enormous magnitude, analogous to the annual equation in the moon's motion, and at the time of periastron passage would probably soon cause the body to come into collision with one of the stars, or be driven off in an orbit analogous to a hyperbola.

Thus, although the above orbit is exact to a very high degree, the system will still deserve the occasional attention of astronomers.

Since the angular motion for many years to come will be extremely slow, observations of distance will be more valuable than angular measures in effecting a further improvement of the elements.

42 COMAE BERENICES = Σ 1728.

$\alpha = 13^{\text{h}} 5^{\text{m}}.1$; $\delta = +18^{\circ} 4'$.
6, orange ; 6, orange.

Discovered by William Struve in 1827.

OBSERVATIONS.

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1827.83	189.5	obl.	2-1	Struve	1853.09	194.2	0.62	4	Dawes
1829.40	191.6	0.64	3	Struve	1853.35	194.1	0.61	14-12	Madler
1833.37	170.7?	obl.	1	Struve	1853.40	190.8	0.57	3	O. Struve
1834.43	228.3	obl.	1	Struve	1854.38	194.1	0.60	1	O. Struve
1835.39	11.2	—	4	Struve	1854.39	193.6	0.61	8-7	Madler
1836.41	10.2	0.30	3	Struve	1854.39	192.8	0.55	5	Dawes
1837.40	11.0	0.39	6	Struve	1855.38	198.7	0.55	2-1	Madler
1838.41	11.5	0.36	3	Struve	1855.44	189.1	0.62	2	O. Struve
1839.42	12.2	0.59	—	Galle	1856.40	192.7	0.52	5-4	Madler
1840.45	15.7	0.55	3	O. Struve	1856.42	192.0	0.78	3	Winnecke
1840.74	18.5	0.4 $\pm$	3	Dawes	1856.96	192.5	0.47	6	Secchi
1841.40	14.7	0.32	12-5	Madler	1857.39	198.3	0.50	3-1	Madler
1841.41	14.5	0.49	2	O. Struve	1857.49	187.7	0.44	2	O. Struve
1842.40	13.9	0.32	3	O. Struve	1858.40	196.3	0.4 $\pm$	6	Madler
1842.45	15.6	—	4	Madler	1858.44	188.5	0.38	2	O. Struve
1842.53	single	—	—	Dawes	1859.36	215.8	0.2 $\pm$	3	Madler
1843.28	single	—	1	Madler	1859.37	single	—	—	O. Struve
1843.45	single	—	—	Dawes	1860.34	3.5?	0.2 $\pm$	1	Dawes
1844.32	189.5	—	2	Madler	1861.37	10.7	—	2	Madler
1845.47	single	—	—	O. Struve	1861.40	182.8	0.50	—	Winnecke
1846.40	66.8?	obl.?	3	O. Struve	1861.42	15.6	0.43	2	O. Struve
1847.42	195.5	0.20	1	O. Struve	1862.26	9.1	cuneo	7	Dembowski
1848.42	192.7	0.27	3	O. Struve	1862.37	16.5	—	2	Madler
1849.42	188.6	0.42	3	O. Struve	1862.40	11.6	0.54	2	O. Struve
1850.39	191.4	0.48	3	O. Struve	1862.42	2.9	—	—	Oblomievsky
1850.99	193.3	0.40	1	Madler	1863.25	11.0	0.5 $\pm$	1	Dawes
1851.27	191.3	0.35	1	Madler	1863.44	9.3	0.55	1	O. Struve
1851.42	187.0	0.49	4	O. Struve	1864.42	10.9	0.3 $\pm$	2	Secchi
1851.96	194.5	0.45	3-2	Madler	1864.42	12.5	0.51	3	O. Struve
1852.42	191.0	0.54	6-5	Madler	1864.43	13.4	0.45	1	Dawes
1852.43	190.9	0.56	3	O. Struve	1865.53	13.9	0.25 $\pm$	2	Secchi
1852.45	12.2	0.48	—	Fearnley	1865.57	9.5	cuneo	5	Dembowski
					1865.59	13.7	0.54	6	Englemann
					1866.64	8.5	0.40	3	O. Struve

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1867.32	21.4	—	1	Winlock	1881.25	192.2	0.70	2	Bigourdan
1867.32	24.7	—	1	Searle	1881.25	190.9	0.60	4-3	Doberck
1867.47	13.0	0.36	2	O. Struve	1881.37	193.0	0.64	4	Burnham
1867.77	14.8	cuneo	2	Dembowski	1881.38	191.6	0.6 $\pm$	5	Schiaparelli
1868.44	15.8	0.21	2	O. Struve	1881.39	192.6	0.53	4	Hall
1869.24	11.6	—	1	Leyton Obs.	1881.41	193.5	0.5 $\pm$	7-0	Perry
1869.40	19 ?	obl.	3	Dunér	1882.35	194.4	1.00	4-2	Seabroke
1869.47	15 ?	obl.?	1	O. Struve	1882.38	191.9	0.54	4	Hall
1870.44	single	—	—	O. Struve	1882.42	191.4	0.6 $\pm$	6	Schiaparelli
1870.45	16	obl.	4	Dunér	1882.46	184.6	0.51	1	O. Struve
1871.40	194.6	obl.	3	Dembowski	1882.93	192.1	0.56	7	Englemann
1871.43	single	—	—	O. Struve	1883.42	193.2	0.50	4	Hall
1872.42	200	obl.	1	O. Struve	1883.42	191.1	0.5 $\pm$	8	Schiaparelli
1872.52	200	obl.	2	Dunér	1883.48	193.4	0.55	5-4	Küstner
1873.36	single	—	1	J. M. Wilson	1883.51	191.5	0.53	2	Perrotin
1873.46	189.0	0.20	2	O. Struve	1884.39	195.8	0.3 $\pm$	4	Schiaparelli
1873.74	200.5	obl.	3	Dembowski	1884.40	189.7	0.36	3	Hall
1874.41	189.2	0.30	2	O. Struve	1885.41	single	—	1	Perrotin
1875.30	192.5	0.5 $\pm$	1	Seabroke	1885.42	single	—	4	Schiaparelli
1875.43	192.2	0.4 $\pm$	10	Schiaparelli	1885.49	10.2	0.35	1	Hall
1875.43	190.4	0.51	5	Dembowski	1886.42	10.0	0.27	3	Hall
1875.46	189.7	0.39	3	O. Struve	1886.51	15.8	0.26	6	Schiaparelli
1875.53	191.5	0.32	7-6	Dunér	1887.42	13.1	0.38	9	Schiaparelli
1876.36	186.4	0.5 $\pm$	1	W. Smith	1887.44	13.6	0.42	4	Hall
1876.38	191.2	0.58	4	Dembowski	1888.27	12.0	0.48	3	Schiaparelli
1876.40	193.4	0.40	4	Hall	1888.40	13.8	0.45	3	Hall
1876.42	188.0	0.50	3	O. Struve	1888.43	8.7	0.42	1	O. Struve
1876.45	193.1	0.5 $\pm$	4	Schiaparelli	1889.08	10.5	0.56	1	Leavenworth
1877.41	190.4	0.52	9-5	Schiaparelli	1889.39	11.8	0.61	1	O. Struve
1877.45	191.4	0.51	5	Dembowski	1889.41	10.9	0.49	5	Schiaparelli
1877.46	186.0	0.47	3	O. Struve	1890.33	9.3	0.70	4	Burnham
1878.37	191.3	0.65	1	O. Struve	1890.43	10.5	0.51	12	Schiaparelli
1878.38	193.7	obl.	3	Jedrzejewicz	1891.44	11.4	0.51	3	Hall
1878.38	189.6	0.51	4	Hall	1891.44	10.7	0.49	9	Schiaparelli
1878.43	190.8	0.57	3	Dembowski	1892.37	11.7	0.47	2-1	Leavenworth
1879.37	192.1	0.68	2	Burnham	1892.40	10.7	0.42	6	Schiaparelli
1879.42	193.2	0.51	4	Hall	1892.44	11.7	0.40	8-6	Bigourdan
1879.42	191.4	0.6 $\pm$	5	Schiaparelli	1893.45	10.2	0.32	5	Schiaparelli
1879.44	190.9	0.65	1	O. Struve	1894.33	0.1	0.25	3	Comstock
1880.36	191.7	0.52	4	Hall	1894.45	16.6	—	1-0	Bigourdan
1880.41	194.3	obl.	4	Jedrzejewicz	1894.46	10.38	0.22	4-5	Schiaparelli
					1895.29	13.9	0.14	3	See

Since the date of discovery this remarkable star has described almost three revolutions. From the first it was given particular attention by WILLIAM and

OTTO STRUVE, and the peculiar and unique character of the system has fully justified the care with which it has been measured. The only previous investigation* of the orbit is that made by OTTO STRUVE and DUBIAGO in 1874 (*Monthly Notices* 1874-5, p. 367). O. STRUVE's elements are as follows:

$$\begin{array}{ll} P = 25.71 \text{ years} & \Omega = 11^{\circ}.0 \\ T = 1869.92 & i = 90^{\circ} \\ e = 0.480 & \lambda = 99^{\circ}.18 \\ a = 0''.657 \end{array}$$

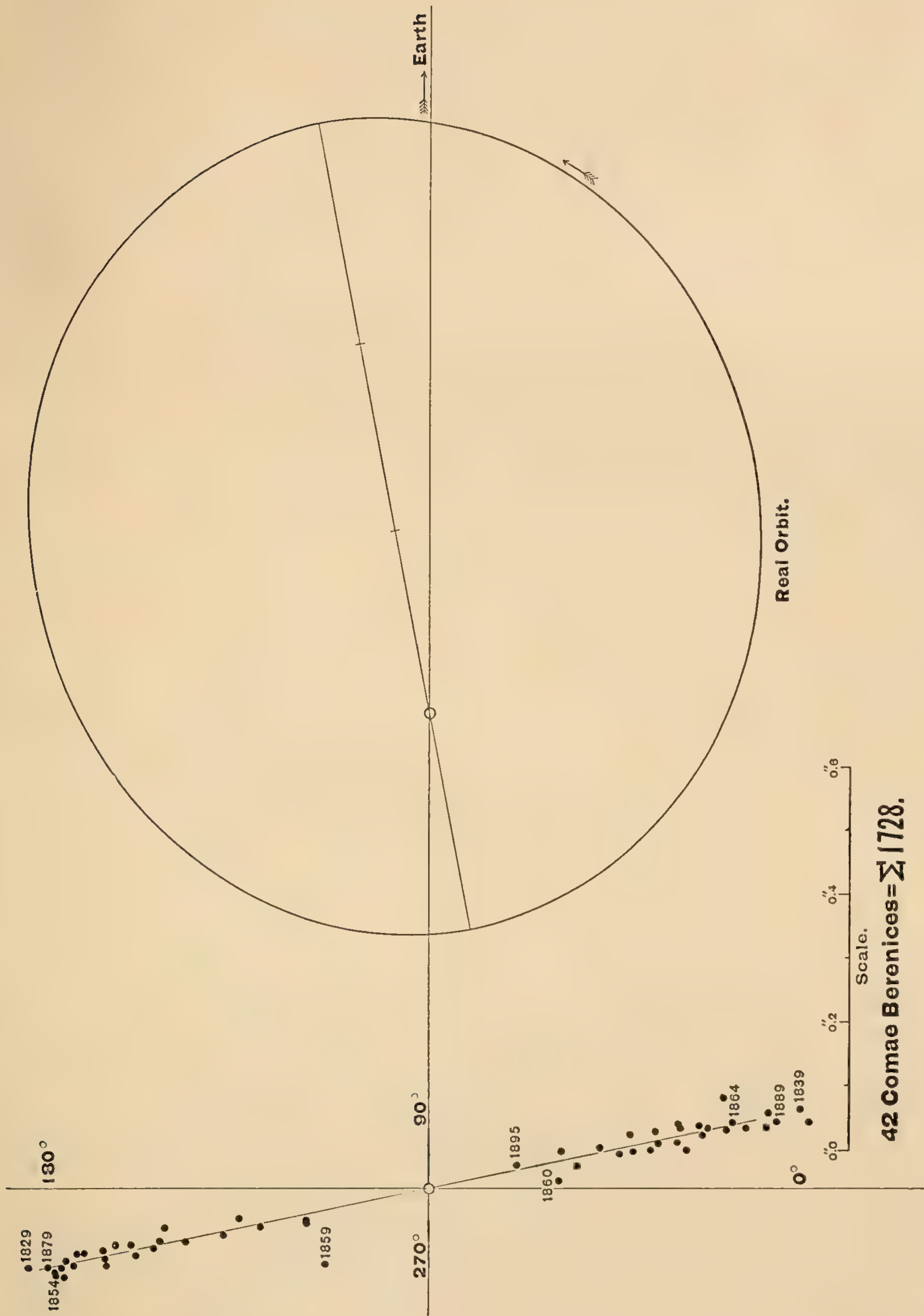
Some three years ago BURNHAM placed at my disposal a list of measures which was nearly complete; I have since added to it such as were omitted, and besides made new observations during 1895. When scrutinized under the fine definition of the 26-inch Clark Refractor of the University of Virginia the pair proved to be excessively close, and with a power of 1300 could only be elongated. The object has now become single in all existing telescopes and can not again be separated until about 1899.

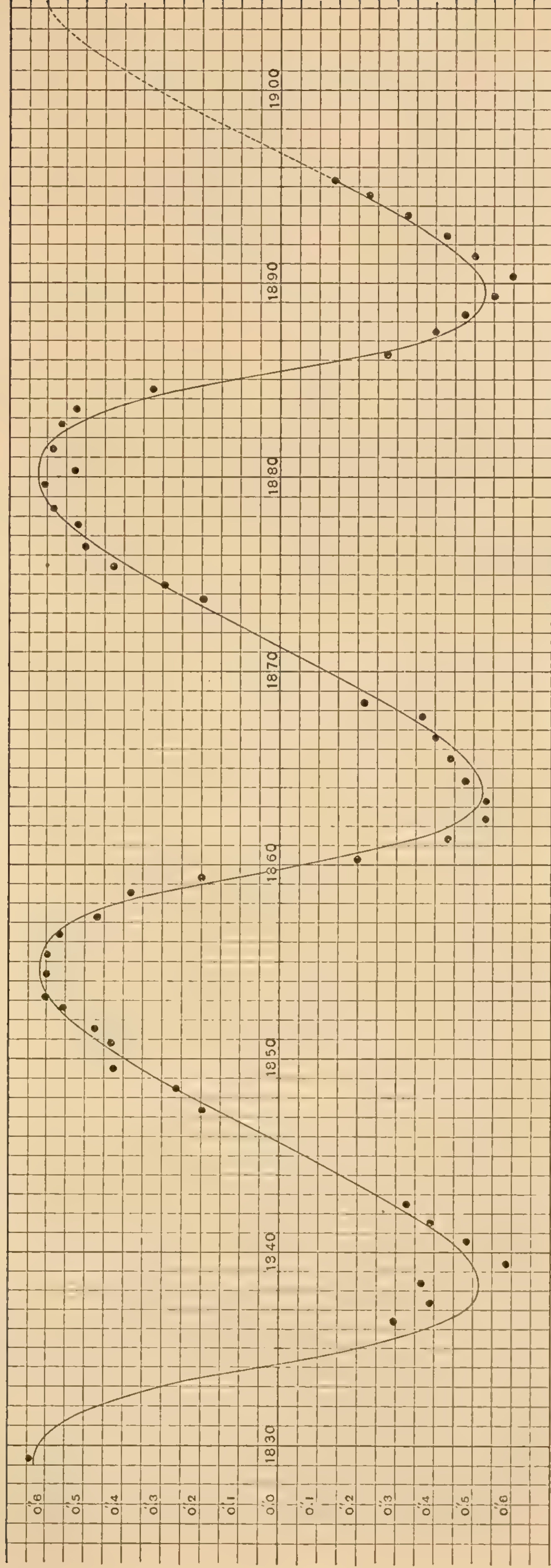
The method followed in the present investigation of the orbit is not very different from that employed by OTTO STRUVE, except that the results are based upon the measures of all reliable observers and are rendered more complete by the observations made since 1874. The list of measures is complete to the occultation of 1896.

It will be seen from an examination of the observations that the motion is to all appearances exactly in the plane of vision, and hence with the exception of the node and inclination, the elements are based wholly on the distances. O. STRUVE's elements are very good, and it would therefore be sufficient to apply differential corrections to his values, but as I had independently discovered a graphical method similar to that employed by him, it seemed of interest to make use of it in deriving approximate values directly from the phenomena. With the elements approximately determined, the observations furnished 52 equations of condition, which were solved for the five unknowns, the weights assigned being proportional to the number of nights. An application of the corrections resulting from the Least Square adjustment gave the following values of the elements:

$$\begin{array}{ll} P = 25.556 \text{ years} & \Omega = 11^{\circ}.9 \\ T = 1885.69 & i = 90^{\circ} \\ e = 0.461 & \lambda = 280^{\circ}.5 \\ a = 0''.6416 & n = \pm 14^{\circ}.0867 \end{array}$$

**Monthly Notices*, June, 1896.





Graphical Illustration of the Motion of 42 Comae Berenices = $\Sigma 1728$.

Apparent orbit:

Length of major axis	= 1".147
Length of minor axis	= 0".00
Angle of major axis	= 11°.9
Angle of periastron	= 11°.9
Distance of star from centre	= 0".054

The apparent motion is shown in the accompanying diagram, to which is added a figure of the real orbit. A graphical illustration of the motion, obtained by taking the x -axis to represent the time, while the ordinates represent the distances, was employed in finding the approximate values of the elements; the curve here traced represents the motion according to the elements as corrected. This orbit of 42 *Comae Berenices* is one of the most exact of double-star orbits, and will never require any but very slight modifications. The period can hardly be in error by more than 0.1 year, while a variation of ± 0.01 in the eccentricity is very improbable.

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1827.83	189.5	191.9	obl.	0.63	- 2.4	+0.01	2-1	Struve
1829.40	191.6	191.9	0.64	—	- 0.3	—	3	Struve
1833.37	170.7?	191.9	obl.	—	-21.2	—	1	Struve
1834.43	228.3	191.9	obl.	—	+36.4	—	1	Struve
1835.39	11.2	11.9	—	—	- 0.7	—	4	Struve
1836.41	10.2	11.9	0.30	0.42	- 1.7	-0.12	3	Struve
1837.40	11.0	11.9	0.39	0.50	- 0.9	-0.11	6	Struve
1838.41	11.5	11.9	0.36	0.51	- 0.4	-0.15	3	Struve
1839.42	12.2	11.9	0.59	0.50	+ 0.3	+0.09	—	Galle
1840.60	17.1	11.9	0.48	0.44	+ 5.2	+0.04	6	O. Struve 3; Dawes 3
1841.40	14.6	11.9	0.40	0.38	+ 2.7	+0.02	14-7	O. Struve 2; Mädler 12-5
1842.43	14.7	11.9	0.32	0.30	+ 2.8	+0.02	7-3	O. Struve 3; Mädler 4-0
1843.36	—	single	—	—	—	—	2	Mädler 1; Dawes —
1844.32	189.5	191.9	—	—	- 2.4	—	2	Mädler
1845.47	—	single	—	—	—	—	—	O. Struve
1846.40	66.8?	191.9	obl.?	—	+54.9	—	3	O. Struve
1847.42	195.5	191.9	0.20	0.18	+ 3.6	+0.02	1	O. Struve
1848.42	192.7	191.9	0.27	0.27	+ 0.8	± 0.00	3	O. Struve
1849.42	188.6	191.9	0.42	0.36	- 3.3	+0.06	3	O. Struve
1850.69	192.3	191.9	0.44	0.45	+ 0.4	-0.01	4	O. Struve 3; Mädler 1
1851.55	190.9	191.9	0.47	0.51	- 1.0	-0.04	8-6	Mädler 1-0; $O\Sigma$ 4; Mädler 3-2
1852.42	191.0	191.9	0.55	0.56	- 0.9	-0.01	9-8	Mädler 6-5; O. Struve 3
1853.28	193.0	191.9	0.60	0.60	+ 1.1	± 0.00	21-16	Dawes 4; Mädler 14-12; $O\Sigma$ 3
1854.39	193.5	191.9	0.60	0.62	+ 1.6	-0.02	14-13	$O\Sigma$ 1; Mädler 8-7; Dawes 5
1855.41	193.9	191.9	0.59	0.61	+ 2.0	-0.02	4-3	O. Struve 2; Mädler 2-1
1856.59	192.4	191.9	0.57	0.57	+ 0.5	± 0.00	14-13	Mädler 5-4; Winn. 3; Secchi 6
1857.44	193.0	191.9	0.47	0.51	+ 1.1	-0.04	5-3	Mädler 3-1; O. Struve 2
1858.42	192.4	191.9	0.39	0.35	+ 0.5	+0.04	8	Mädler 6; O. Struve 2
1859.36	215.8	191.9	0.2 $\pm$	0.14	+23.9	+0.06	3	Mädler
1860.34	3.5?	11.9	0.2 $\pm$	0.12	- 8.4	+0.08	1	Dawes
1861.40	13.1	11.9	0.43	0.34	+ 1.2	+0.09	4-2	Mädler 2-0; O. Struve 2
1862.34	12.4	11.9	0.54	0.46	+ 0.5	+0.08	11-2	Dem. 7-0; Mädler 2-0; $O\Sigma$ 2

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1863.35	10.2	11.9	0.53	0.52	— 1.7	+0.01	2	Dawes 1; O. Struve 1
1864.42	12.3	11.9	0.48	0.51	+ 0.4	—0.03	6-4	Secchi 2-0; O Σ . 3; Dawes 1
1865.56	12.4	11.9	0.44	0.47	+ 0.5	—0.03	13-8	Secchi 2; Dem. 5-0; En. 6
1866.64	8.5	11.9	0.40	0.41	— 3.4	—0.01	3	O. Struve
1867.62	13.9	11.9	0.36	0.33	+ 2.0	+0.03	4-2	O. Struve 2; Dembowski 2-0
1868.44	15.8	11.9	0.21	0.25	+ 3.9	—0.04	2	O. Struve
1869.37	15.2	11.9	obl. ?	—	—	—	5	Ley. 1; Dunér 3; O. Struve 1
1870.45	16.0	11.9	obl.	—	—	—	4	Dunér
1871.41	194.6	191.9	obl.	—	—	—	3-0	Dembowski
1872.47	200.0	191.9	obl.	—	—	—	3	O. Struve 1; Duner 2
1873.60	194.7	191.9	0.20	0.23	+ 2.8	—0.03	5-2	Dembowski 3-0; O. Struve 2
1874.41	189.2	191.9	0.30	0.30	— 2.7	± 0.00	2	O. Struve [Du. 7-6
1875.42	191.3	191.9	0.43	0.40	— 0.6	+0.03	26-25	Sea. 1; Sch. 10; Dem. 5; O Σ . 3;
1876.40	190.4	191.9	0.50	0.47	— 1.5	+0.03	16	Sm. 1; Dem. 4; Hall 4; O Σ . 3;
1877.43	190.9	191.9	0.52	0.53	— 1.0	—0.01	17-13	Sch. 9-5; Dem. 5; O Σ . 3 [Sch. 4
1878.40	191.4	191.9	0.58	0.58	— 0.5	± 0.00	11-8	Jed. 3-0; Hl. 4; Dem. 3; O Σ . 1
1879.40	191.9	191.9	0.61	0.61	± 0.0	± 0.00	12	β . 2; Hall 4; Sch. 5; O Σ . 1
1880.38	193.0	191.9	0.52	0.62	+ 1.1	—0.10	8	Hall 4; Jed. 4 [Perry 7-0
1881.34	192.3	191.9	0.59	0.61	— 0.4	—0.02	26-18	Big. 2; Dk. 4-3; β . 4; Sch. 5; Hl. 4;
1882.52	190.9	191.9	0.56	0.54	— 1.0	+0.02	22-18	Sea. 4-0; Hl. 4; Sch. 6; O Σ . 1; En. 7
1883.46	192.3	191.9	0.52	0.43	+ 0.4	+0.09	19-18	Hl. 4; Sch. 8; Kü. 5-4; Per. 2
1884.40	192.7	191.9	0.33	0.26	+ 0.8	+0.07	7	Schiaparelli 4; Hall 3
1886.46	12.9	11.9	0.27	0.25	+ 1.0	+0.02	9	Hall 3; Schiaparelli 6
1887.43	13.3	11.9	0.40	0.41	+ 1.4	—0.01	13	Schiaparelli 9; Hall 4
1888.33	11.5	11.9	0.47	0.19	— 0.4	—0.02	7-6	Schiaparelli 3; Hall 3; O Σ . 1-0
1889.25	11.1	11.9	0.55	0.52	— 0.8	+0.03	7	Leavenworth 1; Sch. 5; O Σ . 1
1890.38	9.9	11.9	0.60	0.51	— 2.0	+0.09	16	β . 4; Schiaparelli 12
1891.44	11.0	11.9	0.50	0.45	— 0.9	+0.05	12	Hall 3; Schiaparelli 9
1892.40	11.4	11.9	0.43	0.39	— 0.5	+0.04	16-13	Lv. 2-1; Sch. 6; Bigourdan 8-6
1893.45	10.2	11.9	0.32	0.31	— 1.7	+0.01	5	Schiaparelli
1894.41	9.0	11.9	0.23	0.22	— 2.9	+0.01	8	Com. 3; Big. 1-0; Sch. 4-5
1895.29	13.9	11.9	0.14	0.14	+ 2.0	± 0.00	3	See

O Σ 269. $\alpha = 13^h 28^m.3$; $\delta = +35^\circ 46'$.

7.3, yellowish ; 7.7, yellowish.

Discovered by Otto Struve in 1844.

OBSERVATIONS.

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1844.31	218.0	0.33	1	O. Struve	1855.47	223.6	0.27	1	O. Struve
1846.38	231.1	0.39	3	O. Struve	1861.26	242.8	0.33	1	O. Struve
1847.30	222.7	0.25	1	Mädler	1865.50	45	oblonga	1	Dembowski
1847.41	215.1	0.18	1	Mädler	1868.26	semplíce	—	1	Dembowski
1849.47	218.0?	oblong	1	O. Struve	1872.47	257.1	oblong	1	O. Struve
1851.30	222.4	0.20	1	Mädler	1877.26	oblonga in 180°?		1	Dembowski
1851.39	228.9	0.33	1	O. Struve					

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1883.41	61.4	0.22	4	Englemann	1891.49	28.9	0.19	2	Schiaparelli
1885.42	195.	elong.	2	Perrotin	1892.40	215.0	0.21	2	Burnham
1889.52	207.7	0.22	3	Schiaparelli	1894.40	210.5	0.30 $\pm$	1	Comstock
1890.41	26.3	0.22	1	Schiaparelli	1895.41	219.0	0.225	2	Schiaparelli
1891.26	213.4	0.22	3	Burnham	1895.74	235.4	0.44	1	See

Since the epoch of discovery in 1844 the companion has described an entire revolution, but the discordance of the observations renders it difficult to define the exact character of the orbit. The measures are frequently very inconsistent, and the most careful selections are necessary in forming the mean places. During the past few years the system has received merited attention from BURNHAM and SCHIAPARELLI; their measures make known the nature of the motion and enable us to fix the elements with considerable precision. BURNHAM was the first to give a proper interpretation of the earlier observations (*Observatory*, July, 1891), and to find a satisfactory apparent ellipse. GORE afterwards attempted an investigation of the orbit based on the angles only; he found the following elements:

$$\begin{aligned}
 P &= 47.70 \text{ years} & \Omega &= 51^\circ.93 \\
 T &= 1883.12 & i &= 82^\circ.81 \\
 e &= 0.0575 & \lambda &= 43^\circ.51 \\
 a &= 0''.58
 \end{aligned}$$

The exclusive use of angles in deriving the orbits of close and difficult double stars has frequently led to erroneous results, because when the distance is very small it is even more reliable than the angle. The use of distances becomes not only important but also necessary when the orbit is highly inclined, and the companion therefore has an angular motion which is small compared to the errors of observation, as is the case with OY 269. Accordingly in dealing with the orbit of this star we have given rather more attention to the distances than to the discordant and frequently retrograding angles. Using certain selected measures of the best observers we find the elements of OY 269 to be as follows:

$$\begin{aligned}
 P &= 48.8 \text{ years} & \Omega &= 46^\circ.2 \\
 T &= 1882.80 & i &= 71^\circ.3 \\
 e &= 0.361 & \lambda &= 32^\circ.63 \\
 a &= 0''.3248 & n &= +7^\circ.3771
 \end{aligned}$$

Apparent orbit:

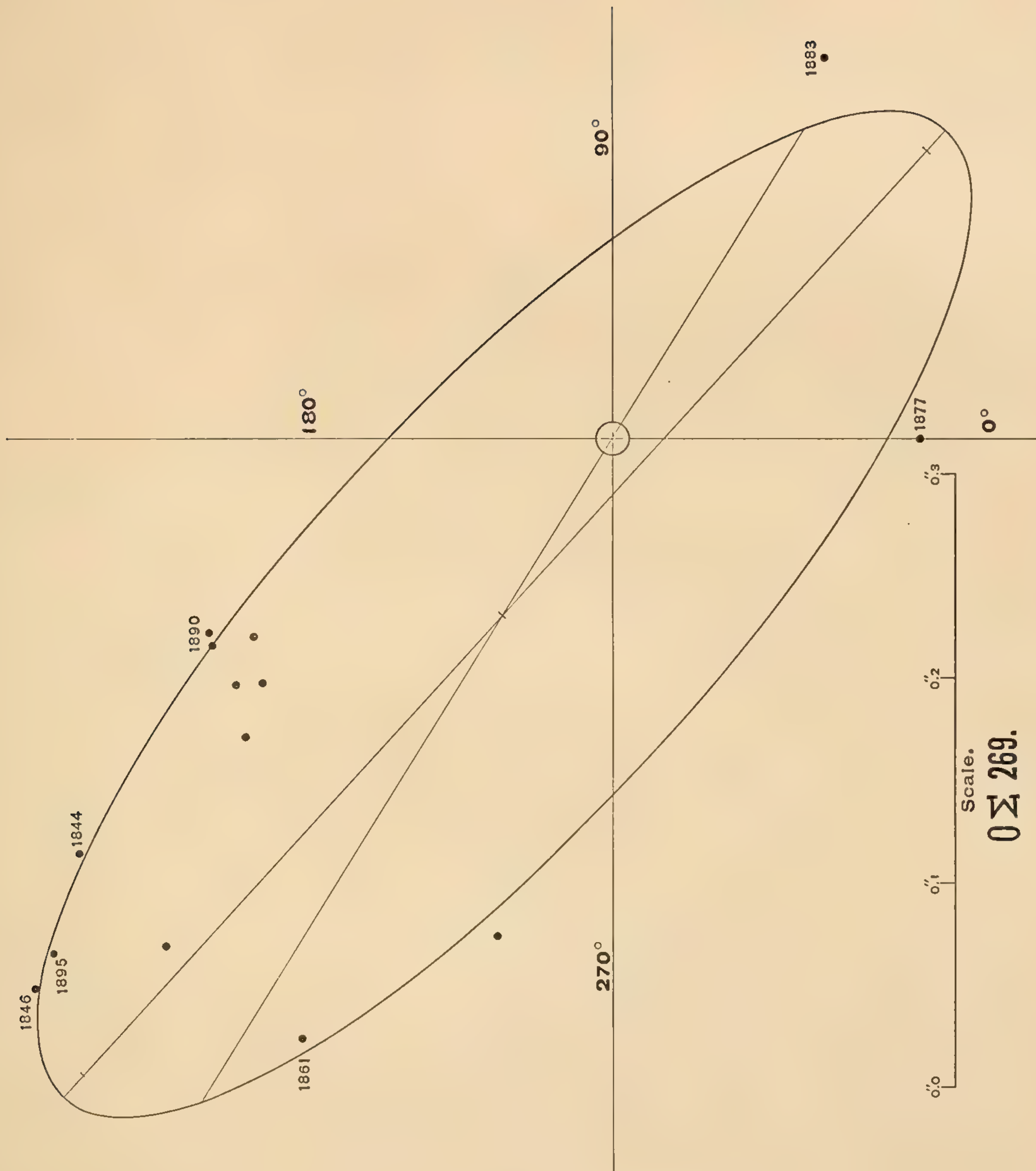
Length of major axis	= 0".64
Length of minor axis	= 0".20
Angle of major axis	= 47°.7
Angle of periastron	= 57°.8
Distance of star from centre	= 0".102

The period here found is undoubtedly very nearly correct, but the other elements are subject to greater uncertainty. However, the observation of ENGLEMAN in 1883 and DEMBOWSKI's estimate in 1877, establish the essential nature of the periastron end of the apparent ellipse, and assure us that no large correction of our apparent orbit will ever be required. The eccentricity is not likely to be altered by more than ± 0.05 , nor can the node and inclination suffer changes which are proportionately larger. Thus it appears that the orbit is very satisfactory for the scant material now available; and while large corrections are not to be anticipated, it will be desirable to improve upon these elements when more good measures are secured. The ephemeris shows that the star will be comparatively easy for a good many years, and it will therefore commend itself to the regular attention of observers.

t	θ_c	ρ_c	t	θ_c	ρ_c
1896.40	222.4	0.37	1899.40	226.9	0.41
1897.40	224.0	0.39	1900.40	228.2	0.41
1898.40	225.5	0.40			

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1844.31	218.0	215.6	0.33	0.30	+ 2.4	+0.03	1	O. Struve
1846.39	223.8	219.9	0.39	0.35	+ 3.9	+0.04	1	O. Struve
1851.34	228.9	227.9	0.33	0.41	+ 1.0	-0.08	1	O. Struve
1861.26	242.8	243.0	0.33	0.34	- 2.0	-0.01	1	O. Struve
1872.47	257.1	298.6	oblong	0.12	-41.5	—	1	O. Struve
1877.26	0.0?	28.0	oblonga	0.19	-28.0	—	1	Dembowski
1883.41	61.4	62.8	0.22	0.16	- 1.40	+0.06	4	Englemann
1889.52	207.7	199.5	0.22	0.17	+ 8.2	+0.05	3	Schiaparelli
1890.41	206.3	205.4	0.22	0.21	+ 0.9	+0.01	1	Schiaparelli
1891.26	213.4	209.5	0.22	0.24	+ 3.9	-0.02	3	Burnham
1891.49	208.9	210.4	0.20	0.24	- 1.5	-0.04	2-1	Schiaparelli
1892.40	215.0	213.6	0.21	0.28	+ 1.4	-0.07	2	Burnham
1895.07	222.9	220.0	0.37	0.35	+ 2.9	+0.02	2	Comstock 1; See 1
1895.41	219.0	220.7	0.23	0.35	- 1.7	-0.12	2	Schiaparelli



25 CANUM VENATICORUM = Σ 1768.

$\alpha = 13^h 33^m$; $\delta = +36^\circ 48'$.
5, white ; 8.5, blue.

Discovered by William Struve in 1827.

OBSERVATIONS.

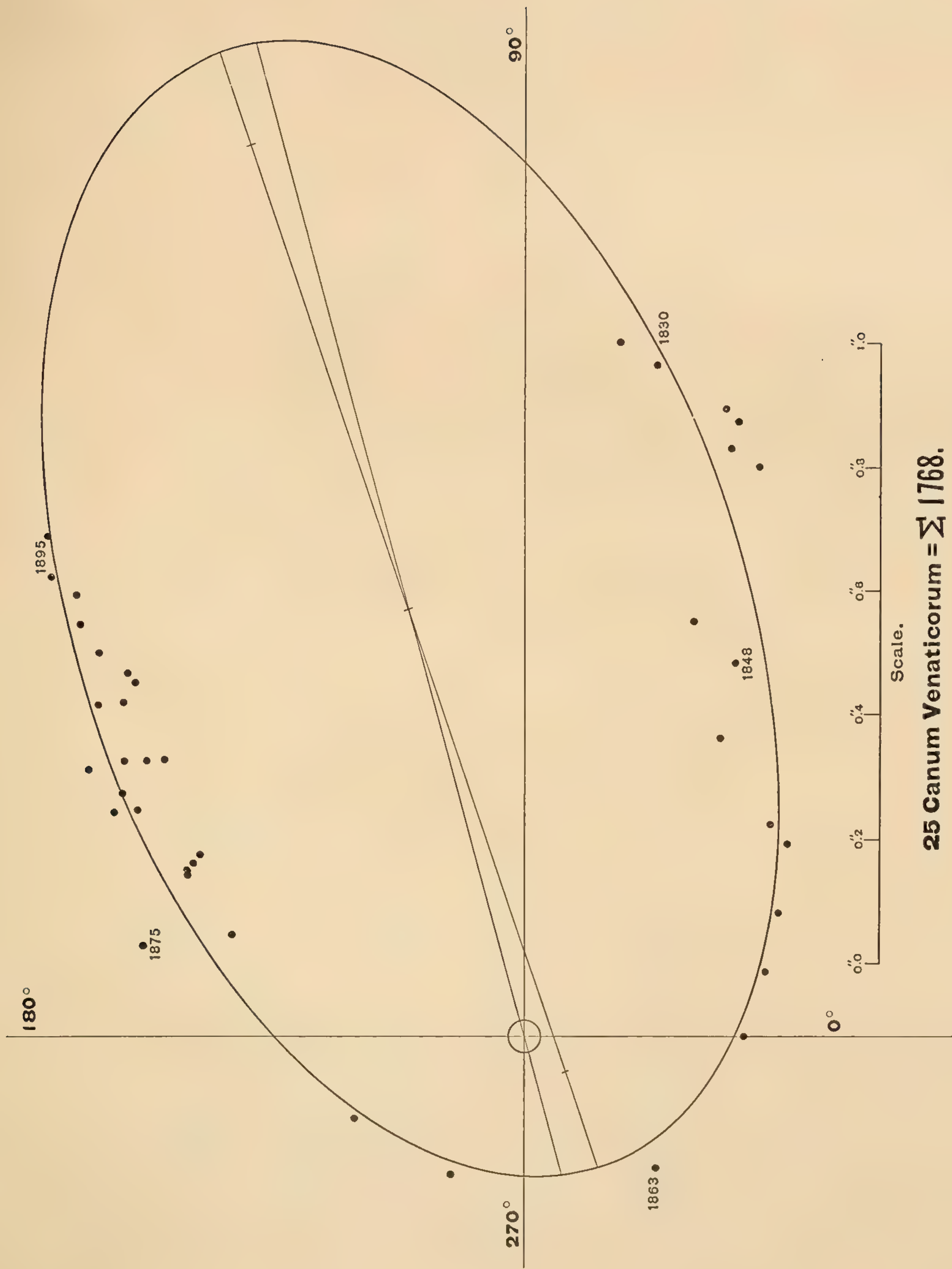
t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1829.89	79.6	1.05	5	Struve	1872.38	round	—	1	W. & S.
1833.12	72.4	1.09	5	Struve	1872.47	58 ?	—	1	O. Struve
1836.50	71.8	1.07	3	Struve	1875.36	single	—	1	Hall
1841.17	72.6	1.01	4	O. Struve	1875.48	167.1	0.63	1	O. Struve
1841.37	70.8	1.00	4-3	Mädler	1875.49	round	—	1	Dunér
1842.35	67.7	0.99	3-1	Dawes	1876.42	doubtful	—	1	Hall
1843.35	70.2	1.02	2	Dawes	1876.45	161.4	0.4 $\pm$	4	Schiaparelli
1843.52	70.5	0.71	3	Mädler	1877.37	154.5	0.4 $\pm$	10	Schiaparelli
1846.80	67.8	0.72	3	O. Struve	1877.54	154.7	0.60	1	O. Struve
1847.71	55.3	0.40	1	Mädler	1878.41	151.8	0.75	4	Dembowski
1849.77	65.6	0.65	3	O. Struve	1879.43	155.7	0.5 $\pm$	5	Schiaparelli
1851.28	56.5	0.39	6-4	Mädler	1879.49	157.5	0.51	5	Hall
1852.32	45.0	0.3 $\pm$	4	Mädler	1880.37	157.5	0.35	2	Hall
1853.32	36.2	0.35 $\pm$	1	Mädler	1880.46	155.0	0.60	2	Burnham
1854.43	36.2	0.35 $\pm$	3	Dawes	1881.24	27.6	—	1	Doberek
1854.78	46.2	0.35 $\pm$	2	Mädler	1881.32	151.6	0.49	1	Bigourdan
1856.49	25.7	oblonga	—	Secchi	1881.40	153.4	0.60 $\pm$	5	Schiaparelli
1858.65	26.7	0.2 $\pm$	2	Mädler	1881.40	157.4	0.53	3	Hall
1859.41	single	—	1	O. Struve	1881.43	155.9	0.41	3	Burnham
1860.36	10-15	0.15 $\pm$	1	Dawes	1882.27	16.0	—	1	Doberek
1861.26	single	—	1	O. Struve	1882.33	149.3	0.75	5	Englemann
1861.58	44.5	—	1	Mädler	1882.43	152.7	0.45	3	Hall
1862.39	single	—	1	O. Struve	1882.45	151.3	0.7 $\pm$	8	Schiaparelli
1862.95	180 ?	—	1	Dembowski	1883.42	147.0	0.59	1	Hall
1863.15	315 ?	—	1	Dembowski	1883.43	151.4	0.80	6	Englemann
1865.44	—	round	1	Dawes	1883.46	149.0	0.7 $\pm$	5	Schiaparelli
1868.13	127 ?	—	1	Dembowski	1883.51	149.2	0.53	2	Perrotin
1869.40	178 ?	—	1	Dunér	1884.33	143.8	—	2	Bigourdan
1870.43	186.	0.1 $\pm$	1	Dunér	1884.42	145.5	0.63	3	Hall
1871.45	47 ?	—	1	Dunér	1885.32	148.2	0.8 $\pm$	9	Schiaparelli
					1885.37	149.1	0.89	3	Perrotin
					1885.54	149.6	0.77	3	Tarrant
					1886.38	143.1	—	1	Perrotin
					1886.45	145.2	0.78	4	Hall
					1886.51	146.7	0.78	4	Schiaparelli

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1887.41	145.8	0.67	4	Hall	1892.17	137.5	0.98	3	Burnham
1887.46	142.7	0.72	9	Schiaparelli	1892.64	140.0	0.95	3-2	Comstock
1888.44	145.8	0.73	3	Hall	1893.50	138.4	0.81	2	Schiaparelli
1888.54	142.9	0.76	5	Schiaparelli	1893.58	138.9	0.89	1	Comstock
1889.48	140.5	0.84	5-4	Schiaparelli	1894.47	138.1	0.86	1	Schiaparelli
1890.42	137.9	0.81	4	Schiaparelli	1895.11	132.6	1.35	3	Barnard
1891.48	141.4	0.80	4	Schiaparelli	1895.20	134.5	1.11	4-5	Barnard
1891.51	143.6	0.93	3	Maw	1895.28	136.4	1.06	3-4	See
					1895.52	137.4	0.90	2	Comstock

The observations of this remarkable system prior to 1840 gave evidence of a slow retrograde motion, and accordingly it received the attention of OTTO STRUVE, MÄDLER, DAWES, and subsequent observers. Up to this time the radius vector has swept over 308° of position-angle, while the distance has diminished from $1''.13$ to $0''.23$ and again increased to about its former value. The data furnished by observation do not suffice to fix the elements of the orbit with great accuracy, but we believe that it is now possible to get a fair approximation to the motion, and that the resulting elements will not be sensibly improved for a great many years.

When the measures of this star are examined it is found that they are far from satisfactory, and therefore we must not expect an agreement such as could be obtained for easier objects, where the components are wider or more nearly equal in magnitude. Some of the recorded measures are so inconsistent that the mean places must be formed with care, and even then the representation of the motion is not entirely satisfactory. The smaller distances have been under-measured, as is clear from the fact that a star of this difficulty could not be seen with small telescopes (such as those used between 1860 and 1875), unless separated by something like $0''.3$. Under these circumstances it seemed proper to increase the measured distances near periastron, in order that when plotted on the diagram of the apparent ellipse they might not convey to the reader an erroneous impression. In the table of computed and observed places, however, we have retained the original values, and it will be seen that the differences are not at all considerable. DOBERCK is the only astronomer who has previously computed an orbit for this pair; using measures up to 1880 he found:

$$\begin{array}{ll}
 P = 119.9 \text{ years} & \Omega = 42^\circ.4 \\
 T = 1863.0 & i = 33^\circ.3 \\
 e = 0.72 & \lambda = 245^\circ.0 \\
 a = 0''.81 &
 \end{array}$$



25 Canum Venaticorum = Σ 1768.

A careful investigation of all the observations leads to the following elements of 25 *Canum Venaticorum*:

$$\begin{array}{ll} P = 184.0 \text{ years} & \Omega = 123^\circ.0 \\ T = 1866.0 & i = 33^\circ.5 \\ e = 0.752 & \lambda = 201^\circ.0 \\ a = 1''.1307 & n = -1^\circ.9565 \end{array}$$

Apparent orbit:

$$\begin{array}{ll} \text{Length of major axis} & = 1''.91 \\ \text{Length of minor axis} & = 1''.08 \\ \text{Angle of major axis} & = 108^\circ.9 \\ \text{Angle of periastron} & = 285^\circ.4 \\ \text{Distance of star from centre} & = 0''.714 \end{array}$$

This orbit is remarkably eccentric, and so far as known is surpassed in this respect by four stars only — γ *Virginis* (0.9), γ *Andromedae* (0.85), γ *Centauri* (0.80) and 99 *Herculis* (0.78). Whatever changes may hereafter be required in these results, it is certain that the eccentricity will remain conspicuous, and will not be varied sensibly from the value here obtained. The period, however, remains uncertain by perhaps 25 years, so that the motion of the system is not so well determined as could be desired. An ephemeris is appended for the use of observers.

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1827.28	82.4	79.7	1.13	1.15	+ 2.7	-0.02	1	Struve
1830.54	78.9	77.1	1.10	1.09	+ 1.8	+0.01	4-3	Struve
1833.12	72.4	74.8	1.06	1.04	- 2.2	+0.02	5-4	Struve
1836.50	71.1	71.5	1.05	0.96	- 0.4	+0.09	2	Struve
1841.37	70.8	65.6	1.00	0.85	+ 5.2	+0.15	4-3	Mädler
1842.35	67.7	64.2	0.99	0.83	+ 3.4	0.06	3-1	Dawes
1846.80	67.8	56.7	0.72	0.71	+11.1	+0.01	3	O. Struve
1848.74	60.5	52.6	0.53	0.66	+ 3.9	-0.13	4	Mädler 1; O. Struve 3
1851.28	56.5	47.3	0.39	0.60	+ 9.2	-0.21	6-4	Mädler
1852.82	40.6	40.1	0.35 $\pm$	0.54	+ 0.5	-0.19	5-1	Mädler
1854.43	36.2	35.2	0.35 $\pm$	0.50	+ 1.0	-0.15	3	Dawes
1857.57	26.2	19.8	0.2 $\pm$	0.41	+ 6.4	-0.20	3-2	Secchi 1-0; Mädler 2
1860.36	15 $\pm$	356.9	0.15 $\pm$	0.33	+18.1	-0.18	1	Dawes
1862.95	0 ?	330.4	—	0.28	+29.6	—	1	Dembowski
1863.15	315 ?	328.4	oblonga	0.27	-13.4	—	1	Dembowski
1868.76	242.5	236.5	—	0.24	+ 6.0	—	2	Dembowski 1; Dunér 1
1870.94	206.5	205.2	elong.	0.29	-24.8	—	2	Dunér
1872.47	238 ?	190.1	—	0.35	+48 ?	—	1	O. Struve
1875.48	167.1	171.3	0.63	0.47	- 4.2	+0.16	1	O. Struve
1876.45	161.4	167.2	0.5 $\pm$	0.51	- 5.8	-0.01	4-1	Schiaparelli
1877.45	154.6	163.6	0.60	0.55	- 9.0	+0.05	11-1	Schiaparelli 10-0; O. Struve 1
1878.41	151.8	160.6	0.75	0.58	- 8.8	+0.17	4	Dembowski
1879.46	156.6	157.7	0.60	0.62	- 5.9	-0.02	10-1	Schiaparelli 5-1; Hall 5-0
1880.41	156.3	155.2	0.60	0.66	- 0.2	-0.06	4-2	Burnham 2; Hall 2-0

<i>t</i>	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	<i>n</i>	Observers
1881.40	153.4	153.3	0.60	0.69	+ 0.1	−0.09	5	Schiaparelli
1882.39	150.3	151.0	0.72	0.73	− 0.7	−0.01	13	Englemann 5; Schiaparelli 8
1883.45	149.1	149.0	0.75	0.76	+ 0.1	−0.02	14–11	Hl. 1–0; En. 6; Sch. 5; Per. 2–0
1884.42	145.5	147.4	0.66	0.80	− 1.9	−0.14	3–1	Hall
1885.41	149.0	145.8	0.82	0.82	+ 3.2	±0.00	15	Sch. 9; Perrotin 3; Tarrant 3
1886.48	146.0	144.2	0.78	0.86	+ 1.8	−0.08	8	Hall 4; Schiaparelli 4
1887.46	142.7	143.0	0.73	0.88	+ 1.3	−0.15	13–9	Schiaparelli 9
1888.49	144.3	141.5	0.75	0.92	+ 2.8	−0.17	8	Hall 3; Schiaparelli 5
1889.48	140.5	140.7	0.84	0.94	− 0.2	−0.10	5–4	Schiaparelli
1890.42	137.9	139.3	0.84	0.97	− 1.4	−0.13	4–3	Schiaparelli
1891.50	142.5	138.1	0.87	1.00	+ 4.4	−0.13	7	Schiaparelli 4; Maw 3
1892.17	137.5	137.5	0.97	1.02	± 0.0	−0.05	6–5	Burnham 3
1893.54	138.6	136.1	0.92	1.05	+ 2.5	−0.13	3	Schiaparelli 2; Comstock 1
1894.47	138.1	135.4	0.86	1.07	+ 2.7	−0.21	1	Schiaparelli
1895.20	133.9	134.7	1.11	1.09	− 0.8	+0.02	7–5	Barnard
1895.28	136.4	134.6	1.06	1.09	+ 1.8	−0.03	3–4	See

EPHEMERIS.					
<i>t</i>	θ_c	ρ_c	<i>t</i>	θ_c	ρ_c
1896.50	134.0	1.11	1899.50	131.6	1.17
1897.50	133.2	1.13	1900.50	130.9	1.19
1898.50	132.4	1.15			

α CENTAURI.

$\alpha = 14^h\ 32^m.6$; $\delta = -60^\circ\ 25'$.
1, orange yellow ; 2, orange yellow.

Discovered by Father Richaud at Pondicherry, India, December, 1689.

OBSERVATIONS.									
<i>t</i>	θ_o	ρ_o	<i>n</i>	Observers	<i>t</i>	θ_o	ρ_o	<i>n</i>	Observers
1690.0	—	—	1	Richaud	1834.33	217.33	17.83	1	Herschel†
1709.5	—	—	1	Feuillée	1834.45	218.78	17.50	2	Herschel
1752.20	218.73	20.51	—	Lacaille	1835.08	218.80	17.33	1	Herschel
1761.5	—	15.6	1	Maskelyne	1835.89	219.59	17.02	11–1	Herschel
1822.00	209.6	28.75	—	Fallows*	1836.61	220.26	16.76	1	Herschel
1824.00	215.41	22.45	35+	Brisbane	1837.22	220.65	16.39	4	Herschel
1826.01	213.18	22.45	—	Dunlop	1840.00	223.2	14.74	—	Maclear
1830.01	215.03	19.95	—	Johnson	1846.21	232.4	10.96	3	Jacob
1831.00	215.97	22.56	—	Taylor*	1846.80	234.3	9.56	4	Jacob
1832.16	216.35	19.85	—	Johnson and Taylor*	1847.09	235.7	9.33	2–3	Jacob
1833.0	217.45	18.67	7±	Henderson	1847.36	234.5	9.31	3	Jacob
					1848.00	237.93	8.05	13–12	Jacob

*Taken on the authority of SIR JOHN HERSCHEL.
†HERSCHEL'S means have been formed anew.

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1849.63	244.5	6.23	—	Jacob	1854.63	283.44	—	3	Powell
1849.94	245.25	6.96	1	Maclear	1854.66	282.81	4.43	5	Maclear
1849.97	245.42	7.04	3-2	Maclear	1854.93	285.88	3.96	5-4	Maclear
1850.10	246.63	7.01	1	Maclear	1854.96	288.02	—	2	Powell
1850.17	245.75	7.08	6	Maclear	1855.06	289.32	—	10	Powell
1850.20	245.85	6.84	3	Maclear	1855.23	290.19	4.38	3	Maclear
1850.31	247.07	6.75	4	Maclear	1855.29	292.60	—	5	Powell
1850.37	247.52	6.52	7	Jacob	1855.33	293.8	4.11	10	Powell
1850.38	245.74	7.12	1	Maclear	1855.36	291.96	4.38	4	Maclear
1850.41	242.0	7.78	15	Gilliss	1855.54	294.73	—	5	Powell
1850.61	248.84	6.58	3	Maclear	1856.02	301.02	3.99	11-6	Powell
1850.64	249.1	6.20	7	Jacob	1856.02	302.13	3.85	7-6	Maclear
1850.92	250.27	5.88	6	Jacob	1856.10	303.06	3.88	18	Jacob
1850.94	251.84	6.02	3	Maclear	1856.38	306.92	4.05	1	Maclear
1851.02	251.05	5.88	8	Jacob	1856.51	309.84	3.93	10-9	Jacob
1851.08	252.50	6.12	3	Maclear	1856.91	311.26	4.21	4	Mann
1851.20	252.13	5.94	10-8	Jacob	1856.94	311.88	—	11	G. Maclear
1851.33	253.92	6.02	5	Maclear	1856.95	310.78	4.05	6	Mann
1851.56	254.42	5.88	3	Maclear	1856.96	315.77	3.96	10-9	Jacob
1851.70	256.38	5.27	8	Jacob	1857.15	318.19	4.02	15	Jacob
1851.94	256.58	5.80	3	Maclear	1857.39	320.60	4.47	2-1	Maclear
1851.94	258.2	5.11	9-8	Jacob	1857.86	326.48	4.14	14	Jacob
1851.99	258.85	5.08	8-7	Jacob	1858.17	330.51	4.39	5	Jacob
1852.25	259.02	5.72	3	Maclear	1858.23	339.42	5.09	3	Maclear
1852.27	261.07	5.03	7	Jacob	1859.34	339.71	5.18	15-12	Powell
1852.38	261.88	4.94	6	Jacob	1859.43	343.44	5.10	5	Mann
1852.43	261.67	5.27	5	Maclear	1859.52	341.8	4.92	4	Powell
1852.53	264.16	5.00	4	Jacob	1859.97	346.08	5.00	3	Mann
1852.56	262.8	5.03	—	Maclear	1860.05	346.55	—	1	G. Maclear
1852.58	262.89	5.18	7-9	Maclear	1860.09	345.4	5.65	17-13	Powell
1852.73	262.45	4.95	5-2	Maclear	1860.18	349.34	5.52	4-1	Maclear
1852.79	263.31	—	4	Maclear	1860.35	348.87	—	3	Maclear
1853.05	267.67	4.55	—	Jacob	1860.48	348.7	5.68	1	Powell
1853.13	266.54	4.84	4-6	Maclear	1861.05	351.08	6.07	10-9	Powell
1853.15	268.33	4.59	—	Jacob	1861.09	353.65	6.09	3	Maclear
1853.34	268.72	4.87	5	Maclear	1861.31	353.03	6.21	7	Powell
1853.50	271.03	4.68	6	Maclear	1861.58	354.26	6.32	5-3	Powell
1853.58	272.17	4.57	2-1	Mann	1862.0	0.0	10.0	—	Ellery*
1853.58	270.10	—	—	Powell	1862.20	357.84	6.80	7	Powell
1853.92	275.19	4.44	4-3	Maclear	1862.47	0.0	—	—	Ellery†
1854.00	276.63	4.21	—	Jacob	1862.56	1.38	7.55	3	Maclear
1854.03	276.85	—	7	Powell	1863.03	1.4	7.2	6-4	Powell
1854.24	278.98	4.62	4	Maclear	1863.75	5.2	8.5	—	Ellery
1854.25	279.06	4.16	2	Jacob					
1854.26	279.62	—	4	Powell					

* Apparently a rough "guess."

† From transit observations.

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1864.11	5.7	7.85	7-5	Powell	1878.16	116.98	1.77	1	Russell
1864.72	—	8.1	—	Ellery	1878.22	119.82	1.95	3	Russell
1865.56	17.3	9.95	1	Ellery*	1878.28	127.37	1.77	1	Russell
1866.06	11.1	9.3	3	Powell	1878.38	139.10	2.40	—	Maxwell Hall
1868.17	—	9.2	—	Ellery	1879.25	174.40	3.41	—	Ellery
1868.18	—	9.6	—	Ellery	1879.47	173.55	3.41	2	Hargrave
1868.38	13.59	10.29	2	Mann	1880.18	183.9	5.22	4	Tebbutt
1868.51	21.8	11.02	5	Ellery*	1880.39	185.2	5.56	3	Tebbutt
1869.13	17.97	10.4	2	Powell	1880.45	184.98	5.52	1	Russell
1870.1	20.45	10.24	13-12	Powell	1881.28	189.88	5.07	1	Hargrave
1870.61	21.8	10.09	5-4	Powell	1881.54	190.13	7.52	1	Hargrave
1870.65	—	10.2	—	Ellery	1881.65	193.15	7.94	2	Tebbutt
1870.65	24.7	10.45	3	Ellery*	1882.00	194.44	8.23	18	Gill
1870.75	22.53	10.46	4	Russell	1882.22	191.6	8.70	1	Tebbutt
1871.05	23.01	9.89	11	Powell	1882.50	195.82	9.12	52	Elkin
1871.31	23.7	9.8	7	Powell	1884.19	199.0	11.96	—	Russell
1871.48	22.91	10.22	2	Russell	1884.43	199.5	12.32	—	Russell†
1871.51	24.2	9.41	1	Ellery	1884.53	199.80	12.93	6	Tebbutt
1872.47	25.31	9.73	2	Russell	1885.56	200.8	14.05	4-3	Tebbutt
1872.55	24.1	10.36	1	Ellery	1886.27	202.5	14.89	5	Pollock
1873.16	—	8.3	—	Ellery	1886.38	200.4	14.74	1	Russell
1873.33	28.1	9.50	1	Russell	1886.52	201.2	15.19	1	Russell
1874.15	30.5	8.0	—	Ellery	1886.55	201.02	14.87	4	Pollock†
1874.47	30.0	7.97	2	Russell	1886.56	202.42	15.13	10	Pollock
1874.85	34.17	—	—	Lindsay	1886.58	201.7	15.18	3	Russell
1875.02	34.21	6.82	—	Seeliger	1886.60	201.41	15.16	4	Tebbutt
1875.94	39.3	6.68	1	Ellery*	1887.39	202.3	16.06	3-5	Tebbutt
1876.41	46.97	4.35	2	Russell	1887.43	202.08	15.83	6-5	Pollock
1876.61	51.05	4.15	2	Ellery	1887.60	202.35	16.28	3-2	Tebbutt
1876.90	64.3	4.94	1	Ellery	1887.72	202.16	16.18	2	Tebbutt
1876.94	51.2	4.5	1	Ellery	1887.74	203.0	15.73	4	Pollock
1877.14	64.4	3.30	—	Maxwell Hall	1888.30	203.4	16.87	3	Tebbutt
1877.25	69.1	3.13	5	Ellery	1888.63	202.93	17.12	1	Tebbutt
1877.52	72.77	2.60	2-1	Russell	1889.45	204.5	17.91	3	Pollock
1877.56	77.25	2.11	3	Russell	1890.41	205.2	18.58	2	Tebbutt
1877.57	80.50	2.13	2-3	Gill	1890.47	204.75	18.66	4-3	Sellors
1877.59	81.74	1.90	3	Russell	1890.60	205.05	19.06	3-2	Sellors
1877.63	81.49	1.94	3-1	Gill	1890.74	204.6	18.69	1-3	Tebbutt
1877.82	97.12	1.85	2-3	Gill	1891.43	205.62	19.15	5-4	Sellors
1877.89	101.12	1.62	2	Gill	1891.56	207.17	19.25	4-2	Tebbutt

* From transit observations.

† From $\Delta\alpha$ and $\Delta\delta$.

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1891.57	205.3	19.24	2	Sellors	1893.21	206.75	20.22	2-1	W.H.Pickering
1891.64	206.4	19.35	3-6	Tebbutt	1893.34	206.4	19.92	8	Sellors
1892.30	206.45	19.52	2	Gill & Finlay*	1893.42	206.73	20.32	6-4	Tebbutt
1892.40	205.46	19.73	5-4	Sellors	1893.49	206.5	20.24	8	Sellors
1892.45	205.53	19.75	7-4	Tebbutt	1893.50	206.75	20.53	4-2	Tebbutt
1892.58	205.83	19.73	8-5	Tebbutt	1894.47	207.2	20.58	6	Sellors
1892.76	206.9	19.96	1	W.H.Pickering	1894.78	208.0	20.72	19-11	Tebbutt
1893.21	206.9	20.04	1	A. E. Douglass	1895.55	207.8	20.97	16-10	Tebbutt

In attempting to investigate the orbit of α *Centauri* it seems desirable to review briefly the work already done on this celebrated system.

The record left us by RICHARD does not throw much light upon the nature of the orbit, but is of considerable historical interest:

“Regardant à l’occasion de la Comète plusieurs fois les pieds du *Centaure* avec une lunette d’environ douze pieds, je remarquai que le pied le plus oriental et le plus brillant étoit une double étoile aussi bien que le pied de la croisade; avec cette différence que dans la croisade, une étoile paraît avec la lunette notablement éloignée de l’autre; au lieu qu’ au pied du *Centaure*, les deux étoiles paraissent même avec la lunette presque se toucher; quoique cependant on les distingue aisément.”†

The next record of α *Centauri* was made by FATHER FEUILLÉE, who observed at Lima, Peru, July 4, 1709; in his *Journal des Observations, &c.*, Paris, 1714, tome I, p. 425, we find the following account:

“Sur les deux heures du matin, en attendant que je pusse observer l’émergence du premier satellite de *Jupiter*, que des nuages me cachèrent, j’observai avec une lunette de 18 pieds l’étoile de la premier grandeur qui est au pied boréal du devant du *Centaure*; je trouvai cette étoile composée de deux, dont l’une est de la troisième grandeur et l’autre de la quatrième. Celle de la quatrième grandeur est la plus occidentale, et leur distance est égale au diamètre de cette étoile.”

From this rather indefinite observation POWELL infers that the distance of the components in 1709 was about 10", and attaches considerable importance to the remark that the companion was “the more westerly” (la plus occidentale). Unfortunately the language is rather ambiguous, and we can not tell whether FEUILLÉE meant that the companion was really to the west of the central star, or whether it merely appeared to the west in the inverted field of view. As

* By photography.

† Publications of the Royal Academy of Sciences, Paris, 1692; or *Monthly Notices*, 1884-5, p. 18.

α *Centauri* was low in the southwest when the observation was made, it is also possible that the remark may have arisen, as MR. ROBERTS has observed, from the position of the heavens at that instant rather than the position-angle of the companion. In any case it follows from the orbit here deduced that the position-angle was $24^{\circ}.3$, and the distance $10''.07$.

The third observation of α *Centauri* was made by LACAILLE at the Cape of Good Hope in 1752. While determining the positions of southern stars he observed the components of α *Centauri*, and from the resulting $\Delta\alpha$ and $\Delta\delta$ we find the values of ρ and θ given in the list of measures. The observations of LACAILLE were first printed in the *Cœlum Australe Stelliferum*, which was published at Paris in 1763, and reprinted in 1847 by the British Association for the Advancement of Science, under the auspices of a Committee composed of HERSCHEL, HENDERSON and BAILY. LACAILLE's observations appear to be as good as could be expected from the instruments and methods employed.

In 1761 α *Centauri* was observed on one night by MASKELYNE while at the island of St. Helena; by means of a rough divided-object-glass micrometer he found a distance of $15''.6$.

The observations made early in the present century by FALLOWS, BRISBANE, DUNLOP, JOHNSON, TAYLOR and HENDERSON, rest on measures of $\Delta\alpha$ and $\Delta\delta$. The observation of FALLOWS was made with a small and defective Altitude and Azimuth Instrument, and is entirely erroneous. For a long time this measure was very misleading to computers, as it indicated an eccentricity of about 0.96. The results of BRISBANE, DUNLOP, JOHNSON, TAYLOR and HENDERSON are likewise unworthy of any high degree of confidence. The first observations of conspicuous worth are the micrometrical measures made by SIR JOHN HERSCHEL at the Cape of Good Hope. The measures of HERSCHEL taken in conjunction with others recently made expressly for the purpose have enabled us to determine the orbit of α *Centauri* with a degree of precision which appears extraordinary when we consider the character of the observations. It will be found on inspecting the list of measures that many of them are vitiated by sensible errors of observation, which are partly systematic and partly accidental. We must remember, however, in judging of the value of results that α *Centauri* is a very bright star, so that the images are unusually large; and hence if the telescope is not practically perfect, and the atmospheric conditions favorable, we could hardly expect that the measures will be very accordant. It is also to be remembered that the southern observers are not specialists in double-star work, and hence we can not expect results such as could be obtained by the skill of a BURNHAM or a STRUVE. Nevertheless, the measures of α *Centauri*

taken as a whole, will enable us to obtain one of the best orbits yet deduced for any binary, and we may gratefully acknowledge our deep obligation to the southern observers, who amid many difficulties have measured this star with care and assiduity.

In the list of measures given above will be found all the records which are of any value. The observations of T. MACLEAR, G. MACLEAR and W. MANN, which were made about the middle of the century, are taken from DR. ELKIN'S *Inaugural Dissertation*, in which they were first printed; the number of nights was kindly supplied by DR. ELKIN in a private letter. Most of the other measures are taken from the *Memoirs* and *Monthly Notices* of the Royal Astronomical Society. In this connection I take occasion to acknowledge my special obligations to MESSRS. TEBBUTT, PICKERING, DOUGLASS, RUSSELL, SELLORS, GILL and FINLAY for securing sets of measures expressly for this investigation; also to thank HERR HANS LUDENDORFF of the Royal Observatory, Berlin, for confirming from original sources the measures of LACAILLE, BRISBANE, DUNLOP and JOHNSON.

Most of the orbits determined before 1875 have now only historical interest, and among those more recently determined only three are approximately correct; namely, those of ROBERTS (*A.N.*, 3175), SEE (*M.N.*, Dec., 1893), and DOBERCK (*A.N.*, 3330). The following table of the elements found by previous computers is essentially complete:

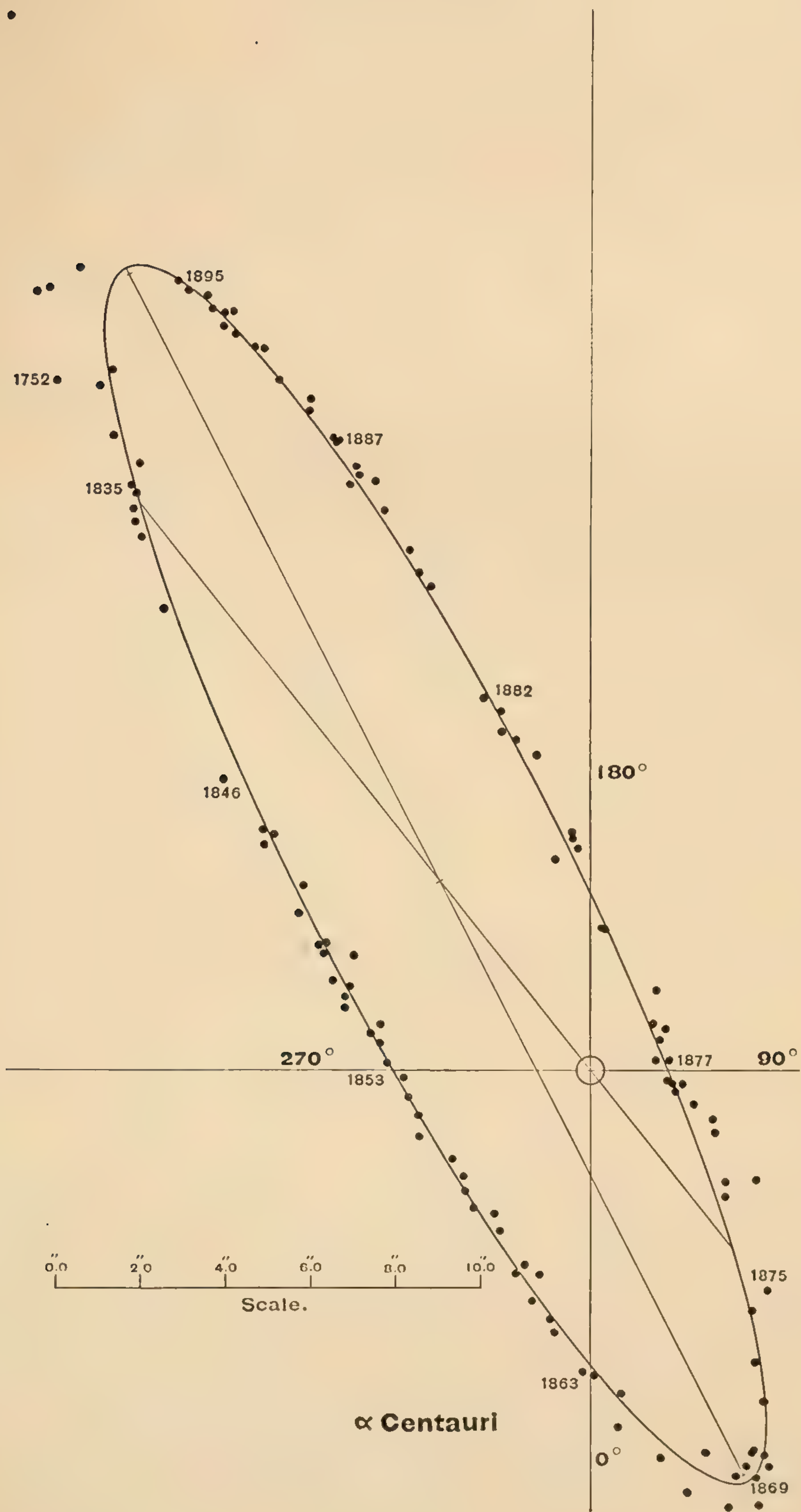
<i>P</i>	<i>T</i>	<i>e</i>	<i>a</i>	Ω	<i>i</i>	λ	Authority	Source
^{yrs.} 77.0	1851.50	0.950	15.5	86.12	47.77	291.37	Jacob, 1848	Mem. R.A.S., XVII, p. 88
79.0	1863.25	0.818	—	—	—	32.7	Jacob	A.N., XLIV, p. 43
80.94	1859.42	0.7752	13.57	16.7	62.9	26.03	Hind, 1851	
75.3	1858.012	0.966	30.	177.83	77.83	—	Powell, 1854	Mem. R.A.S., XXIV, 93
82.59	1857.012	0.969	31.76	2.6	77.3	27.65	Powell, 1854	Mem. R.A.S., XXIV, 93
77.81	1871.345	0.7033	20.575	22.35	80.95	58.43	Copeland, 1869	
76.25	1874.2	0.63944	20.13	24.3	81.22	59.2	Powell, 1870	M.N., XXX, 192
85.042	1874.85	0.6673	21.797	21.8	82.3	59.53	Hind, 1877	M.N., XXXVII, 97
88.536	1875.12	0.5332	18.45	25.23	79.4	45.97	Doberck, 1877	A.N., 3330
77.42	1875.97	0.5260	17.50	25.78	79.53	54.83	Elkin, 1880	Dissertation, p. 8
76.222	1875.951	0.5158	17.33	25.51	79.25	54.98	Downing, 1884	M.N., XLIV, 239
87.438	1875.45	0.544	18.89	25.83	79.78	48.98	Powell, 1886	M.N., XLVI, 337
80.34	1875.74	0.526	17.20	25.22	79.53	52.5	Gill 1882	Mem. R.A.S., XLVIII, 15
81.185	1875.715	0.52865	17.71	25.1	79.36	52.02	Roberts, 1893	A.N., 3175
81.07	1875.62	0.520	17.705	25.45	79.74	51.56	See, 1893	M.N., Dec. 1893
79.123	1876.02	0.51184	18.45	25.42	79.23	52.88	Doberck, 1895	A.N., 3330
83.565	1875.57	0.52252	18.165	25.9	79.32	49.42	Doberck, 1895	A.N., 3330

After careful study of all the observations we have formed mean places and reduced them for precession to 1900.0. These places are given in the

accompanying table, which also contains the comparison resulting from the elements found below.

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1690.00	—	258.94	—	6.67	—	—	1	Richaud
1709.50	—	23.86	—	9.94	—	—	1	Feuillée
1752.2	217.84	217.21	20.51	18.36	+0.63	+2.15	—	Lacaille
1822.0	209.05	211.17	28.75	22.06	-2.12	+6.69	—	Fallows
1824.0	214.88	212.21	22.15	21.28	+2.67	+1.17	35+	Brisbane
1826.01	212.66	213.07	22.45	21.26	-0.41	+1.19	—	Dunlop
1830.01	214.54	215.01	19.95	19.95	-0.47	± 0.00	—	Johnson
1831.0	215.49	215.77	22.56	19.28	-0.28	+3.28	—	Taylor
1832.16	215.87	216.47	19.85	18.68	-0.60	+1.17	—	Johnson and Taylor
1833.0	216.98	217.03	18.67	18.42	-0.05	+0.25	7 $\pm$	Henderson
1834.33	216.87	217.92	17.83	17.68	-1.05	+0.15	1	Herschel
1834.45	218.32	217.99	17.50	17.67	+0.33	-0.17	2	Herschel
1835.08	218.35	218.47	17.33	17.63	-0.12	-0.30	2-1	Herschel
1835.89	219.14	219.07	17.02	17.06	+0.07	-0.04	11-1	Herschel
1836.61	219.82	219.67	16.76	16.43	+0.15	+0.33	1	Herschel
1837.22	220.21	220.18	16.39	16.17	+0.03	+0.22	4	Herschel
1840.0	222.78	222.89	14.74	14.42	-0.11	+0.32	—	Maclear
1846.21	232.02	232.87	10.96	9.70	-0.85	+1.26	3	Jacob
1846.80	233.93	234.33	9.56	9.18	-0.40	+0.38	4	Jacob
1847.09	235.33	235.21	9.33	8.90	+0.12	+0.43	2-3	Jacob
1847.36	234.13	235.87	9.31	8.76	-1.74	+0.55	3	Jacob
1848.00	237.57	237.80	8.05	8.35	-0.23	-0.30	13-12	Jacob
1849.63	244.15	243.97	6.23	7.12	+0.18	-0.89	—	Jacob
1849.95	244.98	245.48	7.00	6.83	-0.50	+0.17	4-3	Maclear
1850.20	244.97	246.55	6.92	6.57	-1.58	+0.35	14	Maclear
1850.38	246.28	247.91	6.82	6.46	-1.53	+0.36	8	Jacob 1; Maclear 7
1850.41	241.65	247.96	7.78	6.44	-6.31	+0.34	15	Gilliss
1850.62	248.62	249.22	6.39	6.32	-0.60	+0.07	10	Maclear 3; Jacob 7
1850.93	250.7	250.48	5.95	6.10	+0.22	-0.15	10-9	Jacob 7-6; Maclear 3
1851.10	251.55	251.45	5.98	6.04	+0.10	-0.06	21-19	Jacob 8; Maclear 3; Jacob 10-8
1851.44	253.33	253.90	5.95	5.75	-0.57	+0.20	8	Maclear 5; Maclear 3
1851.87	256.14	256.55	5.53	5.53	-0.41	± 0.00	11	Jacob 8; Maclear 3
1851.95	258.18	257.19	5.09	5.48	-0.99	-0.39	17-15	Jacob 9-8; Jacob 8-7
1852.33	260.58	259.95	5.24	5.28	+0.63	-0.04	21	Maclear 3; Jacob 7; Jacob 6; Maclear 5
1852.64	262.78	262.40	5.03	5.01	+0.38	+0.02	20-15	Ja. 4; Mac. —; Mac. 7-9; Mac. 5-2; Mac. 4
1853.27	268.73	268.18	4.69	4.75	+0.55	-0.06	19-18	Ja. —; Mac. 4-6; Ja. —; Mac. 5; Mac. 6;
1853.75	272.32	272.61	4.44	4.50	-0.29	-0.06	4-3	Powell —; Maclear 4-3 [Mann 2-1
1854.16	277.91	277.60	4.33	4.36	+0.31	-0.03	17-6	Jacob —; Po. 7-0; Mac. 4; Ja. 2; Po. 4-0
1854.79	284.72	285.35	4.19	4.16	-0.63	+0.03	15-9	Powell 3-0; Mac. 5; Mac. 5-4; Po. 2-0
1855.25	291.26	290.90	4.29	4.20	+0.36	+0.09	32-17	Po. 10-0; Mac. 3; Po. 5-0; Po. 10; Mac. 4
1856.13	302.97	302.33	3.94	4.08	+0.64	-0.14	42-31	Po. 5; Po. 11-6; Mac. 7-6; Ja. 18; Mac. 1
1856.51	309.84	307.72	3.93	4.10	+2.12	-0.17	10-9	Jacob
1856.94	312.11	312.80	4.07	4.16	-0.69	-0.09	31-19	Mann 4; G. Mac. 11-0; Mann. 6; Ja. 10-9
1857.27	319.09	317.57	4.24	4.24	+1.52	± 0.00	17-16	Jacob 15; Maclear 2-1
1857.86	326.18	326.15	4.14	4.42	+0.03	-0.28	14	Jacob
1858.17	330.22	328.09	4.39	4.50	+2.13	-0.11	5	Jacob
1859.33	340.6	339.49	5.12	5.06	+1.11	+0.06	23-20	Maclear 3; Powell 15-12; Mann 5
1859.52	341.52	341.38	4.92	5.15	+0.14	-0.23	4	Powell
1859.97	345.8	344.89	5.00	5.40	+0.91	-0.40	3	Mann
1860.27	347.78	346.70	5.62	5.56	+1.08	+0.06	26-15	G. Mac. 1; Po. 17-13; Mac. 4-1; Mac. 3-0;
1861.07	352.09	351.96	6.08	6.05	+0.13	+0.03	13-12	Powell 10-9; Maclear 3 [Po. 1
1861.44	353.38	354.15	6.26	6.25	-0.77	+0.01	12-10	Powell 7; Powell 5-3
1862.41	359.47	358.15	7.17	7.10	+1.22	+0.07	10	Powell 7; Ellery —; Maclear 3



t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1863.03	1.14	2.64	7.2	7.34	-1.50	-0.14	6-4	Powell
1863.75	4.95	4.43	8.5	7.73	+0.52	+0.77	-	Ellery
1864.11	5.55	5.74	7.85	7.96	-0.19	-0.11	7-5	Powell
1864.72	—	7.70	8.1	8.38	—	-0.28	3±	Ellery
1865.56	17.06	10.14	9.95	8.75	+7.92	+1.20	1	Ellery
1866.06	10.86	11.50	9.3	9.22	-0.74	+0.08	3	Powell
1868.38	13.37	15.70	10.29	10.08	-2.33	+0.21	2	Mann
1868.51	21.58	16.09	11.02	10.13	+5.49	+0.89	5	Ellery
1869.13	17.75	18.45	10.4	10.40	-0.70	±0.00	2	Powell
1870.1	20.24	20.42	10.24	10.30	-0.18	-0.06	13-12	Powell
1870.61	21.59	21.47	10.09	10.28	+0.12	-0.19	5-4	Powell
1870.65	24.5	21.55	10.32	10.28	-0.05	+0.05	3-5±	Ellery
1870.75	22.33	21.78	10.46	10.26	+0.55	+0.20	4	Russell
1871.18	23.15	22.74	9.84	10.18	+0.41	-0.34	18	Powell 11; Powell 7
1871.49	23.35	23.30	9.82	10.08	+0.05	-0.26	3	Russell 2; Ellery 1
1872.51	24.51	25.07	10.04	9.60	-0.56	+0.44	3	Russell 2; Ellery 1
1873.25	27.91	28.34	8.90	8.90	-0.43	±0.00	1-3±	Ellery -; Russell 1
1874.31	30.07	31.00	7.98	7.60	-0.93	+0.38	3-4±	Ellery -; Russell 2
1875.02	34.04	34.45	6.82	6.50	-0.41	+0.32	-	Seeliger
1875.94	39.13	40.50	6.68	4.86	-1.37	+1.82	1	Ellery
1876.41	46.80	45.93	4.35	3.98	+0.87	+0.37	2	Russell
1876.61	50.89	50.09	4.15	3.56	+0.80	+0.59	2	Ellery
1876.92	57.09	55.86	4.72	3.00	+1.23	+1.72	2	Ellery
1877.14	64.4	61.50	3.30	2.60	+2.90	+0.70	-	Maxwell Hall
1877.25	68.94	69.95	3.13	2.45	-1.01	+0.68	5	Ellery
1877.52	72.61	77.41	2.60	2.10	-4.80	+0.50	2-1	Russell
1877.56	77.09	79.70	2.11	2.01	-1.61	+0.10	3	Russell
1877.57	80.34	80.21	2.13	2.00	+0.13	+0.13	2-3	Gill
1877.59	81.58	81.37	1.90	1.98	+0.21	-0.08	3	Russell
1877.63	81.33	83.58	1.94	1.92	-2.25	+0.02	3-1	Gill
1877.82	96.97	96.24	1.85	1.75	+0.73	+0.10	2-3	Gill
1877.89	100.97	101.15	1.62	1.70	-0.18	-0.08	2	Gill
1878.16	116.83	122.39	1.77	1.67	-5.56	+0.10	1	Russell
1878.22	119.67	127.51	1.95	1.68	-7.84	+0.27	3	Russell
1878.28	127.12	131.01	1.77	1.71	-3.89	+0.06	1	Russell
1878.38	138.95	138.22	2.40	1.78	+0.73	+0.62	-	Maxwell Hall
1879.25	174.25	172.21	3.41	3.12	+1.96	+0.29	-	Ellery
1879.47	173.41	176.35	3.41	3.56	-2.94	-0.15	2	Hargrave
1880.18	183.76	184.97	5.22	4.93	-1.21	+0.29	4	Tebbutt
1880.39	185.06	186.70	5.56	5.36	-1.64	+0.20	3	Tebbutt
1880.46	184.84	187.05	5.52	5.53	-2.21	-0.01	1	Russell
1881.28	189.75	191.72	5.07	7.11	-1.97	-2.04	1	Hargrave
1881.54	190.00	192.77	7.52	7.64	-2.77	-0.12	1	Hargrave
1881.65	193.02	193.18	7.94	7.82	-0.16	+0.12	2	Tebbutt
1882.00	194.44	194.33	8.23	8.45	+0.11	-0.22	18	Gill
1882.22	194.48	194.87	8.70	8.80	-0.39	-0.10	1	Tebbutt
1882.50	195.82	196.03	9.12	9.29	-0.21	-0.17	52	Elkin
1884.19	198.89	199.37	11.96	12.04	-0.48	-0.08	-	Russell
1884.43	199.39	199.55	12.32	12.38	-0.16	-0.06	-	Russell
1884.53	199.69	199.65	12.93	12.53	+0.04	+0.40	6	Tebbutt
1885.56	200.7	200.82	14.05	13.84	-0.12	+0.21	4-3	Tebbutt
1886.27	202.4	201.60	14.89	14.73	+0.80	+0.16	5	Pollock
1886.38	200.3	201.70	14.74	14.78	-1.40	-0.04	1	Russell
1886.54	201.46	201.85	15.06	15.01	-0.39	+0.05	15	Russell 1; Pollock 4; Pollock 10
1886.59	201.46	202.91	15.17	15.07	-1.45	+0.10	7	Russell 3; Tebbutt 4
1887.41	202.1	202.64	15.95	15.96	-0.54	-0.01	9-10	Tebbutt 3-5; Pollock 6-5
1887.69	202.30	202.87	16.09	16.24	-0.57	-0.14	9-8	Tebbutt 3-2; Tebbutt 2; Pollock 4
1888.30	203.32	203.34	16.87	16.87	-0.02	±0.00	3	Tebbutt

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1888.63	202.85	203.62	17.12	17.14	-0.77	-0.02	1	Tebbutt
1889.45	204.43	204.22	17.91	17.81	+0.21	+0.10	3	Pollock
1890.49	204.93	204.89	18.77	18.64	+0.04	+0.13	9-7	Tebbutt 2; Sellors 4-3; Sellors 3-2
1890.74	204.53	205.05	18.69	18.85	-0.42	-0.16	1-3	Tebbutt
1891.55	206.01	205.52	19.25	19.28	+0.49	-0.03	14	Sel. 5-4; T. 4-2; Sel. 2; T. 3-6
1892.30	206.45	205.97	19.52	19.73	+0.48	-0.18	2	Gill and Finlay
1892.43	205.47	206.04	19.74	19.83	-0.57	-0.09	12-8	Sellors 5-4; Tebbutt 7-4
1892.67	205.87	206.18	19.84	19.93	-0.31	-0.09	9-6	Tebbutt 8-5; Pickering 1
1893.25	206.70	206.50	20.06	20.21	+0.20	-0.16	11-10	Douglass 1; Pickering 2-1; Sellors 8
1893.47	206.66	206.59	20.36	20.30	+0.07	+0.06	18-14	Tebbutt 6-4; Sellors 8; Tebbutt 4-2
1894.62	207.6	207.21	20.65	20.81	+0.39	-0.16	25-17	Sellors 6; Tebbutt 19-11
1895.55	207.8	207.67	20.97	21.09	+0.13	-0.12	16-10	Tebbutt

In dealing with this orbit it seems probable that the graphical method will be superior to any process involving a least-square adjustment, because of the undoubted existence of sensible systematic errors in the observations. An adjustment based on both angles and distances will eventually be desirable, but before this definitive determination can be made with advantage, it will be necessary to have an additional revolution. In the present state of the observations it is wholly useless to apply corrections of a very minute character. Basing the work upon all the best observations we find the following elements of α Centauri:

$$\begin{aligned}
 P &= 81.1 \text{ years} & \Omega &= 25^\circ.15 \\
 T &= 1875.70 & i &= 79^\circ.30 \\
 e &= 0.528 & \lambda &= 52^\circ.00 \\
 a &= 17''.70 & n &= +4^\circ 43' 89.54
 \end{aligned}$$

Apparent orbit:

$$\begin{aligned}
 \text{Length of major axis} &= 32''.18 \\
 \text{Length of minor axis} &= 6''.16 \\
 \text{Angle of major axis} &= 27^\circ.25 \\
 \text{Angle of periastron} &= 38^\circ.65 \\
 \text{Distance of star from centre} &= 5''.90
 \end{aligned}$$

If we adopt the parallax of GILL and ELKIN ($0''.75$), we find that the major semi-axis of the orbit is 23.6 astronomical units. It follows that the combined mass of the components is 2.00 times the mass of the sun and earth.

Thus we see that the companion of α Centauri moves in an orbit with a major axis which is about a mean between those of *Uranus* and *Neptune*. But owing to the eccentricity of the orbit the distance at periastron (11.2) only slightly surpasses that of *Saturn* from the sun, while at apastron it extends considerably beyond *Neptune* (36.0).

According to preliminary researches of STONE in 1875, it was found that the masses of the two components are sensibly equal. MR. A. W. ROBERTS has

recently made a very careful determination of this mass-ratio, and finds (*A.N.*, 3313) that the masses of α^2 and α^1 (the companion) are as $51:49 \pm \frac{1}{5}\%$ of the amount. A very similar result was obtained by DR. ELKIN in his *Inaugural Dissertation*, and hence we may conclude that in this case the relative masses are known with almost the desired precision.

MR. ROBERTS has also made a careful discussion of the parallax of α *Centauri* from the meridian observations of 1879-81 and obtained (*A.N.*, 3324) results which confirm the work of GILL and ELKIN with the heliometer. Using both right ascensions and declinations MR. ROBERTS finds:

$$\pi = +0''.71 \pm 0''.05.$$

Our knowledge of this system is therefore far more accurate than that of any other system in the heavens, and it does not seem possible that the results here obtained will ever be sensibly altered. But as some refinement is still possible this glorious object will always merit the attention of observers.

02285.

$$\alpha = 14^h 41^m.7 \quad ; \quad \delta = +42^\circ 48'.$$

7.5, yellowish ; 7.6, whitish.

Discovered by Otto Struve in 1845.

OBSERVATIONS.

<i>t</i>	θ_o	ρ_o	<i>n</i>	Observers	<i>t</i>	θ_o	ρ_o	<i>n</i>	Observers
1845.80	72.2	0.61	3	O. Struve	1887.60	202.2	0.24	4	Schiaparelli
1847.96	72.2	0.42	3	Mädler	1888.61	187.5	0.22	3	Schiaparelli
1852.71	58.4	0.49	5	Mädler	1889.52	193.2	0.22	1	Schiaparelli
1855.84	53.9	0.51	3	O. Struve	1891.30	168.7	0.24	3	Burnham
1857.50	65.5	0.40	1	Secchi	1891.49	159.2	0.20	1	Schiaparelli
1865.53	36.0	—	1-0	Dembowski	1892.30	162.2	0.24	3-2	Burnham
1876.40	350.0	0.3±	1	Burnham	1893.46	156.0	0.24	1	Burnham
1881.50	—	doubtful	1	Burnham	1893.51	158.8	—	1-0	Bigourdan
1883.84	258.3	0.22	5	Englemann	1894.47	136.8	—	1-0	Bigourdan
1885.40	225.0	elong.	1	Perrotin	1895.32	147.3	0.30	3	See
					1895.56	143.2	0.35	1	Schiaparelli

This close double star was measured by OTTO STRUVE several times during the few years following its discovery.* The other early measures were by MÄDLER and SECCHI, while in later years the pair has been measured only by ENGLEMAN, SCHIAPARELLI, BURNHAM and the writer. Thus, only a small number of observations are available for the determination of an orbit, but it happens that these are distributed so as to give a fairly good set of elements.

The star has always been a difficult object, and hence the measures are necessarily less accurate than in case of easier pairs. BURNHAM was the first to attempt an investigation of the orbit (*Sidereal Messenger*, June, 1891). His apparent ellipse and the resulting elements are not very different from those found in this paper. MR. GORE has since attempted an orbit by a very different process, and obtained results of a wholly different character (*Monthly Notices*, April, 1893). These two sets of elements are :

GORE	BURNHAM
$P = 118.57$ years	62.1
$T = 1881.93$	1885.3
$e = 0.58$	0.429
$a = 0''.46$	$0''.387$
$\Omega = 107^\circ.0$	$54^\circ.3$
$i = 45^\circ.7$	$44^\circ.3$
$\lambda = 161^\circ.4$	$180^\circ.0$

Using all the measures, and basing the work on both angles and distances, I find the following elements of OY285 :

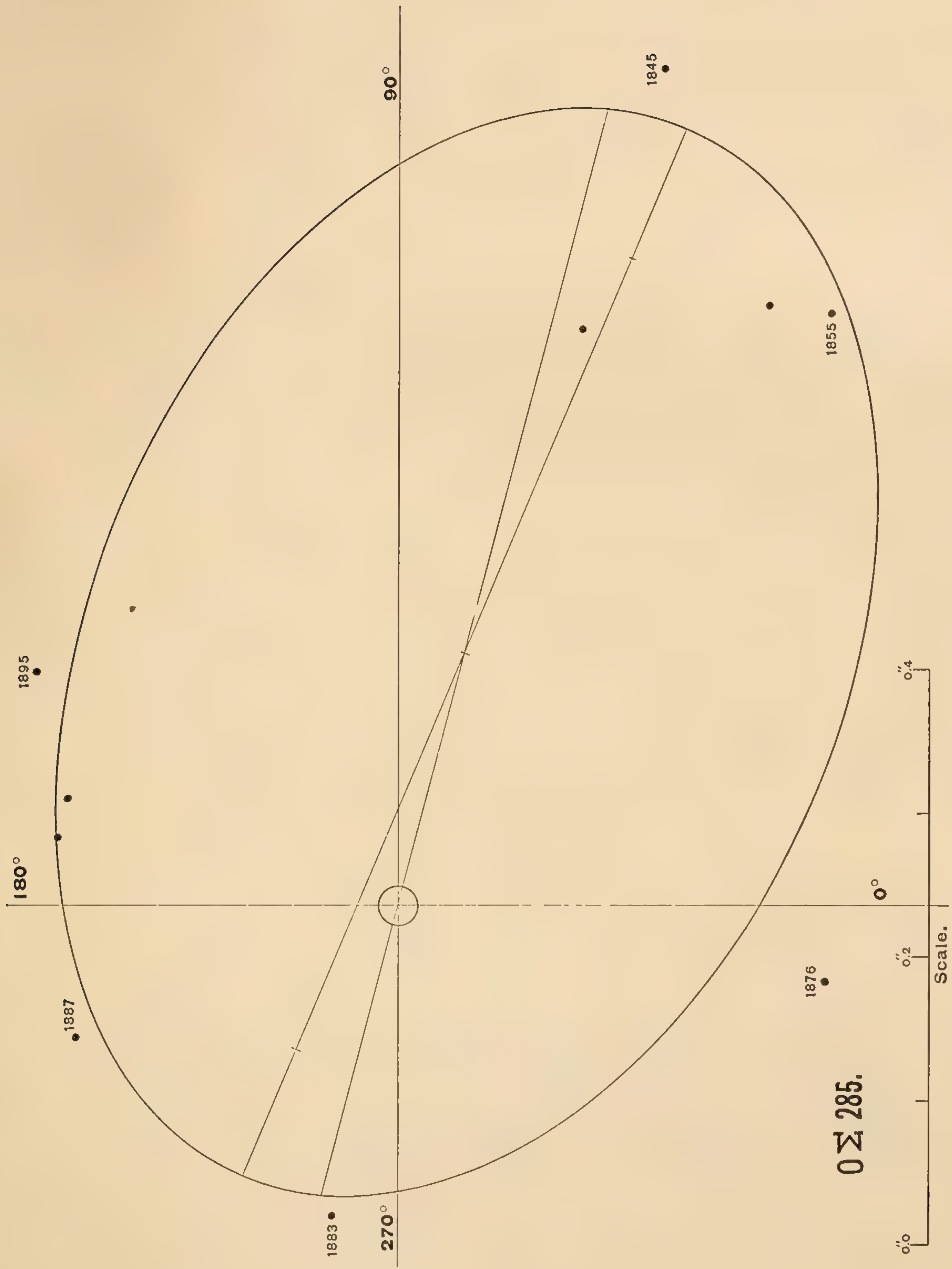
$P = 76.67$ years	$\Omega = 62^\circ.2$
$T = 1882.53$	$i = 41^\circ.95$
$e = 0.470$	$\lambda = 162^\circ.23$
$a = 0''.3975$	$n = -4^\circ.6953$

Apparent orbit:

Length of major axis	$= 0''.788$
Length of minor axis	$= 0''.522$
Angle of major axis	$= 67^\circ.1$
Angle of periastron	$= 255^\circ.3$
Distance of star from center	$= 0''.182$

The following table of computed and observed places shows that the measures are represented as well as could be expected in the case of an object of this difficulty.

* *Astronomical Journal*, 356.



COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1845.80	72.2	73.2	0.61	0.57	- 1.0	+0.04	3	O. Struve
1847.96	72.2	70.0	0.42	0.57	+ 2.2	-0.15	3	Madler
1852.71	58.4	62.9	0.49	0.56	- 4.5	-0.07	5	Madler
1855.84	53.9	58.0	0.51	0.54	- 4.1	-0.03	3	O. Struve
1857.50	65.5	55.1	0.40	0.52	+10.4	-0.12	1	Secchi
1865.53	36.0	38.4	—	0.42	- 2.4	—	1	Dembowski
1876.40	350.0	357.4	0.3 ±	0.24	- 7.4	+0.06	1	Burnham
1881.50	—	267.6	doubtful	0.20	—	—	1	Burnham
1883.84	258.3	241.0	0.22	0.21	+17.3	+0.01	5	Englemann
1887.60	202.2	203.6	0.24	0.22	- 1.4	+0.02	4	Schiaparelli
1891.30	168.7	170.1	0.24	0.24	- 1.4	±0.00	3	Burnham
1892.30	162.2	162.0	0.24	0.25	+ 0.2	-0.01	3-2	Burnham
1893.46	156.0	153.2	0.24	0.26	+ 2.8	-0.02	1	Burnham
1895.32	147.3	142.0	0.30	0.28	+ 5.3	+0.02	3	See

The only large residual is that of ENGLEMANN, whose small telescope would necessarily render his observations subject to considerable uncertainty. Indeed, he gives the angle as $78^{\circ}.3$, but I have assumed that he really saw the companion, and have therefore changed the angle by 180° . The estimate of 36° for the position-angle in 1865.53 is very nearly correct, and leaves no doubt that the elongation observed by DEMBOWSKI was real.

When I measured the object recently with the 26-inch refractor of the Leander McCormick Observatory in Virginia, the stars were not separated, except on one night, and hence the difficulty of the pair will doubtless account for the error in angle. The star is slowly separating, and ought to be observed annually. The following is an ephemeris for the next five years.

t	θ_c	ρ_c	t	θ_c	ρ_c
1896.40	135.6	0.30	1899.40	122.3	0.35
1897.40	130.7	0.32	1900.40	118.4	0.36
1898.40	126.5	0.33			

The comparatively long period of this close star may probably be construed to mean that the system is very remote from the *Earth*, otherwise the mass would be excessively small. The eccentricity of the orbit is fairly well defined, and is near the mean value of this element among double stars.

ξ BOÖTIS = Σ 1888. $\alpha = 14^h 46^m.8$; $\delta = +19^\circ 31'$.

4.5, yellow ; 6.5, purple.

Discovered by Sir William Herschel, April 19, 1780.

OBSERVATIONS.

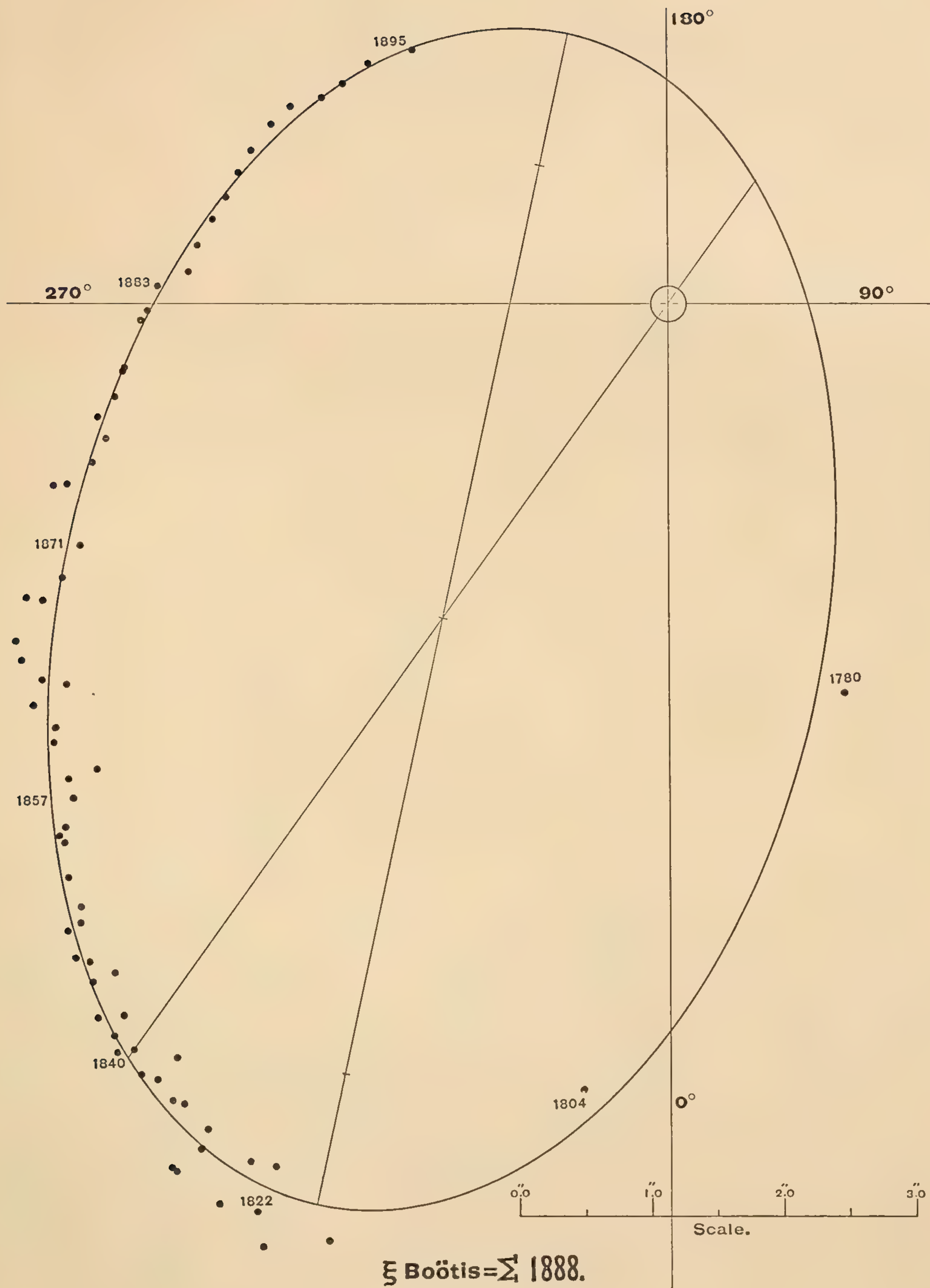
t	θ_o °	ρ_o "	n	Observers	t	θ_o °	ρ_o "	n	Observers
1780.69	24.1	3.23	1	Herschel	1841.06	325.1	7.03	5	O. Struve
1791.39	nf	—	1	Herschel	1841.42	323.4	7.27	3	Dawes
1792.30	355.7	—	1	Herschel	1841.43	324.7	7.10	4	Madler
1795.32	354.9	—	1	Herschel	1841.65	322.1	6.72	—	Kaiser
1802.25	352.9	—	1	Herschel	1842.30	322.7	7.03	2	Dawes
1804.25	353.9	6 $\pm$	1	Herschel	1842.40	323.4	6.88	3-1	Madler
1821.20	342.4	9.25	1	H. and So.	1843.33	322.7	6.70	1	Dawes
1822.69	335.8	7.54	—	Struve	1843.35	322.4	6.81	7-5	Madler
1823.30	—	6.67	—	Amici	1843.58	323.8	6.91	7	Schlüter
1823.34	340.2	8.42	1	H. and So.	1843.68	322.2	6.64	—	Kaiser
1825.37	337.0	7.78	4	South	1844.36	321.6	6.90	3	Madler
1828.54	336.0	7.18	2	Herschel	1845.36	320.9	6.81	8-6	Madler
1829.46	334.2	7.22	4	Struve	1845.37	322.3	6.12	—	Hind
1830.29	333.7	7.62	5-4	Herschel	1845.40	318.6	6.76	28	Morton
1831.40	331.2	7.30	5	Bessel	1846.29	320.4	6.69	5	Madler
1832.40	331.1	7.14	2	Struve	1846.46	319.2	6.75	20	Morton
1833.23	330.7	7.54	2	Herschel	1847.37	319.4	6.68	6	Madler
1834.44	330.4	7.54	3	Dawes	1847.44	318.8	6.80	2	Dawes
1835.43	329.0	7.07	5	Struve	1847.63	317.7	6.48	—	Mitchell
1835.45	330.4	7.63	3-2	Madler	1847.82	319.4	6.53	3	O. Struve
1836.37	329.1	7.52	1	Madler	1848.28	318.0	6.63	5-4	Madler
1836.49	328.2	7.09	4	Struve	1848.50	317.9	6.71	2	Dawes
1837.31	327.0	6.79	—	Encke	1850.77	316.5	6.56	1	Madler
1838.22	326.7	6.97	—	Madler	1851.11	317.4	6.56	5	Fletcher
1838.47	327.1	6.85	2	Struve	1851.49	316.1	6.21	5	Madler
1838.54	326.5	7.26	—	Galle	1852.30	316.6	6.51	32	Miller
1839.41	325.8	7.07	—	Galle	1852.56	315.3	6.22	15-13	Madler
1840.26	325.1	6.70	34-25 _{obs.}	Kaiser	1853.44	314.4	6.31	8-7	Madler
1840.43	324.1	7.16	3	Dawes	1853.54	313.4	6.23	3	O. Struve
					1854.46	312.0	6.26	3	Dawes
					1854.48	312.4	6.07	5-4	Madler
					1854.75	311.7	5.99	8	Dembowski
					1855.38	311.7	6.07	2	Madler
					1855.42	310.5	6.00	3	Secchi

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1856.39	312.4	5.89	4-3	Mädler	1870.38	293.0	5.41	2	Main
1856.45	310.8	5.95	8	Dembowski	1870.46	295.8	4.66	—	Leyton Obs.
1856.45	311.9	6.76	2	Luther	1870.56	294.4	4.95	1	Dunér
1856.55	311.7	6.00	3	Winnecke	1871.35	292.8	4.93	2	Main
1856.88	310.0	6.02	12	Secchi	1871.49	293.5	4.73	4	Dunér
1857.40	311.2	5.76	5	Mädler	1871.82	290.9	4.75	9	Dembowski
1857.42	310.0	5.90	1	Dawes	1873.19	286.7	4.62	4	O. Struve
1857.56	308.9	5.90	2	Dembowski	1873.39	286.0	4.93	1	Main
1858.36	308.2	5.76	5	Dembowski	1873.43	287.0	4.84	1	Lindstedt
1858.38	307.8	5.93	12	Morton	1873.48	286.6	4.71	1	Leyton Obs.
1858.54	309.9	5.65	7	Mädler	1873.91	287.8	4.62	8	Dembowski
1859.39	309.4	5.57	3	Mädler	1874.22	289.2	5.0	—	Gledhill
1861.29	305.0	5.52	35	Powell	1874.36	283.9	4.92	4	Main
1861.50	307.1	5.79	10-9	Mädler	1874.43	287.3	4.71	2-1	Leyton Obs.
1861.57	305.0	5.78	5	O. Struve	1874.44	288.4	4.72	5	W. & S.
1862.15	303.4	5.93	6	Auwers	1875.34	286.5	4.76	4	Main
1862.33	305.9	5.68	1	Main	1875.48	283.9	4.43	1	O. Struve
1862.47	304.1	5.59	4	O. Struve	1875.36	285.4	4.60	—	Gledhill
1862.51	302.9	—	—	Auwers	1875.38	286.3	—	—	Nobile
1862.51	302.2	—	—	Winnecke	1875.40	284.3	4.41	5	Schiaparelli
1862.65	306.4	5.27	2	Mädler	1875.51	286.6	4.45	4	Dunér
1863.15	303.0	5.59	14	Dembowski	1875.90	284.7	4.43	8	Dembowski
1863.28	302.4	5.79	—	Leyton Obs.	1876.34	284.8	4.31	5	Doberck
1863.56	302.0	5.67	5	O. Struve	1876.43	283.4	4.64	3	Hall
1864.46	303.4	5.32	1	Englemann	1876.58	282.0	4.19	1	O. Struve
1864.87	301.6	5.44	16	Dembowski	1877.24	282.9	4.70	3	Doberck
1865.33	301.6	5.61	3	Englemann	1877.45	283.0	4.35	5	Jedrzejewicz
1865.77	300.8	5.41	4	Secchi	1877.45	280.7	4.23	5	Schiaparelli
1866.39	298.5	5.59	2-4	Leyton Obs.	1877.54	279.4	4.21	1	O. Struve
1866.44	299.6	5.20	—	Kaiser	1877.93	280.9	4.26	8	Dembowski
1866.43	299.8	5.24	2-1	Englemann	1878.40	281.3	4.62	4	Goldney
1866.50	298.0	5.81	3-2	Searle	1878.42	277.4	4.32	2	Hall
1866.50	299.2	6.27	3-2	Winlock	1878.45	281.2	4.13	3	Doberck
1866.86	299.0	5.30	11	Dembowski	1878.52	278.8	4.01	5	Schiaparelli
1867.30	298.4	5.64	1	Winlock	1878.54	279.4	4.13	1	O. Struve
1867.42	296.7	5.43	2	Searle	1879.51	277.6	4.10	6	Schiaparelli
1868.40	294.7	5.33	1	Main	1879.52	275.7	4.18	5	Hall
1869.09	295.4	5.09	4	O. Struve	1880.16	278.8	4.28	5	Franz
1869.47	295.6	5.07	5	Dunér	1880.48	276.0	4.19	3	Jedrzejewicz
1869.56	292.4	5.35	3	Main	1880.51	276.3	3.97	3	Schiaparelli
1869.61	298.8	5.42	1	Leyton Obs.	1881.40	269.2	4.04	3	Hall
					1881.50	273.2	3.87	3	Schiaparelli
					1881.60	273.3	4.03	3	Seabroke

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1882.33	267.6	4.73	1	Glasenapp	1887.43	256.0	3.54	3	Hall
1882.42	270.4	3.99	3	Hall	1887.50	257.0	3.31	12	Schiaparelli
1882.50	271.4	3.86	7	Schiaparelli	1888.25	250.2	3.51	1	Glasenapp
1883.43	267.1	3.90	3	Hall	1888.42	251.9	3.40	3	Hall
1883.47	268.1	3.72	9	Schiaparelli	1888.54	255.0	3.15	2	O. Struve
1883.50	269.4	3.72	3	Jedrzejewicz	1888.62	253.9	3.51	2	Maw
1883.52	267.6	4.14	3	Seabroke	1889.31	250.5	3.83	2	Glasenapp
1883.57	268.1	3.79	4	Perrotin	1889.48	249.1	3.40	3	Hall
1884.42	262.8	4.30	2	Glasenapp	1889.61	249.9	3.31	3	Maw
1884.45	266.6	3.65	6	Englemann	1890.41	246.2	3.15	3	Maw
1884.45	266.1	3.71	2	Perrotin	1890.43	246.3	3.21	3	Hall
1884.49	266.3	3.58	9	Schiaparelli	1890.53	244.4	3.47	2	Hayn
1884.50	266.2	3.56	1	O. Struve	1891.44	241.0	3.26	5-4	See
1885.37	264.3	3.44	3	Tarrant	1891.45	242.4	3.18	3	Hall
1885.37	261.4	3.68	3	Hall	1891.48	243.4	3.18	4	Maw
1885.44	262.9	3.51	4	Perrotin	1892.32	240.0	3.08	3	Leavenworth
1885.44	262.1	3.55	5	deBall	1892.41	239.4	3.11	3	Maw
1885.48	263.1	3.61	12	Schiaparelli	1892.49	238.3	2.91	3	Comstock
1885.55	263.1	3.61	7	Englemann	1893.47	235.8	2.96	3	Maw
1885.64	263.6	3.63	4	Jedrzejewicz	1894.53	231.2	2.90	3	Maw
1886.40	259.6	3.56	3	Perrotin	1895.49	226.4	2.88	3	Comstock
1886.43	259.3	3.59	3	Hall	1895.70	223.8	2.57	4	See
1886.51	260.2	3.49	7	Schiaparelli	1895.73	224.4	2.65	2	Moulton
1886.60	259.4	3.32	6	Englemann					

The stars of this system are somewhat unequal in magnitude, and are moreover distinguished by very striking colors. The principal star is yellow, while the companion is reddish purple; and hence the appearance of the system, so far as it depends on contrast in color and inequality of the components, is very similar to those of 70 *Ophiuchi* and η *Cassiopeae*.* The early observations of HERSCHEL established the physical connection of the stars, and since the time of STRUVE the measures are both sufficiently numerous and sufficiently exact to give the position of the companion with the desired precision. In spite of the fact that since 1780 an arc of only about 170° has been described, we are enabled by the favorable shape of this arc to make a very satisfactory determination of the elements. The companion is now approaching periastron, and in the course of a few years the motion will become very rapid. For the next fifteen years this system will deserve special attention from observers, as the part of the apparent ellipse swept over by the companion during this interval

* *Astronomische Nachrichten*, 3334.



will be the most critical, and measures secured near periastron will enable us to render the orbit exact to a very high degree.

The following table gives the elements of this interesting system published by previous computers:

P	T	e	a	Ω	i	λ	Authority	Source
yr.			"	°	°	°		
117.14	1779.958	0.59374	12.56	0.0	80.1	101.0	Herschel, 1833	Mem. R.A.S. vol. VI.
160.695	1761.71	0.454	5.591	172.7	52.7	315.2	Mädler	Hand.D.S. p.304[p.149
168.91	1779.75	0.7822	9.95	11.4	71.6	96.4	Hind, 1872	M.N., vol. XXXII, p.250
140.64	1767.76	0.641	5.425	11.6	48.4	124.15	Winogradsky '72	Gore's Catalogue
127.97	1770.44	0.6781	4.813	12.02	37.9	130.9	Doberck, 1876	A.N. 2118
127.35	1770.69	0.7081	4.86	26.37	36.9	117.77	Doberck, 1877	A.N. 2129

From an investigation of all the observations we are led to the following elements of ξ Boötis :

$$\begin{aligned}
 P &= 128.0 \text{ years} & \Omega &= 10^\circ.5 \\
 T &= 1903.90 & i &= 52^\circ.28 \\
 e &= 0.721 & \lambda &= 239^\circ.25 \\
 a &= 5''.5578 & n &= -2^\circ.8125
 \end{aligned}$$

Apparent orbit :

$$\begin{aligned}
 \text{Length of major axis} &= 9''.07 \\
 \text{Length of minor axis} &= 5''.76 \\
 \text{Angle of major axis} &= 167^\circ.7 \\
 \text{Angle of periastron} &= 144^\circ.7 \\
 \text{Distance of star from centre} &= 2''.94
 \end{aligned}$$

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
	°	°	"	"	°	"		
1780.69	24.1	35.3	3.23	2.18	-11.2	+1.05	1	Herschel
1792.30	355.7	2.2	—	5.24	-6.5	—	1	Herschel
1795.32	354.9	358.5	—	5.71	-3.6	—	1	Herschel
1802.25	352.9	351.9	—	6.48	+1.0	—	1	Herschel
1804.25	353.9	350.1	6±	6.66	+3.8	-0.66	1	Herschel
1821.20	342.4	337.8	9.25	7.33	+4.6	+1.92	1	Herschel and South
1822.69	335.8	336.8	7.54	7.34	-1.0	+0.20	—	Struve
1823.32	340.2	336.4	7.55	7.35	+3.8	+0.20	1.2±	Herschel and So. 1; Amici 0.2±
1825.37	337.0	335.1	7.78	7.35	+1.9	+0.43	4	South
1828.54	336.0	332.9	7.18	7.33	+3.1	-0.15	2	Herschel
1829.46	334.2	332.2	7.22	7.31	+2.0	-0.09	4	Struve
1830.29	333.7	331.6	7.62	7.30	+2.1	+0.32	5-4	Herschel
1831.40	331.2	330.9	7.30	7.29	+0.3	+0.01	5	Bessel
1832.40	331.1	330.2	7.14	7.27	+0.9	-0.13	2	Struve
1833.23	330.7	329.7	7.54	7.25	+1.0	+0.29	2	Herschel
1834.44	330.4	328.8	7.54	7.22	+1.6	+0.32	3	Dawes
1835.43	329.0	328.0	7.07	7.19	+1.0	-0.12	5	Struve
1836.49	328.2	327.2	7.09	7.16	+1.0	-0.07	4	Struve
1837.31	327.0	326.6	6.79	7.13	+0.4	-0.34	—	Encke
1838.41	326.8	325.8	7.03	7.09	+1.0	-0.06	2+	Mädler — ; Σ .2; Galle —
1839.41	325.8	325.2	7.07	7.06	+0.6	+0.01	—	Galle
1840.34	324.1	324.4	6.93	7.02	-0.3	-0.09	3-6±	Kaiser 34-25 obs.; Dawes 3

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1841.39	323.1	323.6	7.03	6.97	- 0.5	+0.06	7-12+	$O\Sigma$. 0-5; Da. 3; Mä. 4; Ka. -
1842.35	323.0	322.8	6.95	6.93	+ 0.2	+0.02	5.3	Dawes 2; Mädler 3-1
1843.48	322.8	322.0	6.77	6.88	+ 0.8	-0.11	15-13±	Mä. 7-5; Da. 1; Schl. 7; Ka. -
1844.36	321.6	321.3	6.90	6.83	+ 0.3	+0.07	3	Mädler
1845.38	320.6	320.4	6.56	6.78	+ 0.2	-0.22	-	Mä. -; Hi. -; Mo. 28 obs.
1846.37	319.8	319.6	6.72	6.73	+ 0.2	-0.01	8±	Mädler 5; Morton 20 obs.
1847.56	318.8	318.6	6.62	6.67	+ 0.2	-0.05	11+	Mä. 6; Da. 2; Mit. -; $O\Sigma$. 3
1848.39	318.0	317.9	6.67	6.62	+ 0.1	+0.05	7-6	Mädler 5-4; Dawes 2
1850.77	316.5	315.9	6.56	6.48	+ 0.6	+0.08	1	Mädler
1851.30	316.7	315.4	6.44	6.44	+ 1.3	0.00	10	Fletcher 5; Mädler 5
1852.43	316.0	314.4	6.37	6.37	+ 1.6	0.00	18-16±	Miller 32 obs.; Mä. 15-13
1853.49	313.9	313.4	6.27	6.31	+ 0.5	-0.04	11-10	Mädler 8-7; $O\Sigma$. 3
1854.56	312.0	312.3	6.11	6.23	- 0.3	-0.12	16-15	Dawes 3; Mädler 5-4; Dem. 8
1855.40	311.1	311.6	6.03	6.18	- 0.5	-0.15	5	Mädler 2; Secchi 3
1856.56	311.3	310.4	6.12	6.09	+ 0.9	+0.03	29-28	Mä. 4-3; Dem. 8; Winn. 3; Lu. 2;
1857.46	310.0	309.5	5.85	6.03	+ 0.5	-0.18	8	Mä. 5; Da. 1; Dem. 2 [Sec. 12
1858.43	308.6	308.5	5.78	5.96	+ 0.1	-0.18	24	Dem. 5; Morton 12; Mädler 7
1859.39	309.4	307.5	5.57	5.90	+ 1.9	-0.33	3	Mädler
1861.45	305.7	305.2	5.70	5.74	+ 0.5	-0.04	18-17±	Po. 35 obs.; Mä. 10-9; $O\Sigma$. 5
1862.40	304.9	304.1	5.62	5.66	+ 0.8	-0.04	13	Au. 6; Main 1; $O\Sigma$. 4; Mä. 2
1863.33	302.5	303.0	5.68	5.59	- 0.5	+0.09	19+	Dem. 14; Leyton obs. -; $O\Sigma$. 5
1864.67	302.5	301.4	5.38	5.47	+ 1.1	-0.09	17	Englemann 1; Dembowski 16
1865.55	301.2	300.3	5.51	5.41	+ 0.9	+0.10	7	Englemann 3; Secchi 4
1866.52	299.0	299.1	5.57	5.33	- 0.1	+0.24	21-20+	Ley. 2-4; Ka. -; En. 2-1; Sr. 3-2;
1867.36	297.5	297.9	5.54	5.25	- 0.4	+0.29	3	Wlk. 1; Sr. 2 [Wlk. 3-2; Dem. 11
1868.40	294.7	296.5	5.33	5.17	- 1.8	+0.16	1	Main
1869.43	295.5	295.0	5.23	5.08	+ 0.5	+0.15	13	$O\Sigma$. 4; Du. 5; Mä. 3; Ley. 1
1870.47	294.4	293.5	5.01	4.98	+ 0.9	+0.03	3+	Mädler; Leyton -; Dunér 1
1871.55	292.4	291.9	4.80	4.89	+ 0.5	-0.09	15	Mä. 2; Du. 4; Dem. 9 [Dem. 8
1873.48	286.4	288.7	4.74	4.71	- 2.3	+0.03	15	$O\Sigma$. 4; Mä. 1; Ley. 1; Lin. 1;
1874.36	286.5	287.1	4.84	4.63	- 0.6	+0.21	11-10+	Gl. -; Mä. 4; Ley. 2-1; W. & S. 5
1875.45	285.4	285.1	4.51	4.53	+ 0.3	-0.02	22+	Mä. 4; $O\Sigma$. 1; Gl. -; No. -; Sch. 5;
1876.15	283.4	283.3	4.38	4.45	+ 0.1	-0.07	9	Dk. 5; Hl. 3; $O\Sigma$. 1 [Du. 4; Dem. 8
1877.52	281.4	281.2	4.39	4.34	+ 0.2	+0.05	22	Dk. 3; Jed. 5; Sch. 5; $O\Sigma$. 1; Dem. 8
1878.46	279.6	279.4	4.24	4.26	+ 0.2	-0.02	15	Go. 4; Hl. 2; Dk. 3; Sch. 5; $O\Sigma$. 1
1879.52	276.7	277.1	4.14	4.16	- 0.4	-0.02	11	Schiaparelli 6; Hall 5
1880.38	277.0	275.3	4.15	4.09	+ 1.7	+0.06	11	Franz 5; Jed. 3; Sch. 3
1881.50	271.9	272.8	3.98	4.00	- 0.9	-0.02	9	Hall 3; Sch. 3; Sea. 3
1882.46	270.9	270.4	3.93	3.90	+ 0.5	+0.03	10	Hall 3; Schiaparelli 7
1883.50	268.1	268.1	3.85	3.82	0.0	+0.03	22	Hl. 3; Sch. 9; Jed. 3; Sea. 3; Per. 4
1884.47	266.3	265.2	3.65	3.72	+ 1.1	-0.09	18	En. 6; Per. 2; Sch. 9; $O\Sigma$. 1
1885.47	262.9	262.6	3.58	3.61	+ 0.3	-0.06	38	Tar. 3; Hl. 3; Per. 4; Sch. 12; deBall 5;
1886.48	259.6	259.5	3.49	3.57	+ 0.1	-0.08	19	Per. 3; Hl. 3; Sch. 7; En. 6 [En. 7; Jed. 4
1887.47	256.5	256.6	3.43	3.46	- 0.1	-0.03	15	Hall 3; Schiaparelli 12
1888.46	253.0	253.6	3.39	3.37	- 0.6	+0.02	8	Glas. 1; Hl. 3; $O\Sigma$. 2; Maw 2
1889.45	249.8	250.4	3.35	3.29	- 0.6	+0.06	8-6	Glas. 2-0; Hall 3; Maw 3
1890.46	245.6	247.0	3.28	3.21	- 1.4	+0.07	8	Maw 3; Hall 3; Hayn 2
1891.46	242.3	243.3	3.21	3.13	- 1.0	+0.08	12-11	See 5-4; Hall 3; Maw 4
1892.41	239.2	239.5	3.03	3.04	- 0.3	-0.01	9	Lv. 3; Maw 3; Com. 3
1893.47	235.8	235.6	2.96	2.96	+ 0.2	0.00	3	Maw
1894.54	231.2	230.8	2.90	2.86	+ 0.4	+0.04	3	Maw
1895.59	225.1	225.7	2.72	2.75	- 0.6	-0.03	7	Comstock 3; See 4

The table of computed and observed places shows that the set of elements given above is extremely satisfactory, and we may confidently conclude that the general nature of the orbit here obtained will never be materially changed.

It is possible that the period may be varied by so much as one year, and that the eccentricity is uncertain to the extent of about ± 0.02 ; larger alterations in these quantities are not to be expected, and the values of the other elements are correspondingly well determined.

The system of ξ *Boötis* is chiefly remarkable for the great eccentricity of the orbit, and for the wide angular separation of the components. The great length of the major-axis and the comparatively short periodic time would support the belief that the system is not very far from the earth, and this view of relative proximity is rendered the more probable by the brightness of the components. But while these considerations tend to render it probable that the parallax is sensible, such a view is not supported by the small proper motion of the system in space, which is only $0''.161$ per year. We might, therefore, infer that the system is perhaps very remote from the earth, and hence of enormous dimensions, or comparatively near us, with the proper motion mainly in the line of sight. In any case the parallax of this system is particularly worthy of investigation, and it might be determined either by the ordinary process of direct measurement, or by the spectroscopic method (*A.N.*, 3314, or §5, Ch. I.), which here seems likely to be entirely practicable.

The following is an ephemeris for the companion for the next ten years :

t	θ_c	ρ_c	t	θ_c	ρ_c
1896.50	221.2	2.65	1902.50	173.3	1.55
1897.50	216.2	2.53	1903.50	154.7	1.25
1898.50	210.1	2.40	1904.50	125.5	1.03
1899.50	203.4	2.25	1905.50	90.1	1.05
1900.50	195.7	2.06	1906.50	63.2	1.33
1901.50	186.1	1.83			

η CORONAE BOREALIS = Σ 1937.

$\alpha = 15^h 19^m.1$; $\delta = +30^\circ 39'$.
5.5, yellowish ; 6, yellowish.

Discovered by Sir William Herschel, September 9, 1781.

OBSERVATIONS.

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1781.69	30.7	—	1	Herschel	1826.77	35.3	1.07	4	Struve
1802.69	179.7	—	1	Herschel	1829.55	43.2	0.96	2	Struve
1823.27	25.9	1.58	2-1	H. & So.	1830.30	44.5	—	8	Herschel

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1831.34	50.8	—	2	Dawes	1849.44	218.3	0.69	2-1	Dawes
1831.47	52.7	1.02	10-1	Herschel	1849.65	220.3	0.60	3	O. Struve
1831.63	50.6	0.88	3	Struve	1850.50	221.2	0.46	1	W. Struve
1832.50	57.1	0.69	9-2	Herschel	1850.52	230.8	0.49	3	O. Struve
1832.55	56.7	—	1	Dawes	1850.56	235.0	0.7 $\pm$	2	Fletcher
1832.76	56.9	0.79	3	Struve	1850.69	228.8	0.42	3	Mädler
1833.27	61.9	0.72	8-2	Herschel	1851.31	236.8	0.35	3-2	Mädler
1833.39	63.5	—	3	Dawes	1851.42	238.1	0.55	2	Dawes
1834.84	69.1	0.70	1	Struve	1851.56	241.8	0.48	10	O. Struve
1835.41	75.7	0.74	5	Struve	1851.83	234.8	0.31	7-5	Mädler
1836.49	98.8 (Schätzung)	—	1	Mädler	1852.52	250.1	0.5 $\pm$	2	Dawes
1836.52	88.8	0.56	6	Struve	1852.62	261.2	0.43	6	O. Struve
1839.59	119.8	0.5 $\pm$	2	Dawes	1852.67	241.1	0.30	13-11	Mädler
1839.82	132.1	0.76	2	O. Struve	1853.20	257.9	0.4 $\pm$	2	Jacob
1839.82	126.9	0.59	3	W. Struve	1853.37	267.8	0.27	5	Mädler
1840.52	137.2	0.51	5	O. Struve	1853.56	280.9	0.32	5	O. Struve
1840.62	135.9	0.50 $\pm$	2	Dawes	1853.64	273.3	0.44 $\pm$	4	Dawes
1841.42	150.4	0.48	5	Mädler	1853.79	270.4	0.3	1	Mädler
1841.50	149.7	0.52	5	O. Struve	1854.04	285.3	0.5 $\pm$	3	Jacob
1841.65	149.4	0.49	6 1	Dawes	1854.42	301.5	0.47	3	Dawes
1842.26	157.6	0.55	5	Mädler	1854.66	313.2	0.33	4	O. Struve
1842.58	156.6	0.5 $\pm$	2	Dawes	1854.74	317.1	0.26	4-3	Mädler
1842.60	159.1	0.57	2	O. Struve	1855.39	325.6	0.32 $\pm$	2	Secchi
1843.37	166.9	0.57	6	Mädler	1855.50	324.9	0.45	10-6	Winnecke
1843.63	171.6	0.60	7	Mädler	1855.51	322.5	0.15 $\pm$	1-3	Dawes
1844.38	174.0	0.57	3	Mädler	1855.62	330.2	0.40	4	O. Struve
1845.46	179.3	0.58	6	O. Struve	1855.77	330.2	—	2	Mädler
1845.50	186.1	0.59	19	Mädler	1856.35	336.8	0.51	9-6	Winnecke
1845.64	188.3	0.60	1	W. Struve	1856.37	341.7	0.45	1-3	Dawes
1846.61	195.7	0.61	3	O. Struve	1856.39	327.7	0.5 $\pm$	2	Jacob
1846.50	194.0	0.56	14 13	Mädler	1856.51	341.6	0.55	8-4	Winnecke
1847.07	196.6	—	3	Hind	1856.59	344.4	0.47	7	Secchi
1847.24	199.0	0.69	11	Mädler	1856.62	342.6	0.47	3	O. Struve
1847.64	204.0	0.56	5	O. Struve	1857.38	347.2	0.47	2	Mädler
1847.71	204.6	0.62	5	Mädler	1857.45	350.8	0.60	2	Dawes
1848.29	205.7	0.62	3	Mädler	1857.48	351.0	0.58	7	Secchi
1848.34	204.4	0.65	2	Dawes	1857.62	351.8	0.65	4	O. Struve
1848.47	207.4	0.69	1	Dawes	1857.95	355.8	0.6 $\pm$	3	Jacob
1848.62	208.7	0.8 $\pm$	2	W. C. Bond	1858.48	356.5	0.79	1	Winnecke
1848.72	209.8	0.57	2	O. Struve	1858.51	359.2	0.53	3	Secchi
					1858.52	1.1 cuneo.		10	Dembowski
					1858.54	359.6	0.76	5	O. Struve
					1858.61	6.2	0.69	6	Mädler

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1859.39	5.0	0.70	4	Mädler	1870.38	43.6	1.04	8	Dembowski
1859.48	4.5	0.53	4	Secchi	1870.38	47.2	0.98	4-1	Peirce
1859.61	5.9	0.79	4	O. Struve.	1870.44	44.6	1.1	2	Gledhill
1859.62	5.5	0.72	3	Dawes	1870.46	44.1	1.29	—	Leyton Obs.
					1870.47	46.8	1.13	1	Knott
1860.35	8.4	0.87	2	Dawes	1870.51	43.7	0.98	7	Dunér
1861.58	15.8	0.90	3	O. Struve	1870.54	47.2	0.97	3	O. Struve
1861.58	16.5	0.94	6	Mädler	1871.41	47.7	—	—	Leyton Obs.
1862.54	16.4	1.27	3-2	Winnecke	1871.45	47.8	1.09	8	Dembowski
1862.56	16.9	0.71	11	Dembowski	1871.53	47.3	0.88	9	Dunér
1862.58	22.8	0.99	3	Mädler	1871.54	45.7	1.00	5	Knott
1862.76	22.5	0.91	2	O. Struve	1871.56	47.6	1.42	2	Seabroke
					1871.57	46.4	0.95	1	Gledhill
1863.43	20.8	0.81	13	Dembowski	1872.29	47.8	1.29	—	Leyton Obs.
1863.54	23.6	1.10	4	O. Struve	1872.43	51.3	1.03	9	Dembowski
1863.56	19.7	1.07	—	Leyton Obs.	1872.48	51.7	0.92	7	Ferrari
1863.59	23.3	0.83	2	Secchi	1872.49	51.0	1.01	1	W. & S.
					1872.58	51.2	0.84	7	Dunér
1864.44	24.2	0.74	10	Dembowski	1872.59	55.4	0.91	5	O. Struve
1864.46	28.3	1.09	2	Englemann	1873.40	57.1	1.11	3	W. & S.
1865.15	30.1	1.13	5	Englemann	1873.44	56.1	1.04	8	Dembowski
1865.35	29.7	1.14	3	O. Struve	1873.47	56.0	—	1	Leyton Obs.
1865.41	27.4	1.03	9	Dembowski	1873.53	58.0	—	1	Lindemann
1865.44	27.3	1.07	3	Dawes	1873.53	59.0	—	3-0	Möller
1865.50	26.3	0.79	2	Secchi	1873.53	53.9	—	1-0	Romberg
1865.52	30.1	1.59	1	Leyton Obs.	1873.53	57.4	—	1-0	Schwarz
					1873.53	50.3	—	1-0	Wagner
1866.38	32.3	1.40	2	Leyton Obs.	1873.54	54.1	1.00	5-3	Gledhill
1866.44	30.1	1.04	9	Dembowski	1873.54	63.1	—	1-0	Brünnow
1866.54	33.1	1.12	3	Secchi	1873.54	57.4	0.81	4	O. Struve
1866.61	31.4	1.47	4-3	Harvard	1873.72	55.0	1.08	2	Dunér
1866.66	35.5	1.13	4	O. Struve					
					1874.39	58.6	0.99	3	Gledhill
1867.34	35.9	1.07	3	Knott	1874.42	59.5	0.98	8	Dembowski
1867.40	35.6	1.19	3-2	Harvard	1874.43	61.2	0.62	2-1	Leyton Obs.
1867.47	32.6	1.24	2	O. Struve	1874.46	58.2	0.93	2-1	W. & S.
1867.50	33.2	1.04	7	Dembowski	1874.61	64.7	0.83	4	O. Struve
1867.52	31.5	—	1	Leyton Obs.	1875.37	60.7	—	1	Leyton Obs.
1867.62	30.8	0.96	1	Winnecke	1875.41	66.7	0.86	8	Dembowski
1867.69	29.2	1.12	1	Dunér	1875.42	66.1	0.91	4	Schiaparelli
					1875.48	62.5	0.74	1	O. Struve
1868.39	36.0	1.05	7	Dembowski	1875.55	68.7	0.70	11	Dunér
1868.55	41.3	1.05	5	O. Struve					
1868.61	36.0	—	2	Zöllner	1876.38	70.3	0.79	8-2	Doberck
1868.65	37.0	1.15	4	Dunér	1876.44	70.5	0.77	4	Hall
1868.80	35.8	0.88	1	Peirce	1876.45	70.3	0.83	1	Leyton Obs.
					1876.46	74.8	0.84	9	Dembowski
1869.53	40.1	1.03	9	Dunér	1876.51	72.3	0.79	5	Schiaparelli
1869.61	44.7	•—	1	Leyton Obs.	1876.61	73.6	0.66	4	O. Struve

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1877.25	77.7	0.78	1	Copeland	1885.26	—	0.57	1	Copeland
1877.30	82.0	0.69	4-2	Doberck	1885.41	170.1	0.65	4	Hall
1877.36	70.3	—	6	W. & S.	1885.51	171.6	0.57 $\pm$	10	Schiaparelli
1877.42	79.6	0.75	5	Schiaparelli	1885.53	170.7	0.70	5-1	Sea. & Smith
1877.48	81.1	0.78	9	Dembowski	1885.58	170.0	0.61	7	Englemann
1877.53	71.9	1.0 $\pm$	1	Plummer	1886.46	177.0	0.70	5	Hall
1877.56	77.9	0.58	4	O. Struve	1886.49	180.8	0.72	4	Perrotin
1878.41	90.8	0.62	1	Burnham	1886.51	178.6	0.63	3	Tarrant
1878.45	93.3	0.62	3	Doberck	1886.51	181.3	0.80 $\pm$	3-1	Smith
1878.50	91.0	0.60	8	Dembowski	1886.52	178.8	0.66	11	Schiaparelli
1878.53	88.3	0.75	9	Schiaparelli	1886.64	179.1	0.57	8	Englemann
1878.59	87.6	0.57	4	O. Struve	1887.43	186.6	0.82	1	Hough
1878.80	84.4	0.67	1	Pritchett	1887.51	185.6	0.60	15	Schiaparelli
1879.52	102.4	0.62	7	Schiaparelli	1887.63	186.0	0.72	3	Tarrant
1879.54	98.7	0.48	4	Hall	1888.45	195.7	0.62	5	Hall
1880.45	111.9	—	2	Bigourdan	1888.53	199.0	—	1	Copeland
1880.50	116.7	0.52	3-2	Doberck	1888.55	194.8	0.60	14	Schiaparelli
1880.53	115.6	0.50	6	Schiaparelli	1888.63	193.9	0.74	3	O. Struve
1880.59	114.2	oblong	5	Jedrzejewicz	1889.42	182.0	—	1	Hodges
1880.62	114.3	0.46	5	Burnham	1889.50	202.3	0.63	4	Hall
1880.70	114.9	0.76	2	Copeland	1889.52	200.8	0.64	6	Schiaparelli
1881.26	121.3	—	2	Doberck	1889.58	202.1	0.72	1	O. Struve
1881.40	124.9	0.46	4	Hall	1890.43	oblong	—	1	Glaserapp
1881.50	126.9	0.61 $\pm$	4	Schiaparelli	1890.50	210.1	0.64	6	Hall
1881.64	125.8	0.48	1	O. Struve	1890.67	208.2	—	1	Bigourdan
1882.30	134.8	0.55	3-2	Doberck	1891.48	218.4	0.61	3	Hall
1882.45	138.4	0.51	4	Hall	1891.50	213.5	0.67 $\pm$	1	See
1882.50	135.4	0.59	8	Schiaparelli	1891.52	216.8	0.57	8	Schiaparelli
1882.55	141.7	0.50	2	O. Struve	1891.54	222.0	0.75	3	Maw
1882.61	153.2	0.56	6-4	Englemann	1892.44	226.1	0.69	1	H. C. Wilson
1883.48	147.2	0.69	10	Schiaparelli	1892.45	230.1	0.72	2	Leavenworth
1883.51	152.5	0.57	6	Hall	1892.50	230.2	0.57	11	Bigourdan
1883.51	153.2	0.51	7	Englemann	1892.57	229.5	0.57	6	Schiaparelli
1883.56	156.0	0.61	7	Perrotin	1892.65	229.8	0.48	3	Comstock
1883.59	151.6	0.58	3	O. Struve	1893.48	244.7	0.63	1	Maw
1883.64	150.5	0.5 $\pm$	6-5	Jedrzejewicz	1893.48	243.2	0.51	7	Schiaparelli
1884.43	159.4	—	6	Bigourdan	1893.50	242.8	0.50	3	Leavenworth
1884.48	160.1	0.57	3	Hall	1893.52	245.6	0.49	7-6	Bigourdan
1884.52	163.1	0.64	6	Perrotin	1894.48	262.1	0.44	6	Schiaparelli
1884.52	162.0	0.54 $\pm$	6	Schiaparelli	1894.49	261.4	0.44	1	Bigourdan
1884.54	161.7	0.67	1	Pritchett	1895.30	285.0	0.45	3	See
1884.58	158.0	0.58	3	O. Struve	1895.51	285.9	0.30 $\pm$	3	Comstock
1884.64	165.6	0.58	5	Englemann					
1884.66	172.4	—	3	Seabroke					

This beautiful pair proved to be one of the first objects which gave distinct evidence of orbital motion, and the binary character of the system was fully recognized by HERSCHEL in 1803. Since the time of STRUVE the measures are both numerous and satisfactory. The pair is always rather close, but as the components are nearly equal in magnitude, it is generally easy to separate. Numerous orbits have been published by previous computers; the following table of elements is fairly complete.

P	T	e	a	Ω	i	λ	Authority	Source
^{yrs.} 44.242	1806.20	0.26034	0.8325	220.6	37.4	358.63	Herschel, 1833	Mem. R.A.S., VI, 156
43.246	1850.23	0.3376	1.0879	24.3	71.13	261.35	Mädler, 1842	Dorp. Obs., IX, 195
43.310	1815.20	0.3537	1.1912	22.6	71.5	263.17	Mädler, 1842	
42.500	1807.21	0.289	0.9024	20.1	59.47	215.2	Mädler, 1847	Fixt. Syp., I, p. 243
42.501	1805.666	0.4743	1.0125	10.52	65.65	227.17	Villargeau 1842	
66.257	1780.124	0.4695	1.1108	4.42	58.05	194.62	Villargeau 1852	
67.309	1779.338	0.4043	1.2015	9.87	59.32	185.0	Villargeau 1852	A.N., 868
43.115	1850.329	0.2865	0.9567	22.3	60.67	215.48	Winnecke	
41.58	1850.26	0.2625	0.827	26.7	58.0	211.4	Wijkander	
41.576	1850.26	0.2625	0.827	26.7	58.0	215.6	Dunér, 1871	A.N., 1868
40.17	1849.9	0.287	0.985	22.2	60.4	224.1	Flamma'n 1874	Cat. ét. Doub., p. 88
41.562	1850.792	0.2667	0.892	25.72	59.68	218.6	Doberck, 1880	A.N., 2338
41.6	1892.3	0.33	0.86	26.9	55.0	220.5	Comstock, 1893	Proc. Am. Assoc., 1894

Making use of all the measures up to 1895, we find the following elements of η *Coronae Borealis**:

$$\begin{aligned}
 P &= 41.60 \text{ years} & \Omega &= 27^\circ.10 \\
 T &= 1892.50 & i &= 58^\circ.50 \\
 e &= 0.267 & \lambda &= 217^\circ.57 \\
 a &= 0''.9165 & n &= +8^\circ.653846
 \end{aligned}$$

Apparent orbit:

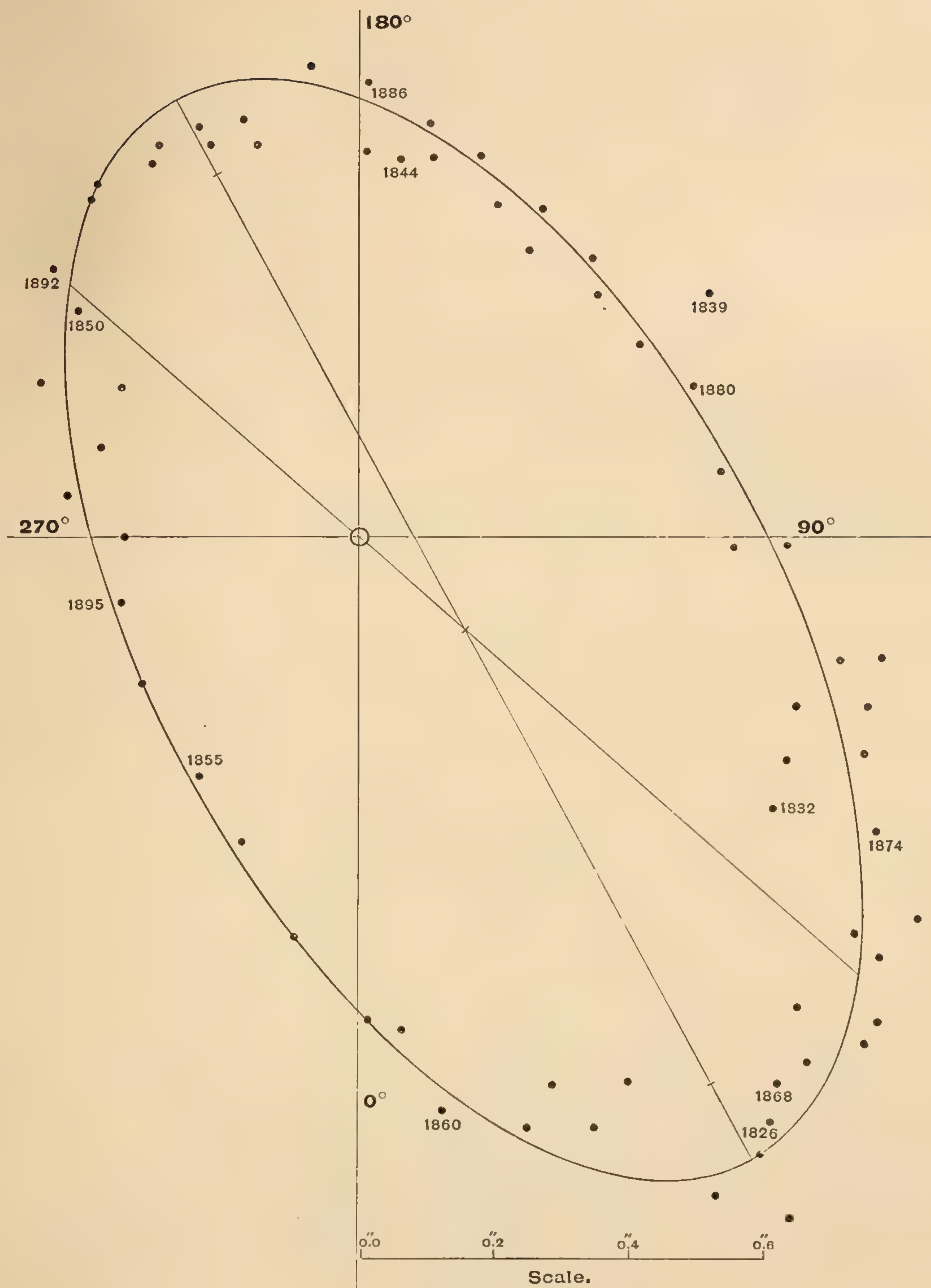
$$\begin{aligned}
 \text{Length of major axis} &= 1''.804 \\
 \text{Length of minor axis} &= 0''.934 \\
 \text{Angle of major axis} &= 28^\circ.7 \\
 \text{Angle of periastron} &= 229^\circ.0 \\
 \text{Distance of star from center} &= 0''.209
 \end{aligned}$$

The accompanying table shows that the motion is well represented, and that the present elements will finally undergo but slight corrections.

* *Astronomische Nachrichten*, 3361.

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1781.69	30.7	27.4	—	1.08	+3.3	—	1	Herschel
1802.69	179.7	174.8	—	0.63	+4.9	—	1	Herschel
1823.27	25.9	27.3	1.58	1.08	-1.4	+0.50	2-1	Herschel and South
1826.77	35.3	37.9	1.07	1.09	-2.6	-0.02	4	Struve
1829.55	43.2	47.0	0.96	1.01	-3.8	-0.05	2	Struve
1831.48	51.4	54.5	0.95	0.92	-3.1	+0.03	15-4	Dawes 2-0; Herschel 10-1; Σ . 3
1832.60	56.9	59.5	0.74	0.86	-2.6	-0.12	13-5	Herschel 9-2; Dawes 1-0; Σ . 3
1833.33	62.7	63.4	0.72	0.82	-0.7	-0.10	11-2	Herschel 8-2; Dawes 3-0
1834.84	69.1	72.5	0.70	0.73	-3.4	-0.03	1	Struve
1835.41	75.7	76.6	0.74	0.70	-0.9	+0.04	5	Struve
1836.52	88.8	85.9	0.56	0.63	+2.9	-0.07	6	Struve
1839.70	125.9	122.2	0.63	0.53	+3.7	+0.10	4	Dawes 2; $O\Sigma$. 2
1840.57	136.0	133.4	0.51	0.53	+2.6	-0.02	7	$O\Sigma$. 5; Dawes 2
1841.52	149.8	146.0	0.50	0.54	+3.8	-0.04	16-11	Mädler 5; $O\Sigma$. 5; Dawes 6-1
1842.48	157.8	157.5	0.54	0.57	+0.3	-0.03	9	Mädler 5; Dawes 2; $O\Sigma$. 2
1843.50	169.2	168.2	0.58	0.60	+1.0	-0.02	13	Mädler 6; Mädler 7
1844.38	174.0	176.4	0.57	0.64	-2.4	-0.07	3	Mädler
1845.46	179.3	184.8	0.58	0.68	-5.5	-0.10	6	O. Struve
1846.61	195.7	194.1	0.61	0.71	+1.6	-0.10	3	O. Struve
1847.42	201.0	200.0	0.63	0.71	+1.0	-0.08	24-21	Hind 3-0; Mädler 11; $O\Sigma$. 5; Mädler 5
1848.49	207.2	207.8	0.66	0.70	-0.6	-0.04	10	Mädler 3; Dawes 2; Dawes 1; Bond 2; $O\Sigma$.
1849.54	219.3	216.0	0.64	0.66	+3.3	-0.02	5-4	Dawes 2-1; $O\Sigma$. 3
1850.59	231.5	225.6	0.54	0.60	+5.9	-0.06	8	$O\Sigma$. 3; Fletcher 2; Mädler 3
1851.53	237.8	235.9	0.42	0.53	+1.9	-0.11	22-19	Mädler 3-2; Dawes 2; $O\Sigma$. 10; Mädler 7-5
1852.60	250.8	253.5	0.41	0.44	-2.7	-0.03	21-19	Dawes 2; $O\Sigma$. 6; Mädler 13-11
1853.51	270.3	272.9	0.35	0.40	-2.6	-0.05	17	Jacob 2; Mädler 5; $O\Sigma$. 5; Dawes 4; Mädler 1
1854.46	304.3	296.5	0.39	0.38	+7.8	+0.01	14-13	Jacob 3; Dawes 3; $O\Sigma$. 4; Mädler 4-3
1855.56	326.6	321.6	0.43	0.43	+5.0	± 0.00	19-13	Sec. 2-0; Winn. 10-6; Da. 1-3; $O\Sigma$. 4; Mä. 2-0
1856.47	339.1	337.7	0.49	0.50	+1.4	-0.01	30-25	Winn. 9-6; Da. 1-3; Ja. 2; Winn. 84; Sec. 7; $O\Sigma$. 3
1857.57	351.3	350.6	0.61	0.61	+0.7	± 0.00	18-16	Mädler 2-0; Dawes 2; Secchi 7; $O\Sigma$. 4; Jacob 3
1858.54	1.3	359.0	0.73	0.70	+2.3	+0.03	24-11	Secchi 3-0; Dembowski 10-0; $O\Sigma$. 5; Mädler 6
1859.52	5.2	5.6	0.74	0.79	-0.4	-0.05	15-11	Mädler 4; Secchi 4-0; $O\Sigma$. 4; Dawes 3
1860.35	8.4	10.1	0.87	0.86	-1.7	+0.01	2	Dawes
1861.58	16.1	15.6	0.92	0.94	+0.5	-0.02	9	$O\Sigma$. 3; Mädler 6
1862.61	19.6	19.7	0.87	1.00	-0.1	-0.13	19-16	Winn. 3-0; Dembowski 11; Mädler 3; $O\Sigma$. 2
1863.53	21.8	22.9	0.95	1.04	-1.1	-0.09	19+	Dem. 13; $O\Sigma$. 4; Leyton Obs. —; Secchi 2
1864.45	26.3	25.9	0.91	1.07	+0.4	-0.16	12	Dembowski 10; Englemann 2
1865.40	28.5	28.9	1.12	1.09	-0.4	+0.03	23	En. 5; $O\Sigma$. 3; Dem. 9; Da. 3; Sec. 2; Ley. 1
1866.52	32.5	32.4	1.23	1.10	+0.1	+0.13	22-21	Leyton Obs. 2; Dem. 9; Sec. 3; Hv. 4-3; $O\Sigma$. 4
1867.50	33.0	35.3	1.10	1.10	-2.3	± 0.00	18-16	Kn. 3; Hv. 3-2; $O\Sigma$. 2; Dem. 7; Ley. 1-0; Du. 1;
1868.59	37.5	38.6	1.03	1.09	-1.1	-0.06	17	Dem. 7; $O\Sigma$. 5; Dunér 4; Peirce 1 [Winn. 1
1869.57	40.7	41.6	1.03	1.06	-0.9	-0.03	10-9	Dunér 9; Leyton Obs. 1-0
1870.45	45.1	44.6	1.07	1.04	+0.5	+0.03	25-22	Dem. 8; Pei. 4-1; Gl. 2; Ley. —; Kn. 1; Du. 7; $O\Sigma$. 3
1871.51	47.1	48.3	1.06	1.00	-1.2	+0.06	25	Ley. —; Dem. 8; Du. 9; Kn. 9; Sea. 2; Gl. 1
1872.47	51.2	52.0	1.00	0.96	-0.8	+0.04	29+	Ley. —; Dem. 9; Fer. 7; W. & S. 1; Du. 7; $O\Sigma$. 5
1873.52	55.9	56.4	1.01	0.90	-0.5	+0.11	22-20	W. & S. 3; Dem. 8; Ley. —; Gl. 5-3; $O\Sigma$. 4; Du. 2
1874.47	60.5	61.0	0.89	0.85	-0.5	+0.03	19-17	Ley. 2-1; Gl. 3; Dem. 8; W. & S. 2-1; $O\Sigma$. 4
1875.44	67.2	66.2	0.82	0.79	+1.0	+0.03	23	Dembowski 8; Schiaparelli 4; Dunér 11
1876.45	71.9	72.6	0.80	0.73	-0.7	+0.07	31-25	Dk. 8-2; Hl. 4; Ley. 1; Dem. 9; Sch. 5; $O\Sigma$. 4
1877.41	77.2	79.4	0.80	0.68	-2.2	+0.12	30-22	Cop. 1; Dk. 4-2; W. & S. 6-0; Sch. 5; Dem. 9; Pl. 1;
1878.55	89.2	89.7	0.64	0.61	-0.5	+0.03	26	β . 1; Dk. 3; Dem. 8; Sch. 9; $O\Sigma$. 4; Pr. 1 [$O\Sigma$. 4
1879.53	100.5	100.2	0.55	0.57	+0.3	-0.02	11	Schiaparelli 7; Hall 4
1880.56	114.5	112.5	0.54	0.54	+2.0	± 0.00	23-20	Big. 2-0; Dk. 3-2; Sch. 6; Jed. 5; β . 5; Cop. 2
1881.44	124.7	123.9	0.51	0.53	+0.8	-0.02	11-9	Dobereck 2-0; Hall 4; Schiaparelli 4; $O\Sigma$. 1
1882.49	140.7	137.8	0.54	0.53	+2.9	+0.01	23-20	Dobereck 3-2; Hall 4; Sch. 8; $O\Sigma$. 2; En. 6-4
1883.55	151.8	150.9	0.58	0.55	+0.9	+0.03	39-38	Sch. 10; Hl. 6; En. 7; Per. 7; $O\Sigma$. 3; Jed. 6-5 [Sea. 3-0
1884.54	163.5	162.5	0.60	0.58	+1.0	+0.02	33-24	Big. 6-0; Hl. 3; Per. 6; Sch. 6; Pr. 1; $O\Sigma$. 3; En. 5;



η Coronae Borealis = Σ 1937.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers.
1885.46	170.6	171.7	0.63	0.62	-1.1	+0.01	26-23	Cop. 0-1; Hl. 4; Sch. 10; Sea. & Sm. 5-1; En. 7
1886.52	179.3	181.1	0.68	0.66	-1.8	+0.02	34-32	Hall 5; Per. 4; Tar. 3; Sm. 3-1; Sch. 11; En. 8
1887.51	186.1	189.0	0.71	0.69	-2.9	+0.02	19	Hough 1; Schiaparelli 15; Tarrant 3
1888.54	195.8	196.5	0.65	0.71	-0.7	-0.06	23-22	Hall 5; Copeland 1-0; Schiaparelli 14; $O\Sigma$. 3
1889.53	201.7	203.7	0.66	0.71	-2.0	-0.05	11	Hall; Schiaparelli 6; $O\Sigma$. 1
1890.53	209.1	211.4	0.64	0.69	-2.3	-0.05	7-6	Hall 6; Bigourdan 1-0
1891.51	217.6	219.1	0.65	0.64	-1.5	+0.01	15	Hall 3; See 1; Schiaparelli 8; Maw 3
1892.50	229.1	229.1	0.61	0.58	± 0.0	+0.03	23	H.C.W. 1; Lv. 2; Big. 11; Sch. 6; Com. 3
1893.49	244.1	241.8	0.53	0.50	+2.3	+0.03	18-17	Maw 1; Schiaparelli 7; Lv. 3; Big. 7-6
1894.49	261.8	259.3	0.44	0.43	+2.5	+0.01	7	Schiaparelli 6; Bigourdan 1
1895.51	285.9	282.7	0.37	0.38	+3.2	-0.01	3-6	See 0-3; Comstock 3

The uncertainty in the period does not surpass 0.1 year, and an alteration of the eccentricity amounting to ± 0.01 is not probable. It seems, however, that there are occasional systematic errors in the angles, and hence careful measurement should be continued. It will not be many years before a definitive determination of the elements of this interesting binary can be advantageously undertaken. The following is a short ephemeris for the use of observers.

t	θ_c	ρ_c	t	θ_c	ρ_c
1896.50	306.9	0.39	1899.50	353.8	0.64
1897.50	327.7	0.45	1900.50	1.6	0.73
1898.50	342.9	0.54			

 μ^2 BOÖTIS = Σ 1938.

$\alpha = 15^h 20^m.7$; $\delta = +37^\circ 43'$.
6.5, white ; 8, white.

Discovered by Sir William Herschel, September 10, 1781.

OBSERVATIONS.

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1782.68	357.2	—	1	Herschel	1833.02	319.3	1.00	3-1	Herschel
1802.86	346.2	—	—	Herschel	1833.39	319.8	1.15	1	Dawes
1822.21	330.7	—	2	Struve	1833.85	319.7	1.19	3	Struve
1823.41	333.7	1.65	3	H. & So.	1835.55	318.6	1.10	3	Struve
1825.46	333.53	1.43	5	South	1835.65	309.1	—	1	Madler
1826.77	327.0	1.38	2	Struve	1836.45	310.1	—	2	Madler
1829.73	324.0	1.24	2	Struve	1836.65	315.1	1.06	3	Struve
1830.24	324.1	0.85	2	Herschel	1837.37	314.9	1.0 $\pm$	1	Dawes
1831.36	321.7	1.14	1	Herschel	1837.70	315.0	0.9	—	Struve
					1839.83	310.4	—	—	W. Struve

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1840.39	306.0	0.83	3	Dawes	1857.38	239.2	0.35	2	Mädler
1840.46	313.8	0.98	3	O. Struve	1857.52	231.7	0.55	1	Secchi
1841.47	308.7	0.82	2	Mädler	1857.65	237.9	0.58	3	O. Struve
1841.66	303.2	0.86	6.3	Dawes	1858.56	225.9	0.45	1	Secchi
1842.23	303.8	0.85	3	O. Struve	1858.56	228.3	0.57	3	O. Struve
1842.40	305.2	0.72	3	Mädler	1858.57	236.0	0.32	4	Mädler
1842.40	300.9	0.85 $\pm$	3	Dawes	1859.39	226.4	0.42	3-2	Mädler
1842.66	304.9	0.78	2	Mädler	1860.95	211.3	0.58	3	O. Struve
1843.57	301.5	0.76	10	Mädler	1861.58	215.1	0.42	2	Mädler
1844.39	299.2	0.71	2	Mädler	1862.56	202.9	0.3?	3	Dembowski
1845.54	295.8	0.64	10	Mädler	1862.63	217.7	0.4 $\pm$	1	Mädler
1846.40	291.8	0.64	12-11	Mädler	1863.38	195.8	0.55	12	Dembowski
1846.68	287.1	0.57	4	O. Struve	1863.63	195.8	0.75	—	Leyton Obs.
1847.08	281.3	—	2	Hind	1864.41	193.0	0.51	4	Knott
1847.30	286.5	0.65 $\pm$	4	Dawes	1864.48	189.5	cuneo.	5	Dembowski
1847.38	288.1	0.55	15-13	Mädler	1865.45	184.8	0.53	10	Dembowski
1848.37	282.4	0.42	2	Mädler	1865.46	190.1	0.48 $\pm$	3	Dawes
1848.52	280.0	0.65	4	Dawes	1865.72	197.9	—	1	Leyton Obs.
1848.52	282.9	0.56	3-4	W. C. Bond	1865.78	187.5	0.57	5	Englemann
1849.44	276.2	0.68	2	Dawes	1866.40	179.2	0.60	3	O. Struve
1850.46	272.7	0.53	2	O. Struve	1866.41	196.4	0.85	3-2	Leyton Obs.
1850.69	276.7	0.40	3-2	Mädler	1866.48	181.2	0.50	7	Dembowski
1851.28	264.9	0.32	3	Mädler	1866.54	180.3	in cont.	1	Secchi
1851.42	266.6	0.52	2	Dawes	1867.48	175.8	0.60	6	Dembowski
1851.48	262.7	0.44	3	O. Struve	1868.38	174.2	0.53	5	Dembowski
1851.77	263.4	0.31	4	Mädler	1869.49	171.1	0.53	6	Dunér
1852.52	262.2	0.55 $\pm$	1	Dawes	1869.54	167.5	0.54	2	O. Struve
1852.60	261.3	0.41	10	Mädler	1870.39	165.8	0.62	7	Dembowski
1852.65	268.2	0.49	3	O. Struve	1870.44	164.0	—	1	Gledhill
1853.23	265.1	0.45 $\pm$	2	Jacob	1870.52	163.9	0.59	4	Dunér
1853.34	256.2	0.33	4	Mädler	1870.65	170.8	—	—	Leyton Obs.
1853.71	254.6	0.5 $\pm$	1	Dawes	1871.43	161.2	0.61	7	Dembowski
1853.77	256.6	0.40	2	Mädler	1871.54	160.8	0.67	5	Dunér
1854.05	253.7	0.5 $\pm$	2	Jacob	1871.57	167.9	0.76	1	Seabroke
1854.41	249.3	0.47	3	Dawes	1871.65	158.4	0.5 $\pm$	1	Gledhill
1854.70	247.2	0.44	4	Mädler	1872.29	167.5	—	—	Leyton Obs.
1855.11	247.2	0.53	4	O. Struve	1872.35	163.4	0.35 $\pm$	2	W. & S.
1855.52	256.9	0.42	2	Mädler	1872.44	154.1	0.65	8	Dembowski
1856.42	236.5	0.45	1	Secchi	1872.46	152.0	0.6 $\pm$	4	Knott
1856.57	242.1	0.59	2	O. Struve	1872.52	158.0	0.55	2	Dunér

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1873.09	158.2	0.63	4	O. Struve	1883.50	115.0	0.70	2	Hall
1873.34	151.0	0.52 $\pm$	3-2	W. & S.	1883.57	117.5	0.76	6	Englemann
1873.41	151.0	0.67	7	Dembowski	1883.59	112.9	0.75	2	Perrotin
1873.48	155.8	—	1	Leyton Obs.	1883.63	110.2	0.64	1	O. Struve
1873.47	152.3	0.48 $\pm$	2	Gledhill	1884.48	113.8	0.69	3	Hall
1874.22	150.7	0.58	2	Gledhill	1884.51	112.3	0.74 $\pm$	4	Schiaparelli
1874.44	149.1	0.7	1	W. & S.	1884.62	110.2	0.86	2	O. Struve
1874.44	147.8	0.81	6	Dembowski	1884.67	119.9	—	4	Seabroke
1874.54	155.4	—	1	Leyton Obs.	1885.40	110.8	0.75	2	Perrotin
1875.41	141.9	0.69	8	Dembowski	1885.49	105.8	1.00 $\pm$	3-1	Smith
1875.47	143.3	0.64 $\pm$	4	Schiaparelli	1885.49	110.1	0.79	3	Tarrant
1875.52	146.7	0.80	1	Dunér	1885.49	111.3	0.71	4	Hall
1876.35	143.6	—	2	Doberck	1885.50	109.4	0.89	4	Schiaparelli
1876.44	145.4	0.73	4	Hall	1885.63	116.9	0.85	7-6	Englemann
1876.46	138.2	0.70	8	Dembowski	1885.70	110.6	0.7 $\pm$	6	Jedrzejewicz
1877.24	138.5	0.75	5	Schiaparelli	1886.49	106.7	—	2	Smith
1877.38	131.6	0.56	4-2	Doberck	1886.51	107.3	0.65	3	Hall
1877.42	136.9	0.71	7	Dembowski	1886.51	106.0	0.72	2	Perrotin
1877.49	145.3	0.73	4	W. & S.	1886.54	107.7	0.74	2	Schiaparelli
1877.62	143.0	0.67	1	O. Struve	1886.78	106.2	0.7 $\pm$	5	Jedrzejewicz
1878.41	136.2	0.68	1	Burnham	1887.44	105.4	0.70	4	Hall
1878.49	137.6	0.62	4	Doberck	1887.55	99.0	—	1	Smith
1878.52	132.0	0.62	6	Dembowski	1887.56	103.0	0.74	6	Schiaparelli
1878.53	132.7	0.63 $\pm$	5	Schiaparelli	1888.45	100.0	0.60	4	Hall
1878.58	137.7	0.63	1	O. Struve	1888.59	101.5	0.75	5-3	Schiaparelli
1879.51	128.6	0.79	4	Schiaparelli	1888.91	103.1	0.73	2	Tarrant
1879.54	133.3	0.73	4	Hall	1888.69	101.6	0.87	1	O. Struve
1880.18	128.7	0.78	5	Burnham	1889.35	97.8	0.73	3	Maw
1880.40	129.6	0.64	1	Hall	1889.42	96.2	1.00	1	Hodges
1880.50	130.1	0.70	4	Doberck	1889.52	98.7	0.84	3	Schiaparelli
1880.53	126.7	0.79	4	Schiaparelli	1890.50	107.8	(0.85)	2	Glasenapp
1880.65	122.6	0.7 $\pm$	4	Jedrzejewicz	1891.49	95.4	0.80 $\pm$	2	Schiaparelli
1881.26	126.9	—	4	Doberck	1891.53	94.7	0.74 $\pm$	2	See
1881.38	126.0	0.63	4	Burnham	1892.42	92.6	0.82	1	Collins
1881.50	121.6	0.78	4	Schiaparelli	1892.58	89.1	0.74	4	Comstock
1881.50	123.7	0.62	6-4	Bigourdan	1893.47	88.0	0.98	4	Bigourdan
1881.50	121.9	0.62	3	Hall	1893.49	88.6	0.77	2	Maw
1881.63	122.4	0.72	1	O. Struve	1894.48	85.6	1.19	1	Callandreau
1882.32	125.0	0.75	2-1	Doberck	1894.50	86.0	1.05	5	Bigourdan
1882.43	121.7	0.64	3	Hall	1894.59	85.4	0.75	1	H. C. Wilson
1882.52	120.4	0.79	4	Schiaparelli	1895.31	83.5	0.84	3	See
1882.53	121.9	0.77	4	Englemann	1895.52	83.9	0.64	3	Comstock
1882.55	116.9	0.64	1	O. Struve					
1883.47	114.3	0.87	4	Schiaparelli					

When the observations of 1782 were compared with those of 1802, the physical character of the system was fairly indicated.* Since the time of STRUVE it has been carefully followed by the best observers, and accordingly the material now available for an orbit is highly satisfactory. The companion is only slightly smaller than the principal star, and is therefore never very difficult to measure. In all parts of the orbit the pair is sufficiently wide to be seen with a six-inch telescope, but as the minimum distance of $0''.49$ in angle 230° was passed in 1858, it is not surprising that the observers on either side of this epoch, with few exceptions, have made their observed distances too small. Thus, although the measures of different observers are not infrequently affected by systematic errors of sensible magnitude, yet by combining the best measures into mean positions for each year, we obtain a set of places which give an orbit that seems likely to be very near the truth.

Some of the elements hitherto published are as follows :

P	T	e	a	Ω	i	λ	Authority	Source
^{yrs.} 146.649	1851.57	0.8529	1.320	94.7	49.4	87.1	Mädler, 1847	Fixt. Syst., I, 252
182.6	1866.0	0.491	1.165	166.1	47.5	23.0	Winagr., 1872	
314.34	1860.88	0.5641	1.761	163.2	41.9	54.4	Hind, 1872	M.N., vol. XXXII, p. 250
200.4	1865.2	0.51	—	172.0	45.0	20.1	Wilson, 1872	Handb. D.S., p. 313
198.93	1865.5	0.4957	—	169.0	46.4	23.6	Klinkerfues	Handb. D.S., p. 313
290.07	1863.51	0.6174	1.500	183.0	44.4	17.7	Dobereck, 1875	A.N., 2026
280.29	1860.51	0.5974	1.47	173.7	39.9	20.0	Dobereck, 1878	A.N., 2194
266.0	1862.55	0.5668	1.057	166.7	35.2	40.9	Pritchard, "	Ox. Obs., No. 1, p. 64

From an investigation of all the observations which appear to be reliable, we find the following elements of μ^2 Boötis:

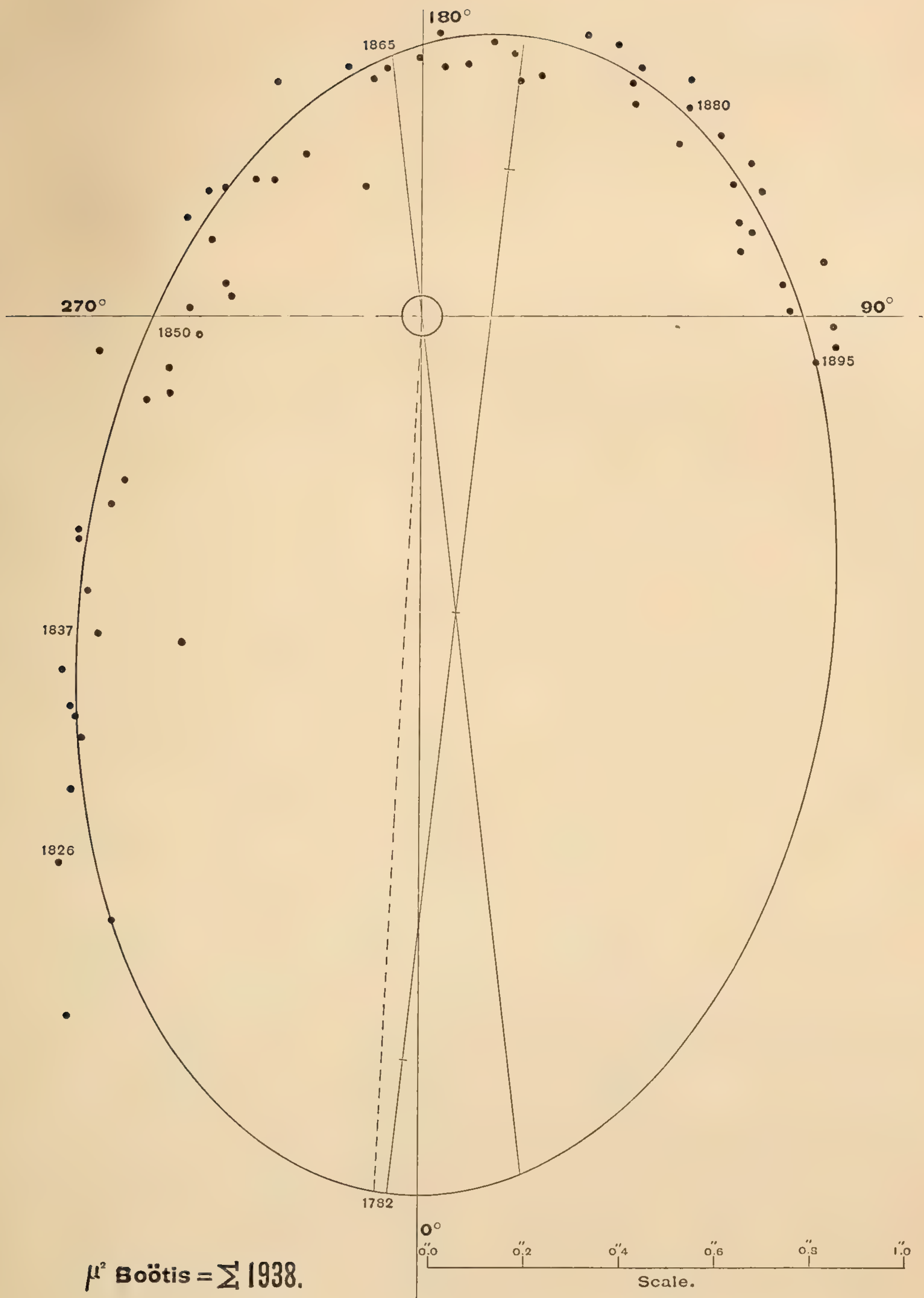
$$\begin{aligned}
 P &= 219.42 \text{ years} & \Omega &= 163^\circ.8 \\
 T &= 1865.30 & i &= 43^\circ.9 \\
 e &= 0.537 & \lambda &= 329^\circ.75 \\
 a &= 1''.2679 & n &= -1^\circ.6407
 \end{aligned}$$

Apparent orbit:

$$\begin{aligned}
 \text{Length of major axis} &= 2''.656 \\
 \text{Length of minor axis} &= 1''.480 \\
 \text{Angle of major axis} &= 173^\circ.5 \\
 \text{Angle of periastron} &= 186^\circ.7 \\
 \text{Distance of star from centre} &= 0''.638
 \end{aligned}$$

An examination of the computed and observed places, given in the following table; seems to justify the conclusion that the elements found above will

* *Astronomische Nachrichten*, 3309.



not be materially changed by future investigation. Thus, the period will hardly be varied by so much as ten years, while the resulting alterations in the eccentricity, inclination and other elements will be relatively inconsiderable.

TABLE OF COMPUTED AND OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1782.68	357.2	353.9	—	1.83	+3.3	—	1	Herschel
1802.86	346.2	343.6	—	1.68	+2.6	—	—	Herschel
1822.21	330.7	330.8	—	1.38	—0.1	—	2	Struve
1823.41	333.7	329.0	1.65	1.33	+4.7	+0.32	3	Herschel and South
1825.46	333.5	326.2	1.43	1.26	+6.3	+0.17	5	South
1826.77	327.0	325.9	1.38	1.25	+1.1	+0.13	2	Struve
1829.73	324.0	323.4	1.24	1.20	—0.9	+0.04	2	Struve
1830.24	324.1	322.1	0.85	1.17	+2.0	—0.32	2	Herschel
1831.36	321.7	320.9	1.14	1.14	+0.8	0.00	1	Herschel
1833.42	319.6	318.1	1.11	1.09	+1.5	+0.02	7-5	Herschel 3-1; Dawes 1; Σ . 3
1835.55	318.6	315.1	1.10	1.03	+3.5	+0.07	3	Struve
1836.65	315.1	313.4	1.06	1.00	+1.7	+0.06	3	Struve
1837.53	315.0	311.8	0.95	0.97	+3.2	—0.02	—	Dawes 1; Σ . —
1840.42	309.9	306.7	0.91	0.89	+3.2	+0.02	6	Dawes 3; $O\Sigma$. 3
1841.66	303.2	304.1	0.86	0.85	—0.9	+0.01	6-3	Dawes
1842.32	302.4	302.6	0.85	0.83	—0.2	+0.02	6	$O\Sigma$. 3; Dawes 3
1843.57	301.5	299.5	0.76	0.80	+2.0	—0.04	10	Mädler
1844.39	299.2	297.3	0.71	0.77	+1.9	—0.06	2	Mädler
1846.68	287.1	291.3	0.57	0.71	—3.2	—0.14	4	O. Struve
1847.34	287.3	288.4	0.60	0.68	—1.1	—0.08	19-17	Dawes 4; Mädler 15-13
1848.47	281.7	284.3	0.54	0.65	—2.6	—0.11	9-10	Mädler 2; Dawes 4; Bond 3-4
1849.44	276.2	280.5	0.68	0.63	—4.3	+0.05	2	Dawes
1850.57	274.7	275.6	0.47	0.60	—0.9	—0.13	5-4	$O\Sigma$. 2; Mädler 3-2
1851.49	263.9	271.2	0.40	0.58	—7.3	—0.18	12	Mädler 7; Dawes 2; $O\Sigma$. 3
1852.55	268.2	265.8	0.49	0.55	+2.4	—0.06	3	O. Struve
1853.50	260.9	260.1	0.42	0.53	+0.8	—0.11	4	Jacob 2; Mädler 2
1854.39	250.1	255.5	0.47	0.52	—5.4	—0.05	9	Jacob 2; Dawes 3; Mädler 4
1855.11	247.2	250.8	0.53	0.51	—2.6	+0.02	4	O. Struve
1856.49	239.3	241.1	0.52	0.49	—1.8	+0.03	3	Secchi 1; $O\Sigma$. 2
1857.52	236.3	235.0	0.49	0.49	+1.3	0.00	6	Mädler 2; Secchi 1; $O\Sigma$. 3
1858.56	230.1	228.0	0.45	0.49	+2.1	—0.04	8	Secchi 1; $O\Sigma$. 3; Mädler 4
1859.39	226.4	223.7	0.42	0.49	+2.7	—0.07	3-2	Mädler
1860.95	211.3	212.4	0.58	0.50	—1.1	+0.08	3	O. Struve
1861.58	215.1	207.9	0.42	0.50	—7.2	—0.08	2	Mädler
1862.56	202.9	202.2	0.32	0.52	+0.7	—0.20	3	Dembowski
1863.38	195.8	197.3	0.55	0.53	+1.5	+0.02	12	Dembowski
1864.44	191.2	191.2	0.51	0.54	0.0	—0.03	9	Knott 4; Dembowski 5
1865.56	187.5	184.7	0.53	0.56	+2.8	—0.03	18	Dem. 10; Dawes 3; Englemann 5
1866.47	180.2	181.4	0.55	0.57	—1.2	—0.02	11	$O\Sigma$. 3; Dembowski 7; Secchi 1
1867.48	175.8	175.9	0.60	0.59	—0.1	+0.01	6	Dembowski
1868.38	174.2	171.8	0.53	0.60	+2.4	—0.07	5	Dembowski
1869.51	169.3	166.7	0.54	0.61	+2.6	—0.07	8	Dunér 6; $O\Sigma$. 2
1870.45	164.6	162.7	0.60	0.62	+1.9	—0.02	12-11	Dem. 7; Gledhill 1-0; Dunér 4
1871.54	160.1	158.4	0.59	0.63	+1.7	—0.04	13	Dem. 7; Dunér 5; Gledhill 1
1872.44	156.9	154.8	0.54	0.65	—1.8	—0.11	16	W. & S.; Dem. 8; Kn. 4; Du. 2
1873.38	153.1	151.5	0.57	0.65	+1.6	—0.08	16-15	$O\Sigma$. 4; W. & S. 3-2; Dem. 7; Gl. 2
1874.37	149.2	147.6	0.69	0.66	+1.6	+0.03	9	Gledhill 2; W. & S. 1; Dem. 6
1875.46	144.0	143.5	0.71	0.67	+0.5	+0.04	13	Dem. 8; Schiaparelli 4; Dunér 1
1876.46	138.2	139.4	0.70	0.68	—1.2	+0.02	8	Dembowski
1877.38	137.6	136.5	0.67	0.68	+1.1	—0.01	20-18	Sch. 5; Dk. 4-2; Dem. 7; W. & S. 4
1878.49	134.6	132.7	0.64	0.69	+1.9	—0.05	16	β . 1; Dk. 4; Dem. 6; Sch. 5

<i>t</i>	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	<i>n</i>	Observers
1879.52	131.0	129.3	0.76	0.69	+1.7	+0.07	8	Schiaparelli 4; Hall 4
1880.44	127.7	126.3	0.72	0.70	+1.4	+0.02	17	β . 5; Hl. 1; Dk. 4; Sch. 4; Jed. 4
1881.43	123.8	122.8	0.66	0.70	+1.0	-0.04	21-15	Dk. 4-0; β . 4; Sch. 4; Big. 6-4; Hl. 4
1882.45	121.3	119.6	0.74	0.71	+1.7	+0.03	13-12	Dk. 0-1; Hall 3; Sch. 4; En. 4
1883.53	114.9	116.2	0.77	0.72	-1.3	+0.05	14	Sch. 4; Hall 2; En. 6; Per. 2
1884.49	113.0	113.1	0.72	0.72	-0.1	0.00	7	Hall 3; Schiaparelli 4
1885.52	110.4	110.0	0.77	0.73	+0.4	+0.04	19	Per. 2; Tar. 3; Hl. 4; Sch. 4; Jed. 6
1886.58	106.5	107.3	0.70	0.74	-0.8	-0.04	12	Hall 3; Per. 2; Sch. 2; Jed. 5
1887.50	104.2	104.2	0.72	0.75	0.0	-0.03	10	Hall 4; Schiaparelli 6
1888.65	101.5	101.0	0.69	0.76	+0.5	-0.07	11-9	Hall 4; Schiaparelli 5-3; Tarrant 2
1889.43	97.6	98.7	0.86	0.77	-1.1	+0.09	7	Maw 3; Hodges 1; Schiaparelli 3
1891.51	95.0	93.2	0.77	0.79	+1.8	-0.02	4	Schiaparelli 2; See 2
1892.50	90.9	90.6	0.78	0.80	+0.3	-0.02	5-4	Collins 1; Comstock 4-3
1893.48	88.3	88.2	0.87	0.81	+0.1	+0.06	6	Bigourdan 4; Maw 2
1894.54	85.7	85.6	0.88	0.82	+0.1	+0.06	6	Bigourdan 5; H. C. Wilson 1
1895.31	83.5	83.8	0.84	0.84	-0.3	0.00	3	See

The following is a short ephemeris :

<i>t</i>	θ_c	ρ_c	<i>t</i>	θ_c	ρ_c
1896.50	81.1	0.85	1899.50	74.8	0.89
1897.50	78.9	0.86	1900.50	72.6	0.90
1898.50	76.9	0.87			

OS 298.

$\alpha = 15^h 32^m.4$; $\delta = +40^\circ 9'$.
7, yellowish ; 7.4, yellowish.

Discovered by Otto Struve in 1845.

OBSERVATIONS.

<i>t</i>	θ_o	ρ_o	<i>n</i>	Observers	<i>t</i>	θ_o	ρ_o	<i>n</i>	Observers
1845.50	180.5	1.25	2	O. Struve	1865.53	210.2	1.0	1	Dembowski
1846.28	186.5	1.41	2	Mädler	1866.29	207.0	0.8	1	Dembowski
1847.32	189.6	1.51	2-1	Mädler	1867.61	209.5	0.99	1	Dembowski
1848.46	183.9	1.11	1	O. Struve	1868.52	32.5	0.84	1	O. Struve
1848.68	185.8	1.23	1	Dawes	1869.46	214.1	0.61	3	Dunér
1851.75	191.8	1.40	2	Mädler	1870.26	225.8	separation doubtful	1	Dembowski
1856.58	193.1	1.21	1	O. Struve	1871.63	226.6	contatto?	1	Dembowski
1857.68	196.8	1.24	1	O. Struve	1872.58	235.8	0.58	1	O. Struve
1859.62	197.4	1.13	1	O. Struve	1875.52	84.2	0.53	1	O. Struve
1861.44	13.5	1.16	1	O. Struve	1875.65	265.5	0.37	2	Dembowski

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1876.47	280.8	0.3 cuneo	3	Dembowski	1887.50	142.0	0.39	3	Hall
1877.53	295.9	0.3	5	Dembowski	1887.56	143.0	0.33	6	Schiaparelli
1878.33	130.8	0.27	2	Burnham	1888.54	339.4	0.65	1	O. Struve
1879.46	335.0	0.26	4	Hall	1888.59	153.4	0.42	5	Schiaparelli
1879.49	327.8	0.33	4	Schiaparelli	1889.52	158.1	0.55	3	Schiaparelli
1881.41	175.4	0.35	3	Hall	1891.48	167.3	0.68	3	Hall
1882.47	7.5	0.33	4	Schiaparelli	1891.49	347.5	0.63	1	Schiaparelli
1882.52	359.5	0.30	4-3	Englemann	1892.42	169.9	0.82	1	Collins
1882.55	358.0	0.32	1	O. Struve	1892.47	169.3	0.88	2	Bigourdan
1883.52	22.4	0.31	6	Schiaparelli	1892.59	168.9	0.64	4	Comstock
1883.65	36.7	0.17	3	Englemann	1893.43	351.5	0.91	1	Bigourdan
1884.44	49.0	0.30	2	Perrotin	1893.71	173.6	0.64	1	Comstock
1884.51	57.3	0.31	5	Schiaparelli	1895.54	173.1	0.85	3	Comstock
1885.65	60.9	0.27	7-4	Englemann	1895.56	174.2	0.82	1	Schiaparelli
1886.67	133.7	0.29	2	Schiaparelli	1895.74	179.4	0.95	2	See
1886.68	104.9	0.29	7	Englemann	1895.74	177.2	1.05	1	Moulton

Since the discovery of this binary in 1845, the companion has described substantially an entire revolution. The period is therefore fixed with sufficient precision; indeed, the numerous and satisfactory measures of this pair secured during the last fifty years define the other elements in a manner almost equally satisfactory. The shape of the apparent orbit is such that the pair is never excessively difficult, and yet measurement near periastron, where the distance reduces to $0''.22$, requires a good telescope. The components are of nearly equal brightness, and hence a number of the measures as recorded requires a correction of 180° .

The following orbits of this pair have been published by previous computers:

P	T	e	a	Ω	i	λ	Authority	Source
68.802 yrs.	1812.96	0.4872	0.886	14.63	56.17	342.52	Doberck, 1879	A.N., 2280
70.26	1882.22	0.51	0.83	12.29	50.63	346.15	Dolgorukow, 1883	A.N., 2531
56.653	1882.857	0.5836	0.8835	2.13	65.85	21.9	Celoria, 1888	A.N., 2843
51.0	1883.0	0.577	0.780	4.1	61.2	20.7	See, 1895	Unpublished

An investigation based on all the best observations leads to the following elements of 05 298.

$$\begin{aligned}
 P &= 52.0 \text{ years} & \Omega &= 1^\circ.9 \\
 T &= 1883.0 & i &= 60^\circ.9 \\
 e &= 0.581 & \lambda &= 26^\circ.1 \\
 a &= 0''.7989 & n &= +6^\circ.9231
 \end{aligned}$$

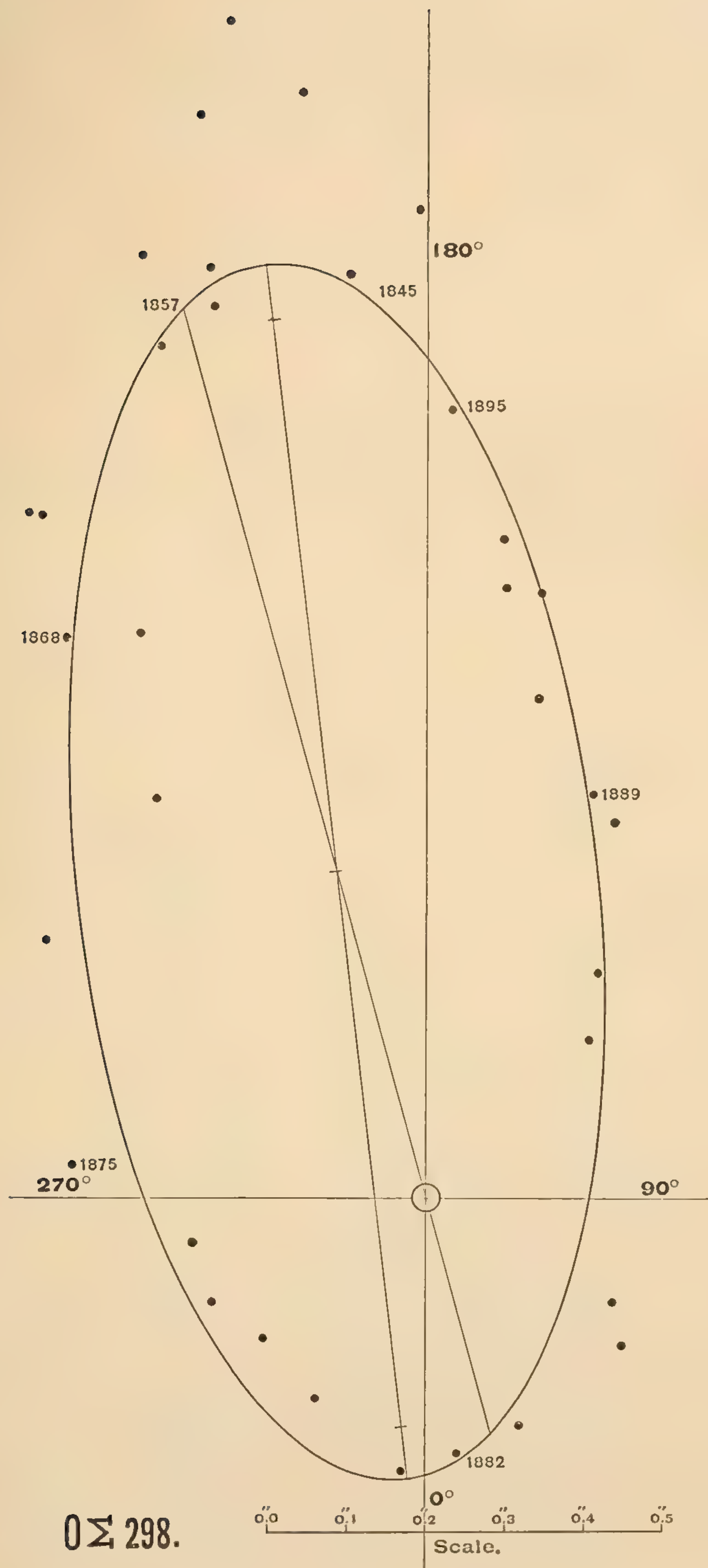
Apparent orbit:

Length of major axis	= 1".546
Length of minor axis	= 0".656
Angle of major axis	= 186°.9
Angle of periastron	= 15°.3
Distance of star from centre	= 0".427

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1845.50	180.5	180.5	1.25	1.07	± 0.0	+0.18	2	O. Struve
1846.28	186.5	181.6	1.41	1.09	+ 4.9	+0.32	2	Mädler
1847.32	189.6	183.1	1.51	1.12	+ 6.5	+0.39	2-1	Mädler
1848.57	184.9	184.8	1.17	1.16	+ 0.1	+0.01	2	O. Struve 1; Dawes 1
1851.75	191.8	188.8	1.40	1.19	+ 3.0	+0.21	2	Mädler
1856.58	193.1	190.2	1.21	1.20	+ 2.9	+0.01	1	O. Struve
1857.68	196.8	196.2	1.24	1.15	+ 0.6	+0.09	1	O. Struve
1859.62	197.4	198.9	1.13	1.11	- 1.5	+0.02	1	O. Struve
1861.44	193.5	201.6	1.16	1.06	- 8.1	+0.10	1	O. Struve
1865.53	210.2	209.1	1.0	0.90	+ 1.1	+0.10	1	Dembowski
1866.29	207.0	210.8	0.8	0.87	- 3.8	-0.07	1	Dembowski
1867.61	209.5	214.2	0.99	0.80	- 4.7	+0.19	1	Dembowski
1868.52	202.5	216.9	0.84	0.75	-14.4	+0.09	1	O. Struve
1869.46	214.1	220.0	0.61	0.71	- 5.9	-0.10	3	Dunér
1870.26	225.8	222.7	—	0.67	+ 3.1	—	1	Dembowski
1871.63	226.6	229.6	—	0.58	- 3.0	—	1	Dembowski
1872.58	235.8	235.3	0.58	0.53	+ 0.5	+0.05	1	O. Struve
1875.57	264.7	263.2	0.45	0.37	+ 1.5	+0.08	3	O. Struve 1; Dembowski 2
1876.47	280.8	275.9	0.3	0.34	+ 4.9	-0.04	3	Dembowski
1877.53	295.9	292.8	0.3	0.33	+ 3.1	-0.03	5	Dembowski
1878.33	310.8	306.4	0.27	0.33	+ 4.4	-0.06	2	Burnham
1879.47	331.4	325.0	0.29	0.34	+ 6.4	-0.05	8	Hall 4; Schiaparelli 4
1881.41	355.4	352.1	0.35	0.36	+ 3.3	-0.01	3	Hall
1882.47	7.5	6.6	0.33	0.34	+ 0.9	-0.01	4	Schiaparelli
1883.57	22.4	26.7	0.31	0.28	- 4.3	+0.03	6	Schiaparelli
1884.47	53.1	53.7	0.31	0.22	- 0.6	+0.09	7	Perrotin 2; Schiaparelli 5
1885.65	60.9	102.4	0.27	0.22	-41.5	+0.05	7-4	Englemann
1886.68	133.7	130.6	0.29	0.29	+ 3.1	± 0.00	2	Schiaparelli
1887.53	142.5	144.1	0.36	0.38	- 1.6	-0.02	9	Hall 3; Schiaparelli 6
1888.56	153.4	153.6	0.53	0.48	- 0.2	+0.05	5-6	O. Struve 0-1; Schiaparelli 5
1889.52	158.1	159.1	0.55	0.56	- 1.0	-0.01	3	Schiaparelli
1891.49	167.4	167.8	0.65	0.74	- 0.4	-0.09	4	Hall 3; Schiaparelli 1
1892.49	169.4	170.8	0.78	0.81	- 0.6	-0.03	7	Collins 1; Bigourdan 2; Com. 4
1893.62	172.5	173.4	0.78	0.88	- 0.9	-0.10	2	Bigourdan 1; Comstock 1
1895.55	173.7	177.3	0.84	0.99	- 3.6	-0.15	4	Comstock 3; Schiaparelli 1
1895.74	178.3	177.6	1.00	1.00	+ 0.7	± 0.00	3	See 2; Moulton 1

The table of computed and observed places shows that these elements are extremely satisfactory. Future observations are not likely to vary the period given above by more than one year, while an error of ± 0.02 in the eccentricity is highly improbable. In spite of the accuracy of the present elements some improvement will ultimately be desirable, and hence astronomers should continue to give this interesting system regular attention. The star will be easy



for a number of years, and observers with small telescopes will find it an important object for measurement.

The following is a short ephemeris:

t	θ_c	ρ_c	t	θ_c	ρ_c
1896.50	178.9	1.03	1899.50	183.3	1.13
1897.50	180.5	1.07	1900.50	184.7	1.15
1898.50	182.0	1.10			

γ CORONAE BOREALIS = Σ 1967.

$\alpha = 15^h 38^m.5$; $\delta = +26^\circ 36'$.
4, yellow ; 7, blue.

Discovered by William Struve in 1826.

OBSERVATIONS.

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1826.75	110.0	0.72	2	Struve	1848.39	297.0	0.39	4	Mädler
1828.98	110.7	0.54	3	Struve	1848.49	292.8	0.4 $\pm$	3	W.C. Bond
1832.21	102.7	0.4 $\pm$	3	Struve	1849.63	289.4	0.50	3	O. Struve
1833.34	105.8	0.4 $\pm$	2	Struve	1850.69	289.9	0.53	3	Mädler
1835.46	simplex	—	3	Struve	1851.33	292.5	0.3 $\pm$	1	Mädler
1836.52	338 ?	obl. ?	4	Struve	1851.50	287.6	0.48	4	O. Struve
1840.51	252. cuneiforme		1	W. Struve	1852.07	285.1	0.57 $\pm$	4	Dawes
1840.78	255. cuneiforme		4	O. Struve	1852.58	296.4	0.46	7-6	Mädler
1841.50	332.3	0.18	10-4	Mädler	1853.01	287.9	0.46	5	O. Struve
1842.49	314.3	0.20	4-1	Mädler	1853.20	294.3	0.5	2	Jacob
1842.80	272.0	0.47	2	Mädler	1853.32	284.5	0.40	4-3	Mädler
1843.30	292.5	0.41	3	O. Struve	1854.40	284.3	0.69	2	Dawes
1843.45	288.9	0.6 $\pm$	1	Dawes	1854.76	291.1	0.4 $\pm$	1	Mädler
1843.48	276.6	0.39	9-2	Mädler	1855.50	semplice	—	—	Secchi
1844.37	286.2 ?	—	1	Mädler	1855.73	292.4	—	1	Mädler
1845.37	292.1	0.45	9	Mädler	1856.37	295.4	0.67	3	Winnecke
1845.61	296.0	0.44	5	O. Struve	1856.59	288.9	0.45	8-7	Secchi
1845.57	292.7	0.43	9-8	Mädler	1856.62	283.8	0.47	6	O. Struve
1846.56	294.2	0.45	11	Mädler	1857.39	286.5	0.32	2	Mädler
1847.29	292.6	0.44	5	O. Struve	1857.52	281.0	0.5 $\pm$	1	Dawes
1847.43	295.1	0.36	11-9	Mädler	1857.52	289.3	0.36	5	Secchi
					1858.51	281.0	cuneo	3	Dembowski
					1858.57	284.1	0.33	4-3	Mädler
					1858.97	284.7	0.46	5	O. Struve

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1859.36	282.6	0.45 $\pm$	1	Dawes	1883.53	142.6	0.16 $\pm$	3	Perrotin
1859.38	290.4	obl.	3	Mädler	1883.57	129.1	0.41	5	Schiaparelli
1861.59	287.7	0.42	3	O. Struve	1883.60	149.3	0.58	1	O. Struve
1862.56	292.9	cuneo	3	Dembowski	1883.64	146.9	0.20	8	Englemann
1862.91	227 ?	doubtful	1	Mädler	1884.52	125	cuneiforme	2	Perrotin
1863.25	semplíce	—	1	Dembowski	1884.53	305.6	0.34	1	Perrotin
1863.64	290.5	0.41	3	O. Struve	1884.53	132.4	0.34	6	Schiaparelli
1865.6	semplíce	—	4	Secchi	1884.61	166.8	0.28	6	Englemann
1865.26	semplíce	—	1	Dembowski	1885.48	round	—	1	Smith
1865.50	280.	<0.5	1	Englemann	1885.54	134.3	0.35	3	Schiaparelli
1865.53	einfach	—	1	Englemann	1885.63	164.6	0.38	10-6	Englemann
1866.30	201.2	—	4	Harvard	1886.51	129.1	0.38	6	Schiaparelli
1866.61	205.3	—	1	Winlock	1886.69	159.9 ?	0.93 ?	8	Englemann
1866.62	286.0	0.43	2	O. Struve	1887.51	126.6	0.38	13	Schiaparelli
1867.75	simple	—	10	Dunér	1887.55	round	—	1	Smith
1868.02	260.2	0.36	2	O. Struve	1888.55	124.3	0.40	16-15	Schiaparelli
1868.72	252	cuneiforme	—	O. Struve	1888.61	132.0	0.85	2	O. Struve
1869.36	280.4	—	1	Leyton Obs.	1889.42	109.2	—	1	Hodges
1872.45	190 ?	—	1	W. & S.	1889.52	122.4	0.41	4	Schiaparelli
1873.38	195 ?	—	1	W. & S.	1890.68	124.1	0.51	1	Bigourdan
1874	simple	—	—	O. Struve	1891.50	120.0	0.5 $\pm$	1	See
1874.56	166.9	—	1	Leyton Obs.	1891.51	122.5	0.42	4	Schiaparelli
1875.40	single	—	1	Hall	1891.51	125.6	0.36	4	Hall
1875.41	165.4	—	1	Leyton Obs.	1891.58	118.8	0.51	1	Bigourdan
1876.32	simple	—	1	Flammarion	1892.44	122.3	0.83 $\pm$	1	H.C. Wilson
1876.	single	—	1	Doberek	1892.44	121.1	0.69	1	Bigourdan
1876.45	single	—	1	Hall	1892.60	122.8	0.47	7	Schiaparelli
1876.81	simple	—	—	Schiaparelli	1892.72	121.9	0.40	3	Comstock
1877.54	163.3	0.44	2	O. Struve	1893.49	120.0	0.52	2	Schiaparelli
1878.60	150.7	0.56	2	O. Struve	1893.50	118.4	0.65	2	Bigourdan
1879.56	single	—	2	Hall	1894.48	119.7	0.53	2	Schiaparelli
1879.81	single	—	5	Burnham	1894.60	121.3	0.60	5-4	Barnard
					1895.30	114.8	0.67	3	See
					1895.55	117.1	0.43	3	Comstock
					1895.61	123.7	0.64	4	Barnard

The components of this remarkable system are of the 4th and 7th magnitudes, and of yellow and bluish colors respectively, so that the object is generally very difficult. STRUVE happened to discover* the companion near the time of its maximum elongation, when the polar coordinates were $\theta = 111^\circ.0$,

* *Astronomical Journal*, 376.

$\rho = 0''.72$. Measures in 1828, 1831 and 1833, showed that both angles and distances were steadily decreasing, and in 1835 the star appeared single under the best seeing. The companion was not again recognized with certainty until 1842, although STRUVE, O. STRUVE and MÄDLER searched for it repeatedly during the intervening period, and occasionally suspected an elongation. But the discordance in the angles of the supposed elongations justify the belief that the phenomena observed were probably nothing more than points of diffraction fringes, or some other kind of spurious images. MÄDLER's observation of $332^\circ.3$ and $0''.18$ at the epoch 1841.50 may be genuine, although at this time the star must have been excessively close. The binary character of the pair was early recognized by STRUVE, who pointed out the particular interest attaching to the system on account of its high inclination. γ *Coronae Borealis* has since been measured by many of the best observers, and yet the stars are so unequal and so close that the errors of observation assume formidable proportions, and render a satisfactory determination of the elements very difficult. The great inclination of the orbit throws nearly all the position-angles into small regions of about 10° on either side, and while the retrograde motion ought to make all angles steadily decrease, we are sometimes confounded by an appearance of direct motion (as from 1859 to 1863) which proves the existence of sensible systematic errors, probably due to the placing of the micrometer wires parallel to the edges of unequal images.

It is equally confusing to find that instead of a steady increase and decrease in the distance, nearly all of the distances are in the immediate neighborhood of $0''.4$; such measures are of course misleading, as the companion cannot be standing still at a constant angle and distance. While, therefore, it is clear that the elements can not lay claim to such accuracy as could be desired, it will yet appear that they are good and even excellent for observations which are so badly vitiated by accidental and systematic errors.

It is obvious that in case of a system whose orbit plane lies nearly in the line of vision, the angles will be practically useless unless measured with the greatest accuracy; yet, in this instance, even when the pair is fairly wide, we frequently find the angles of individual observers differing by so much as 10° , and when the stars are close the uncertainty in angle will amount to at least twice this quantity. On account of such conspicuous errors in angle we have based the present orbit largely upon the distances.

DOBERCK and CELORIA are the only astronomers who have previously attempted an orbit for this pair.

P	T	e	a	Ω	i	λ	Authority	Source
^{yrs.} 95.50	1843.7	0.387	["] 0.75	[°] 111.0	[°] 83.0	[°] 239.0	Doberck, 1877	A.N., Bd., 88
95.5	1843.70	0.350	0.70	110.4	85.2	233.5	Doberck, 1877	A.N., 2123
85.276	1840.508	0.3483	0.631	113.47	81.67	250.7	Celoria, 1889	A.N., 2904

From an investigation of the best observations we find the following elements :

$$\begin{aligned}
 P &= 73.0 \text{ years} & \Omega &= 110^\circ.7 \\
 T &= 1841.0 & i &= 82^\circ.63 \\
 e &= 0.482 & \lambda &= 97^\circ.95 \\
 a &= 0''.7357 & n &= -4^\circ.9315
 \end{aligned}$$

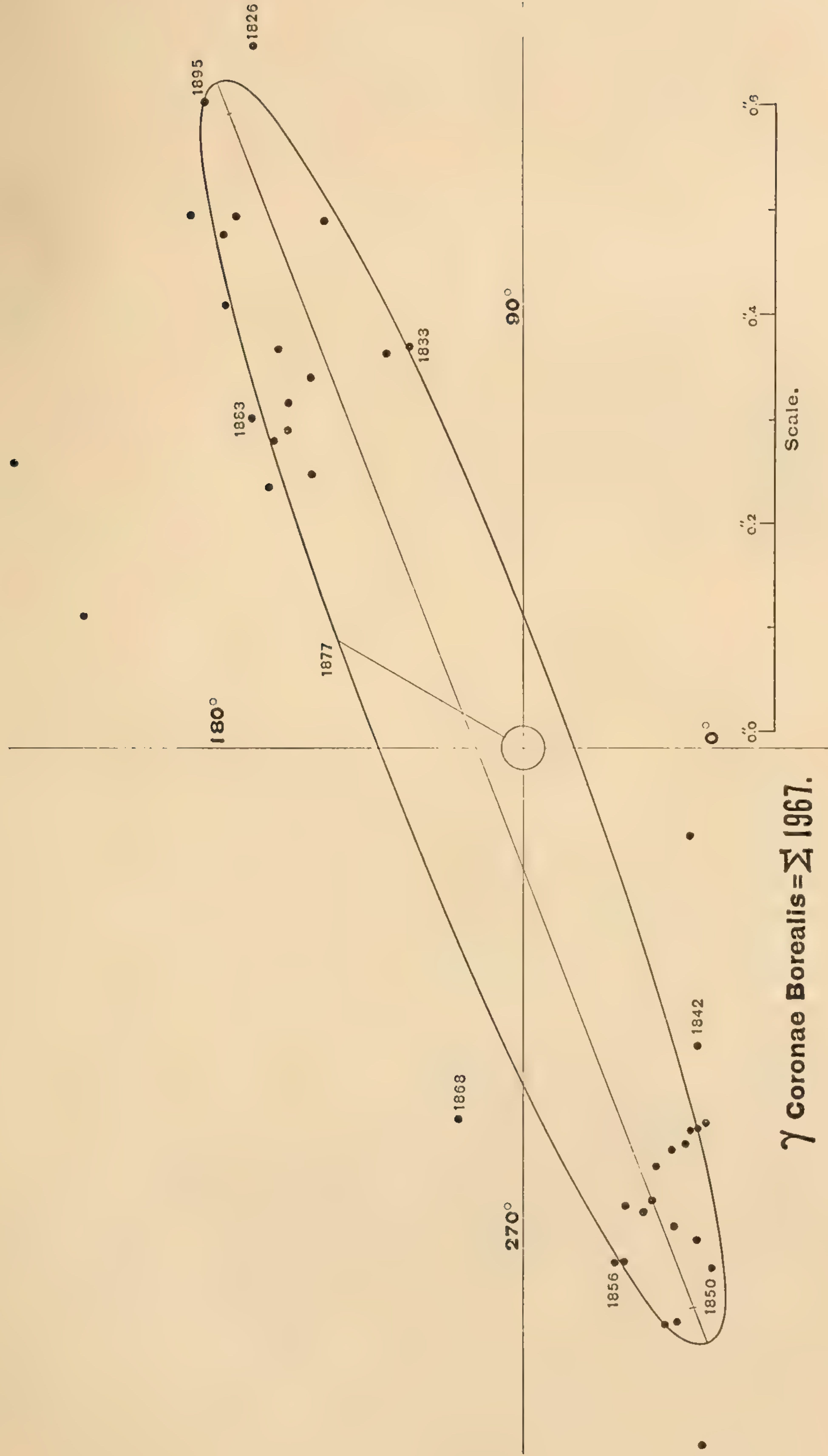
Apparent orbit:

$$\begin{aligned}
 \text{Length of major axis} &= 1''.30 \\
 \text{Length of minor axis} &= 0''.175 \\
 \text{Angle of major axis} &= 111^\circ.3 \\
 \text{Angle of periastron} &= 329^\circ.6 \\
 \text{Distance of star from centre} &= 0''.068
 \end{aligned}$$

The accompanying table shows the agreement of the above elements with the mean places.

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1826.75	[°] 111.0	[°] 114.5	["] 0.72	["] 0.70	[°] - 3.5	["] +0.02	2	Struve
1828.98	110.7	113.5	0.54	0.69	- 2.8	-0.15	3	Struve
1831.68	109.3	111.2	0.4 $\pm$	0.63	- 1.9	-0.23 $\pm$	1	Struve
1833.34	105.8	110.0	0.4 $\pm$	0.57	- 4.2	-0.17 $\pm$	2	Struve
1835.46	simplex	107.4	—	0.44	—	—	3	Struve
1836.52	oblong?	105.5	—	0.37	—	—	4	Struve
1840.78	75	95.1	cune.	0.16	-20.1	—	4	O. Struve
1841.50	332.3	314.8	0.18	0.10	+17.5	+0.08	10-4	Mädler
1842.64	300.4	301.9	0.33	0.21	- 1.5	+0.12	6-3	Mädler
1843.30	292.5	298.6	0.41	0.28	- 6.1	+0.13	3	O. Struve
1844.37	286.2?	295.5	—	0.37	- 9.3	—	1	Mädler
1845.61	296.0	293.1	0.44	0.45	+ 2.9	-0.01	5	O. Struve
1847.36	293.8	290.8	0.40	0.54	+ 3.0	-0.14	16-14	O. Struve 5; Mädler 11-9
1848.44	294.9	289.7	0.4	0.57	+ 5.2	-0.17	7	Mä. 4; W. C. & G. P. Bond 3
1849.63	289.9	288.6	0.50	0.58	+ 1.3	-0.08	3	O. Struve
1850.69	289.9	287.7	0.53	0.59	+ 2.2	-0.06	3	Mädler
1851.50	287.6	287.0	0.48	0.60	+ 0.6	-0.12	4	O. Struve
1852.07	285.1	286.5	0.57 $\pm$	0.60	- 1.4	-0.03 $\pm$	4	Dawes
1853.17	286.2	285.6	0.45	0.59	+ 0.6	-0.14	9-10	O. 5; Ja. 0-2; Mä. 4-3
1854.40	284.3	284.4	0.69	0.58	- 0.1	+0.11	2	Dawes
1855.73	292.4	283.3	—	0.56	+ 9.1	—	1	Mädler
1856.62	283.8	282.4	0.57	0.54	+ 1.4	+0.03	6-9	O. Struve
1857.52	281.0	281.4	0.50	0.52	- 0.4	-0.02	1	Dawes
1858.97	284.7	279.8	0.46	0.48	+ 3.9	-0.02	5	O. Struve
1859.36	282.6	279.4	0.45	0.47	+ 3.2	-0.02	1	Dawes
1861.59	287.7	276.2	0.42	0.41	+11.5	+0.01	3	O. Struve
1862.73	260.0	274.0	cuneo	0.38	-14.0	—	4	Dembowski; Mädler 1
1863.64	290.5	272.3	0.41	0.35	+18.0	+0.06	3	O. Struve



γ Coronae Borealis = Σ 1967.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1865.50	280.	267.7	<0.5	0.30	+12.3	+0.2—	1	Englemann
1866.62	286.0	260.0	0.43	0.24	+26.0	+0.19	2	O. Struve
1868.02	260.2	257.9	0.36	0.22	+ 2.3	+0.14	2	O. Struve
1872.91	192.5	209.0	—	0.13	—16.5	—	2	Wilson & Seabroke
1874.56	166.9	184.8	—	0.14	—17.9	—	1	Leyton Observers
1875.41	165.4	175.3	—	0.14	— 9.9	—	1	Leyton Observers
1877.54	163.3	156.3	0.44	0.18	+ 7.0	+0.26	2	O. Struve
1878.60	150.7	147.0	0.56	0.22	+ 3.7	+0.34	2	O. Struve
1883.57	129.1	130.3	0.41	0.36	— 1.2	+0.05	5	Schiaparelli
1884.53	127.6	128.0	0.33	0.41	— 0.4	—0.08	9-13	Per. 3-1; Sch. 6; En. 0-6
1885.54	134.3	126.8	0.35	0.43	+ 7.5	—0.08	3	Schiaparelli
1886.51	129.1	125.3	0.38	0.46	+ 3.8	—0.08	6	Schiaparelli
1887.51	126.6	124.2	0.38	0.48	+ 2.4	—0.10	13	Schiaparelli
1888.55	124.3	123.0	0.40	0.52	+ 1.3	—0.12	16-15	Schiaparelli
1889.50	119.8	122.0	0.41	0.54	— 2.2	—0.13	5-4	Hodges 1-0; Schiaparelli 4
1890.68	124.1	121.1	0.51	0.57	+ 3.0	—0.06	1	Bigourdan
1891.52	121.7	120.2	0.45	0.59	+ 1.5	—0.14	10	See 1; Sch. 4; Hill 4; Big. 1
1892.55	122.0	119.4	0.60	0.62	+ 2.6	—0.02	12	H. C. Wilson 1; Sch. 7; Com. 3
1893.50	118.4	118.7	0.58	0.64	— 0.3	—0.06	2-4	Bigourdan 2; Schiaparelli 0-2
1894.54	120.4	117.9	0.57	0.66	+ 2.5	—0.09	6	Schiaparelli 2; Barnard 4
1895.42	116.0	117.3	0.69	0.67	— 1.3	+0.02	6-3	See 3; Comstock 3-0

The following is a short ephemeris :

t	θ_c	ρ_c	t	θ_c	ρ_c
	$^{\circ}$	$''$		$^{\circ}$	$''$
1896.50	116.6	0.69	1899.50	115.3	0.70
1897.50	116.0	0.69	1900.50	114.1	0.70
1898.50	115.3	0.70			

According to this orbit previous investigators have materially overestimated the period. While the time of revolution must at present remain slightly uncertain, it does not seem at all probable that this element can surpass 75 years. It follows, therefore, that γ *Coronae Borealis* belongs to the class of unequal binaries with moderately short periods. The inclination and line of nodes here obtained will probably be nearly correct, while the eccentricity is not likely to be varied by so much as ± 0.05 .

Recent distances have been appreciably undermeasured by several observers; the separation of the components is now about $0''.68$, and will not change sensibly for several years. γ *Coronae Borealis* needs further observation, and astronomers should continue to give it regular attention; but owing to the peculiar shape of the apparent orbit great care must be exercised to avoid systematic errors, if the measures are to be of much value in effecting a further improvement of the elements.

ξ SCORPII = Σ 1998. $\alpha = 15^h 58^m.9$; $\delta = -11^\circ 5'$.

5, yellow ; 5.2, yellow.

Discovered by Sir William Herschel, September 9, 1781.

OBSERVATIONS.

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1782.36	188.0	—	1	Herschel	1846.17	23.2	1.00	3-1	Jacob
1825.47	355.3	1.15	3	Struve	1846.47	24.1	0.97	9-8	Mitchell
1828.48	1.0	—	1	Herschel	1847.58	26.0	1.71	1	Mitchell
1830.25	1.4	1.46	4-3	Herschel	1848.54	30.6	1.19	3	Dawes
1831.38	9.4	1.32	2-1	Herschel	1848.54	27.2	0.84	1	Mitchell
1831.48	3.5	1.21	1	Struve	1853.53	46.3	—	1	Dawes
1832.52	4.8	1.24	1	Struve	1855.36	48.2	—	4	Dembowski
1833.37	5.0	1.19	1	Struve	1855.53	53.1	0.46	3	Secchi
1833.39	6.2	1.15	1	Dawes	1856.20	65.5	0.63	3	Jacob
1834.45	8.3	1.24	2-1	Herschel	1856.41	58.1	—	4	Dembowski
1834.45	6.7	1.24	1	Struve	1856.49	70.3	0.36	10-8	Secchi
1834.50	7.1	1.17	4	Dawes	1856.58	69.8	0.47	1	O. Struve
1834.51	14.6	—	3	Mädler	1856.55	59.6	—	2	Winnecke
1835.39	10.6	1.58	5-1	Herschel	1857.68	81.4	0.50	1	Jacob
1835.48	11.0	—	4	Mädler	1858.13	79.4	0.40	1	Jacob
1836.49	9.5	1.02	1	Dawes	1858.22	116.8	0.30	1	Jacob
1836.50	11.0	—	3	Mädler	1862.56	137.9	—	3	Dembowski
1837.33	11.4	—	1	Herschel	1863.44	142.1	—	9	Dembowski
1839.61	16.7	1.28	2	Dawes	1864.45	147.8	0.21	4	Secchi
1840.56	18.6	1.19	3	Dawes	1864.51	150.9	—	10	Dembowski
1840.57	17.2	0.96	1	O. Struve	1865.44	151.4	—	10	Dembowski
1841.48	16.7	1.28	4-3	Mädler	1865.51	155.5	0.35	7	Secchi
1841.57	20.8	0.84	1	O. Struve	1865.55	166.9	0.49	7	Englemann
1841.58	19.0	1.20	3-2	Dawes	1866.46	156.6	0.53	8-3	Dembowski
1841.61	17.7	1.30	2-1	Kaiser	1866.52	161.0	0.40	2-1	Secchi
1842.42	20.4	1.05	4-2	Mädler	1867.45	160.7	0.83	7-4	Dembowski
1842.46	21.6	—	2	Dawes	1868.40	165.0	0.90	7-4	Dembowski
1842.53	21.0	—	1	Kaiser	1868.48	166.5	0.99	1	Knott
1843.40	23.5	1.09	2	Dawes	1869.51	172.5	0.83	6	Dunér
1843.40	23.8	1.16	6-4	Mädler	1869.52	168.2	0.88	5	Dembowski
1843.62	20.8	1.20	11-1	Kaiser	1870.21	168.2	—	1	Gledhill
1844.40	23.7	1.82	3	Mädler	1870.39	169.8	0.89	7-5	Dembowski
					1870.54	173.3	0.88	2	Dunér

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1871.41	173.1	1.06	7-5	Dembowski	1882.27	193.6	1.19	1	Dobereck
1871.49	174.0	1.00	1	Gledhill	1882.43	196.7	1.44	1	Englemann
1871.60	174.8	0.88	5	Dunér	1882.46	192.7	1.12	3	Hall
1872.45	177.3	0.95	1	W. & S.	1882.54	191.8	1.31	5	Schiaparelli
1872.46	176.9	1.12	1	Knott	1882.59	192.1	1.35	3	Frisby
1872.46	173.8	1.12	8-5	Dembowski	1883.45	191.0	1.33	4	Frisby
1872.50	175.8	1.10	2	Ferrari	1883.51	193.9	1.38	2	Hall
1872.53	177.4	0.96	3	Dunér	1883.49	195.3	—	1	Küstner
1873.36	180.4	1.04	1	W. & S.	1883.49	195.5	1.20	3	Englemann
1873.36	176.8	1.19	5-3	Dembowski	1883.52	191.5	1.16	3	Perrotin
1873.68	176.5	1.10	1	Gledhill	1883.55	193.5	1.24	12	Schiaparelli
1874.44	183.1	1.19	1	W. & S.	1884.38	195.8	1.34	4-3	H. C. Wilson
1874.49	178.7	1.05	5	Dembowski	1884.44	195.6	1.46	3	Englemann
1875.44	180.5	1.10	5	Dembowski	1884.50	194.6	1.28	5	Hall
1875.51	182.0	1.18	5	Schiaparelli	1884.53	195.1	1.27	3	Perrotin
1875.51	180.0	1.33	1	W. & S.	1884.54	195.6	1.41	1	H. S. Pr.
1875.56	180.9	0.96	4	Dunér	1884.54	195.0	1.26	9	Schiaparelli
1876.44	185.6	1.04	1	Howe	1885.53	196.2	1.34	8	Schiaparelli
1876.45	181.8	1.21	6	Dembowski	1885.57	198.1	1.38	5	Englemann
1876.52	183.9	1.14	3	Hall	1886.35	197.4	1.19	1	H. C. Wilson
1876.52	183.6	1.18	4	Schiaparelli	1886.46	197.5	1.24	2	Perrotin
1876.54	182.5	1.00 ±	1	Plummer	1886.49	198.6	1.54	2-1	Smith
1876.61	186.5	—	3	Dobereck	1886.51	198.1	1.29	3	Tarrant
1877.43	179.5	0.97	2-1	Dobereck	1886.56	198.0	1.07	3	Hall
1877.43	183.3	1.20	5	Dembowski	1886.63	198.9	1.07	7	Englemann
1877.43	184.1	1.61	1	Upton	1886.55	197.2	1.19	3	Schiaparelli
1877.46	184.9	1.27	1	W. & S.	1887.54	199.6	1.16	9	Schiaparelli
1877.47	187.0	1.12	4-1	Howe	1888.50	200.4	1.24	2	Lv.
1877.55	184.0	1.25	9	Schiaparelli	1888.56	200.6	0.96	2	Hall
1877.55	182.5	1.27	3	Jedrzejewicz	1888.57	201.9	1.14	7	Schiaparelli
1878.46	186.2	1.22	5-4	Dembowski	1889.43	197.5	1.20	2	Hodges
1878.54	186.1	1.31	6	Schiaparelli	1890.39	205.2	—	2	Glasenapp
1879.41	189.2	1.22	5	Howe	1891.46	200.6	1.27	2	Collins
1879.42	186.7	1.44	3	Stone	1891.48	208.7	2.87	1	See
1879.47	187.6	1.45	3	Egbert	1892.53	208.0	1.23	3	Maw
1879.54	189.8	1.07	3	Hall	1892.58	206.5	0.82	4	Comstock
1879.56	186.8	1.29	7	Schiaparelli	1893.46	211.1	1.01	2	Burnham
1879.58	185.6	1.47	2	C. W. Pr.	1893.49	209.5	1.10	1	Schiaparelli
1879.60	188.8	1.16	3	Burnham	1893.51	210.9	0.89	2	Lv.
1879.67	194.5	0.70	3-1	Sea. & Smith	1893.60	209.7	1.07	5	Bigourdan
1880.36	188.8	1.12	2	Egbert	1894.59	207.5	1.0 ±	2-1	Glasenapp
1880.40	189.7	1.17	4-2	Dobereck	1895.31	210.3	1.04	3	See
1880.52	185.7	1.13	1	Frisby	1895.41	213.9	0.91	2	Schiaparelli
1880.54	189.0	1.24	6	Schiaparelli	1895.53	213.4	0.81	3	Comstock
1880.87	189.6	1.10	3	H. S. Pr.					
1881.24	191.3	1.03	1	Dobereck					
1881.40	190.8	1.21	2-1	Bigourdan					

This bright star has been observed with considerable regularity since the time of STRUVE, and much material is now available for the investigation of its orbit. But while the measures are numerous, the considerable southern declination of the object renders them rather difficult, especially for European observers, and hence there is reason to suppose that the results are not free from systematic errors. In the investigation of the orbit we have adopted the usual method, depending on both angles and distances, and, as in case of ζ *Canceri*, have neglected the influence of the third star. This procedure has been adopted by DR. SCHORR in his *Dissertation* on the motion of this system, and is fully justified by the rough and somewhat unsatisfactory state of the measures, which will not yet permit any very fine determination of the elements. Several computers have previously worked on the motion of this system; the following list of orbits is believed to be fairly complete:

P	T	e	a	Ω	i	λ	Authority	Source
^{yrs.} 105.522	1832.611	—	1.287	4.75	70.22	—	Mädler, 1846	
49.048	1860.59	—	1.749	112.7	70.02	78.57	Thiele, 1859	A.N., 1199
95.90	1859.62	0.0768	1.26	12.25	68.7	89.27	Doberck, 1877	A.N., 2121
105.195	1862.32	0.122	1.3093	10.45	67.64	102.63	Schorr 1889	Dissertation, Munich

We find the following elements:

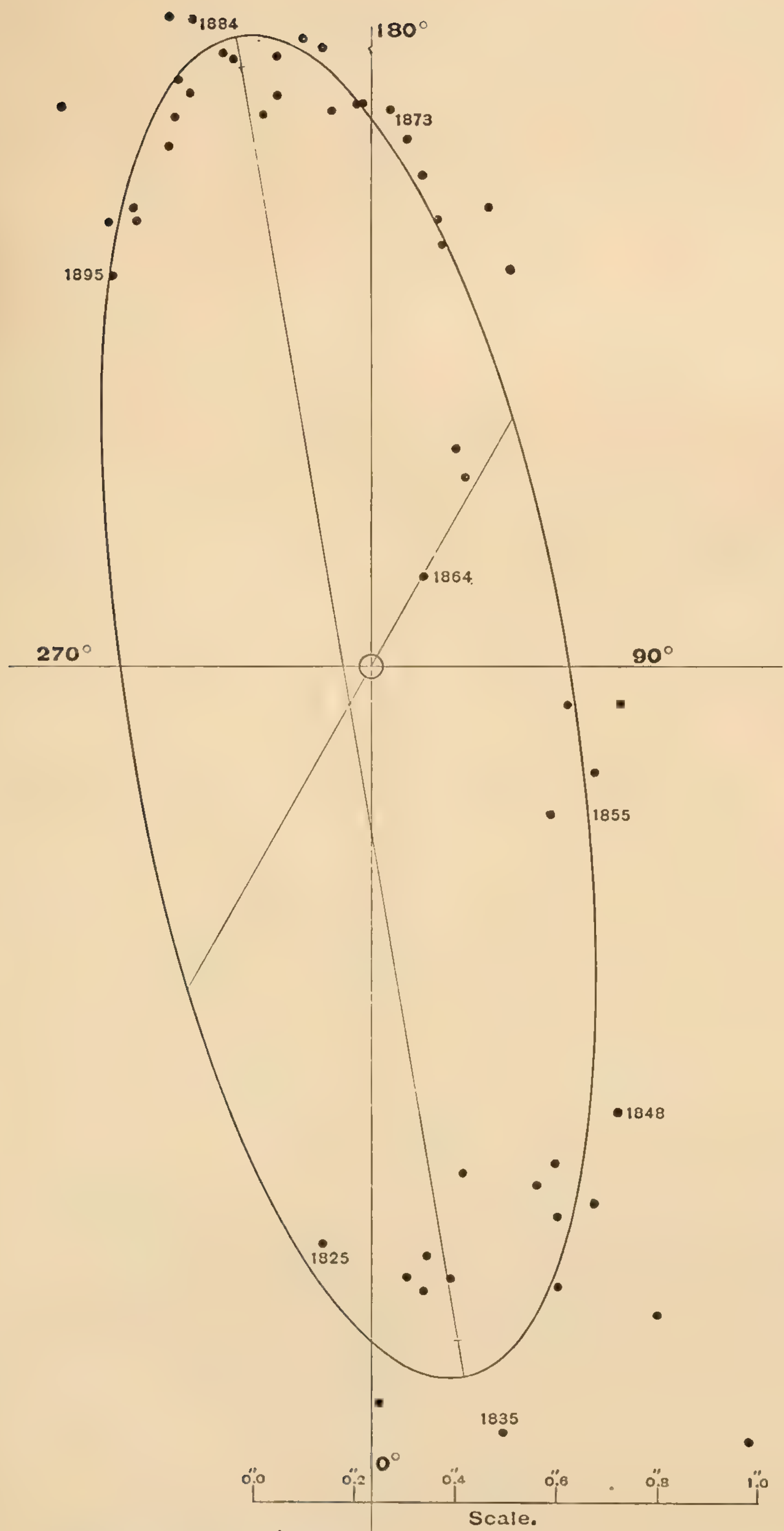
$$\begin{aligned}
 P &= 104.0 \text{ years} & \Omega &= 9^\circ.5 \\
 T &= 1864.60 & i &= 70^\circ.3 \\
 e &= 0.131 & \lambda &= 111^\circ.6 \\
 a &= 1''.3612 & n &= +3^\circ.4616
 \end{aligned}$$

Apparent orbit:

$$\begin{aligned}
 \text{Length of major axis} &= 2''.696 \\
 \text{Length of minor axis} &= 0''.884 \\
 \text{Angle of major axis} &= 9^\circ.6 \\
 \text{Angle of periastron} &= 150^\circ.2 \\
 \text{Distance of star from centre} &= 0''.085
 \end{aligned}$$

The table of computed and observed places shows a very satisfactory agreement, and we may conclude that no very considerable alteration is likely to be made in these elements. But the orbit is so nearly circular and so highly inclined that the definition of λ is not very exact, and in case of this element a larger alteration may be found necessary, when the material shall be sufficient for a definitive determination.

The small eccentricity of this orbit is rather remarkable. Among known binaries there are very few which have such circular orbits, δ *Equulei*, Σ 2173 and μ *Herculis* being the principal objects of this kind, and as most of these orbits are highly inclined, there is still some uncertainty attaching to the eccentricity. It will be necessary to have more exact observations of these stars in



ξ Scorpii AB = Σ 1998.

critical parts of their orbits before this element can be defined with the desired precision.

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1782.36	188.0	197.2	—	—	-9.2	—	1	Herschel
1825.47	355.3	357.2	1.15	1.28	-1.9	-0.13	3	Struve
1830.25	1.4	2.7	1.46	1.39	-1.3	+0.07	4-3	Herschel
1831.48	3.5	4.0	1.21	1.40	-0.5	-0.19	1	Struve
1832.52	4.8	5.2	1.24	1.41	-0.4	-0.17	1	Struve
1833.38	5.6	6.1	1.17	1.41	-0.5	-0.24	2	Struve 1; Dawes 1
1834.47	7.4	7.3	1.22	1.42	+0.1	-0.20	7-6	Herschel 2-1; Σ. 1; Dawes 4
1835.39	10.6	8.3	1.58	1.42	+2.3	+0.16	5-1	Herschel
1836.50	10.3	9.7	1.02	1.40	+0.6	-0.38	4-1	Dawes 1; Mädler 3-0
1839.61	16.7	13.0	1.28	1.35	+3.7	-0.07	2	Dawes
1840.57	17.9	14.0	1.08	1.33	+3.9	-0.25	4	Dawes 3; OΣ. 1
1841.56	18.5	15.7	1.15	1.29	+2.8	-0.14	10-7	Mädler 4-3; OΣ. 1; Dawes 3-2; Kaiser 2-1
1842.42	20.4	16.1	1.05	1.28	+4.3	-0.23	4-2	Mädler 4-2
1843.47	22.7	18.0	1.15	1.22	+4.7	-0.07	19-7	Dawes 2; Mädler 6-4; Kaiser 11-1
1845.36	23.7	20.7	1.40	1.14	+3.0	+0.26	15-12	Mädler 3; Jacob 3-1; Mitchell 9-8
1847.58	26.0	24.7	1.71	1.02	+1.3	+0.69	1	Mitchell
1848.54	28.9	26.6	1.01	0.96	+2.3	+0.05	4	Dawes 3; Mitchell 1
1855.44	50.6	53.5	0.46	0.54	-2.9	-0.08	7-3	Dembowski 4-0; Secchi 3
1856.45	64.7	61.2	0.49	0.50	+3.5	-0.01	20-12	Jacob 3; Dem. 4-0; Sec. 10-8; OΣ. 1; Winn. 2-0
1857.68	81.4	72.8	0.50	0.43	+8.6	+0.07	1	Jacob
1858.13	79.4	78.5	0.40	0.42	+0.9	-0.02	1	Jacob
1864.48	149.3	144.8	0.21	0.52	+4.5	-0.31	14-4	Secchi 4; Dembowski 10-0
1865.50	153.4	154.4	0.42	0.60	-0.1	-0.18	17-14	Dembowski 10-0; Secchi 7; Englemann 7
1866.49	158.8	159.3	0.46	0.66	-0.5	-0.20	10-4	Dembowski 8-3; Secchi 2-1
1867.45	160.7	163.4	0.83	0.72	-2.7	+0.11	7-4	Dembowski
1868.44	165.7	166.9	0.94	0.78	-1.2	+0.16	8-5	Dembowski 7-4; Knott 1
1869.51	170.3	170.2	0.85	0.83	+0.1	+0.02	11	Dunér 6; Dembowski 5
1870.46	171.6	172.6	0.89	0.90	-1.0	-0.01	9-7	Dembowski 7-5; Dunér 2
1871.50	174.0	174.9	0.98	0.96	-0.9	+0.02	13-11	Dembowski 7-5; Gledhill 1; Dunér 5
1872.48	176.2	177.1	1.05	0.02	-0.9	+0.03	15-12	W. & S. 1; Kn. 1; Dem. 8-5; Fer. 2; Du. 3
1873.47	177.9	179.0	1.11	1.06	-1.1	+0.05	7-5	W. & S. 1; Dembowski 5-3; Gledhill 1
1874.46	180.9	180.8	1.12	1.11	+0.1	+0.01	6	W. & S. 1; Dembowski 5
1875.50	181.1	182.4	1.12	1.15	-1.3	-0.03	15	Dembowski 5; Schiaparelli 5; W. & S. 1; Dunér 4
1876.51	184.0	184.0	1.11	1.19	±0.0	-0.08	18-15	Howe 1; Dem. 6; Hl. 3; Sch. 4; Pl. 1; Dk. 3 [Jed. 3
1877.47	184.3	185.4	1.24	1.22	-1.1	+0.02	23-21	Dk. 2-1; Dem. 5; Upton 1; W. & S. 1; Howe 4-1; Sch. 9;
1878.50	186.1	186.8	1.26	1.24	-0.7	+0.02	11-10	Dem. 5-4; Sch. 6 [β. 3; Sea. & S. 3-1
1879.53	188.6	188.2	1.23	1.26	+0.4	-0.03	29-27	Howe 5; Stone 3; Egbert 3; Hl. 3; Sch. 7; Pr. 2;
1880.54	189.3	189.6	1.15	1.27	-0.3	-0.12	15-14	Egbert 2; Dk. 4-2; Frisby 1; Sch. 6; Pr. 3
1881.32	191.0	190.3	1.12	1.28	+0.7	-0.16	3-2	Dobereck 1; Bigourdan 2-1
1882.46	192.5	192.1	1.24	1.28	+0.4	-0.04	12	Dobereck 1; Hall 3; Schiaparelli 5; Frisby 3
1883.50	193.4	193.4	1.26	1.29	±0.0	-0.03	25-24	Frisby 4; Hl. 2; Kü. 1-0; En. 3; Per. 3; Sch. 12
1884.49	195.3	194.7	1.35	1.28	+0.6	+0.07	25-24	H.C.W. 4-3; En. 3; Hl. 5; Per. 3; Pr. 1; Sch. 9
1885.55	197.1	196.0	1.36	1.27	+1.1	+0.09	13	Schiaparelli 8; Englemann 5 [Sch. 3
1886.51	198.0	197.4	1.23	1.25	+0.6	-0.02	21-20	H.C.W. 1; Per. 2; Sm. 2-1; Tar. 3; Hall 3; En. 7;
1887.54	199.6	198.8	1.16	1.23	+0.8	-0.07	9	Schiaparelli
1888.54	201.0	200.3	1.11	1.21	+0.7	-0.10	11	Leavenworth 2; Hall 2; Schiaparelli 7
1889.43	197.5	201.6	1.20	1.18	-4.1	+0.02	2	Hodges
1890.39	205.2	203.1	—	1.15	+2.1	—	2	Glasenapp
1891.47	208.7	204.9	1.27	1.11	+3.8	+0.16	1-2	Collins 0-2; See 1-0
1892.55	207.3	206.8	1.02	1.07	+0.5	-0.05	7	Maw 3; Comstock 4
1893.51	210.3	208.7	1.02	1.03	+1.6	-0.01	10	β. 2; Schiaparelli 1; Leavenworth 2; Bigourdan 5
1894.59	207.5	210.9	1.0 ±	0.99	-3.4	+0.01	2-1	Glasenapp 2-1
1895.42	213.3	212.8	0.93	0.95	+0.5	-0.02	4-6	See 1-3; Comstock 3

The following ephemeris will be useful to observers:

	θ_c °	ρ_c "		θ_c °	ρ_c "
1896.50	216.3	0.88	1899.50	225.6	0.74
1897.50	219.3	0.84	1900.50	229.6	0.70
1898.50	222.4	0.79			

The motion will be rather slow for a good many years, but as the object becomes closer, about 1910, it will deserve the most careful attention.

σ CORONAE BOREALIS = $\Sigma 2032$.

$\alpha = 16^h 11^m$; $\delta = +34^\circ 7'$.
6, yellow ; 7, bluish.

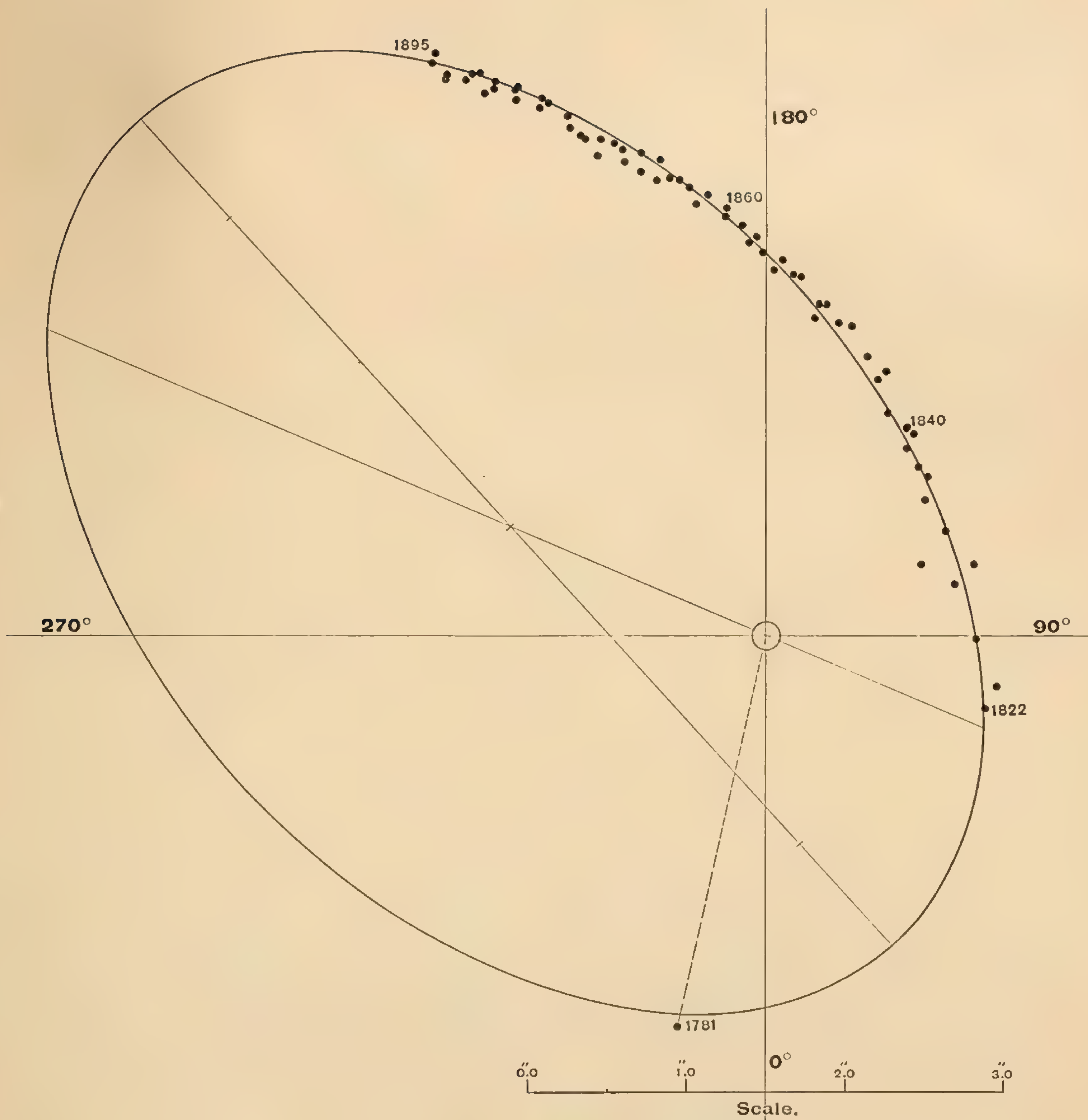
Discovered by Sir William Herschel, August 7, 1780.

OBSERVATIONS.

t	θ_o °	ρ_o "	n	Observers	t	θ_o °	ρ_o "	n	Observers
1781.79	347.5	—	1	Herschel	1836.47	138.5	—	5-0	Mädler
1802.59	348.6?	—	1	Herschel	1836.59	134.7	1.43	6	Struve
1804.74	11.4	—	1	Herschel	1837.47	136.8	—	1	Dawes
1819.62	48.0	—	—	Struve	1837.55	139.9	1.42	5	Struve
1821.30	65.2	—	—	Herschel	1838.45	143.4	1.48	7	Struve
1822.83	71.5	1.44	2-1	H. & So.	1839.52	147.8	1.55	—	Galle
1823.47	72.9	—	—	Herschel	1839.53	144.3	1.60	1	Dawes
1825.44	77.5	1.48	6-3	South	1840.57	147.8	1.66	3	Dawes
1827.02	89.3	1.31	4	Struve	1840.63	149.3	1.54	4	O. Struve
1828.50	92.1	—	6	Herschel	1840.68	145.2	1.53	1	Struve
1830.11	104.9	1.22	3	Struve	1841.48	150.3	1.66	3	Dawes
1830.28	105.1	1.22	9-5	Herschel	1841.56	148.8	1.57	—	Kaiser
1831.36	108.8	1.38	3-2	Herschel	1841.56	152.3	1.60	7	Mädler
1832.52	113.6	1.07	6-1	Herschel	1841.60	153.7	1.56	1	O. Struve
1832.55	115.4	—	3	Dawes	1842.31	156.4	1.81	4	Mädler
1833.26	120.0	1.29	3-2	Herschel	1842.37	153.3	—	1	Dawes
1833.36	120.6	1.30	4	Dawes	1842.73	157.5	1.86	4	Mädler
1834.55	125.6	—	3	Dawes	1843.45	156.8	1.85	6	Mädler
1835.40	134.9	1.3±	4-1	Mädler	1843.47	156.5	1.77	1	Dawes
1835.50	130.5	1.31	5	Struve	1843.68	156.3	1.66	—	Kaiser
					1844.40	160.6	2.05	4	Mädler
					1844.44	157.2	1.53	1	Greenwich
					1845.51	163.1	2.03	20-19	Mädler

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1846.32	162.8	—	—	Hind	1856.39	182.8	2.52	1	Winnecke
1846.36	162.4	2.25	—	Jacob	1856.42	181.9	2.68	6	Dembowski
1846.46	165.1	2.07	11	Mädler	1856.43	182.4	2.45	2	Secchi
1846.68	168.3	1.76	2	O. Struve	1856.57	179.9	2.46	4	O. Struve
1847.44	166.6	2.16	14	Mädler	1856.73	181.2	2.52	3	Jacob
1847.44	166.0	1.88	2	Dawes	1857.39	183.3	2.46	2	Mädler
1847.69	169.6	1.69	1	O. Struve	1857.61	183.6	2.43	2	Secchi
1847.70	166.7	1.33	1	Mitchell	1857.66	183.1	2.53	3	Jacob
1848.41	168.4	2.39	2-1	Mädler	1857.66	180.0	2.52	2	Dembowski
1848.42	171.9	2.2	1	W. C. Bond	1858.01	181.9	2.51	5	O. Struve
1848.53	168.6	1.99	3	Dawes	1858.20	184.0	2.57	3	Jacob
1848.74	170.8	1.91	1	O. Struve	1858.50	184.7	2.69	6	Dembowski
1849.45	170.1	2.09	1	Dawes	1858.54	183.6	2.64	7	Mädler
1849.74	172.3	1.96	3	O. Struve	1859.34	184.9	2.70	20 obs.	Morton
1850.52	168.9	1.99	3	O. Struve	1859.49	185.8	2.69	8-6	Mädler
1850.70	173.0	2.23	2	Mädler	1859.94	186.1	2.62	4	O. Struve
1851.22	174.4	2.32	43 obs.	Fletcher	1860.36	185.5	2.71	2	Dawes
1851.25	174.5	2.34	6	Mädler	1861.55	188.4	2.95	5-3	Mädler
1851.42	173.8	2.26	1	Dawes	1861.58	187.4	2.69	5	O. Struve
1851.63	173.4	2.06	6	O. Struve	1862.71	190.5	3.01	6	Mädler
1851.76	176.2	2.43	9	Mädler	1862.76	189.1	2.77	2	O. Struve
1852.31	176.4	2.38	24-38 obs.	Miller	1862.79	189.3	2.87	1	Scheumann
1852.60	177.5	2.39	12-11	Mädler	1863.09	190.1	2.76	14	Dembowski
1852.63	173.3	2.06	4	O. Struve	1863.60	188.2	2.77	4	O. Struve
1853.14	177.9	2.18	2	Jacob	1864.45	190.5	3.09	2	Englemann
1853.38	177.7	2.46	6	Mädler	1864.95	191.2	2.79	12	Dembowski
1853.63	177.9	2.39	4-3	Dawes	1865.36	191.9	2.94	3	O. Struve
1853.64	—	2.56	1	Argelander	1865.38	191.5	3.08	1	Dawes
1853.64	—	2.47	1	W. Struve	1865.64	194.3	2.93	4	Englemann
1853.66	175.6	2.17	4	O. Struve	1865.72	189.1	—	1	Leyton Obs.
1853.77	178.7	2.65	2	Mädler	1865.74	192.5	30.3	1	v. Fuss
1854.05	177.9	2.25	3	Jacob	1865.81	192.3	2.98	4	Secchi
1854.56	178.5	2.26	3	Dawes	1866.31	190.5	2.82	1	Englemann
1854.66	179.0	2.24	2	O. Struve	1866.42	189.2	3.73	2	Leyton Obs.
1854.67	178.6	2.22	20 obs.	Morton	1866.49	189.6	—	2	Wagner
1854.67	179.8	2.36	5	Dembowski	1866.49	191.3	—	2	Gylden
1854.70	179.4	2.51	5	Mädler	1866.49	192.3	—	2	Smysloff
1855.19	179.9	2.39	3	Dembowski	1866.49	193.2	—	2	Kortazzi
1855.48	180.1	2.43	1	Dawes	1866.55	194.6	3.23	3	Winlock
1855.54	181.6	2.49	6-5	Winnecke	1866.59	193.5	3.22	3-2	Searle
1855.61	180.8	2.31	4	Secchi	1866.63	193.0	3.00	6	O. Struve
1855.61	179.1	2.29	4	O. Struve	1866.68	193.9	2.86	—	Kaiser
1855.78	181.8	2.64	2-1	Mädler	1866.92	193.2	2.88	11	Dembowski

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1867.30	190.2	3.15	1	Searle	1877.46	202.2	3.68	—	W. & S.
1867.31	195.0	2.95	1	Winlock	1877.49	200.1	3.49	7	Schiaparelli
1867.34	194.7	3.0	—	Knott	1877.53	201.6	3.61	5	Jedrzejewicz
1867.37	192.1	3.0	1	Main	1877.58	200.1	3.50	3	O. Struve
1867.72	195.5	2.79	1	Dunér	1878.39	202.3	3.51	2-1	Burnham
1868.29	193.8	3.62	1	Leyton Obs.	1878.50	202.0	3.51	5	Dembowski
1868.58	194.7	2.98	2	O. Struve	1878.51	201.1	3.39	3-2	Doberck
1868.60	194.7	3.14	4	Dunér	1878.53	201.2	3.53	6	Schiaparelli
1868.61	195.5	—	2	Zöllner	1878.57	199.1	3.52	3	O. Struve
1868.88	195.3	2.99	9	Dembowski	1879.45	202.5	3.66	4	Hall
1869.57	195.2	3.60	1	Leyton Obs.	1879.54	202.1	3.68	6	Schiaparelli
1869.63	195.1	3.05	5	Dunér	1880.39	203.0	3.61	1	Burnham
1870.56	196.6	3.18	1	Dunér	1880.55	203.4	3.71	9	Schiaparelli
1870.97	196.8	3.10	12	Dembowski	1881.05	200.6	3.94	5	Hough
1871.41	197.9	3.23	2-3	C. S. Peirce	1881.46	203.0	3.64	3	Hall
1871.42	196.7	3.30	—	Leyton Obs.	1881.70	204.3	3.56	6	Seabroke
1871.54	195.4	3.23	—	Knott	1882.43	202.6	3.75	4	Hall
1871.61	196.5	3.14	3	Dunér	1882.51	203.8	3.79	6	Schiaparelli
1872.29	198.0	3.34	—	Leyton Obs.	1882.52	204.1	3.90	3	O. Struve
1872.57	195.3	3.26	3	O. Struve	1882.65	204.9	—	1	Seabroke
1872.96	198.1	3.20	12	Dembowski	1882.71	205.7	3.92	4	Jedrzejewicz
1873.42	198.4	3.14	—	W. & S.	1883.26	205.4	3.77	6	Englemann
1873.55	200.6	3.64	1	Leyton Obs.	1883.47	204.5	3.77	3	Hall
1873.56	197.6	3.14	2	O. Struve	1883.49	203.2	3.79	4	Perrotin
1873.68	198.9	3.4	—	Gledhill	1883.56	204.6	3.74	12	Schiaparelli
1873.54	197.3	—	1	Müller	1883.63	206.0	3.99	2	Jedrzejewicz
1873.54	201.6	—	1	H. Bruns	1884.48	206.0	3.80	3	Hall
1873.57	199.6	—	1	H. Struve	1884.53	205.8	3.86	3	Perrotin
1874.44	200.5	3.55	1	Main	1884.53	202.4	3.63	2	O. Struve
1874.46	199.2	2.67	2	Leyton Obs.	1884.54	205.4	3.76	11	Schiaparelli
1874.61	199.8	3.41	4	O. Struve	1885.43	205.4	3.88	4	deBall
1874.90	199.1	3.28	11	Dembowski	1885.43	205.7	3.89	3	Hall
1875.42	199.8	2.56	1	Leyton Obs.	1885.54	204.9	3.94	2	Perrotin
1875.46	198.6	3.34	4	Schiaparelli	1885.55	205.8	3.86	9	Schiaparelli
1875.50	200.6	3.47	—	W. & S.	1885.66	206.8	3.93	3	Jedrzejewicz
1875.54	199.6	3.28	5	Dunér	1885.74	207.3	4.09	6	Englemann
1875.65	200.6	3.74	—	Nobile	1886.47	205.6	3.99	5	Perrotin
1876.29	199.3	—	—	Doberck	1886.48	206.9	3.96	6	Hall
1876.45	200.0	3.50	3	Hall	1886.49	208.0	4.01	4	Tarrant
1876.48	200.6	3.28	—	Gledhill	1887.44	205.5	3.99	4	Hall
1876.61	196.3	3.34	3	O. Struve	1887.53	207.1	3.78	7	Schiaparelli
1876.61	200.7	3.45	1	Leyton Obs.	1888.44	206.6	3.92	4	Hall
1877.03	201.0	3.40	11	Dembowski	1888.57	207.4	3.92	8-7	Schiaparelli
1877.33	199.6	3.58	—	Doberck	1888.62	207.8	3.82	3	Maw



σ Coronae Borealis = Σ 2032.

t	θ_0 °	ρ_0 "	n	Observers	t	θ_0 °	ρ_0 "	n	Observers
1889.14	207.7	4.08	2	O. Struve	1893.55	209.3	4.28	4	Bigourdan
1889.52	208.3	4.05	2	Glasenapp	1893.64	209.8	4.24	2	Maw
1889.52	208.8	3.94	1	Schiaparelli	1894.56	209.8	4.09	2	Glasenapp
1890.33	207.8	4.08	3	Burnham	1895.49	210.8	4.28	3	Comstock
1890.69	207.3	4.00	1	Bigourdan	1895.54	210.7	4.16	10	Schiaparelli
1891.49	208.5	—	1	Schiaparelli	1895.59	210.3	4.23	2	Collins
1892.61	209.9	4.06	3	Comstock	1895.59	209.9	4.25	4	Schwarzschild
1892.64	209.4	4.05	2	Schiaparelli	1895.72	208.9	4.26	3	See
1892.64	209.3	4.21	1	Bigourdan					

Since HERSCHEL'S discovery of this star the companion has described an arc* of 223° . The shape of this arc is such that it fixes the apparent ellipse with considerable precision, and enables us to obtain a set of elements which will never be radically changed. It is singular, however, that the periods heretofore obtained for this star are very discordant, and in several instances more than double that found below. Such extraordinary divergence of results may be explained by the lack of sufficient curvature in the arc swept over by the companion at the time the earlier elements were derived, and by the use of injudicious methods in the determination of the orbit.

In this as in most other cases the graphical method based on both angles and distances is superior to analytical methods, and at once enables us to trace the apparent ellipse with the necessary precision. The following table gives a complete summary of the elements found by previous computers who have worked on the motion of this interesting binary.

P	T	e	a	Ω	i	λ	Authority	Source
yr.			"	°	°	°		
286.60	1835.60	0.6112	3.68	138.0	41.25	7.3	Herschel, 1833	Mem. R.A.S., V, p. 205
608.45	1826.60	0.6998	3.92	25.12	29.48	64.63	Mädler	Dorp. Obs., IX, p. 182
478.04	1829.44	0.6406	3.90	0.5	38.93	96.73	Mädler, 1847	Fixt.-Syst., I, p. 240
736.88	1826.48	0.7256	5.194	21.05	25.65	69.4	Hind, 1845	A.N., 551
195.12	1831.17	0.3088	2.72	1.95	46.78	101.95	Jacob, 1855	M.N., XV, p. 180
240.0	1829.7	0.3887	2.94	3.13	45.1	96.88	Powell, 1855	M.N., XV, p. 91
420.24	1825.32	0.5899	2.385	20.73	40.87	65.9	Klinkerfues, 1856	A.N., 990
843.2	1828.91	0.7502	6.001	6.72	29.67	89.3	Doberck, 1875	A.N., 2037
845.86	1826.93	0.7515	5.885	18.35	31.37	71.6	Doberck, 1876	A.N., 2103

Making use of all the observations up to 1895 we find the following elements :

$$\begin{aligned}
 P &= 370.0 \text{ years} & \Omega &= 30^\circ.5 \\
 T &= 1821.80 & i &= 47^\circ.48 \\
 e &= 0.540 & \lambda &= 47^\circ.7 \\
 a &= 3''.8187 & n &= +9^\circ.7297
 \end{aligned}$$

* *Astronomische Nachrichten*, 3339.

Apparent orbit:

Length of major axis	= 7".08
Length of minor axis	= 4".71
Angle of major axis	= 42°.4
Angle of periastron	= 66°.9
Distance of star from centre	= 1".735

There is of course some uncertainty attaching to a period of such great length, but careful consideration of all possible variations of the apparent ellipse convinces me that the value given above is not likely to be varied by more than 25 years, and a change of twice this amount is apparently impossible. The eccentricity is very well determined, and a change of ± 0.04 in the above value is not to be expected.

The distance of the components of σ *Coronae Borealis* is now so great that the companion will move very slowly for the next two centuries. Therefore, so far as the orbit is concerned observations of the pair will be of small value, as very little improvement can be effected for a great many years; but it may still be worth while to secure careful measures of the system, with a view of establishing the regularity of the elliptical motion, and the absence of sensible disturbing influences. There are no irregularities in the measures heretofore secured which are not attributable to errors of observation. The table of computed and observed places shows an agreement which is extremely satisfactory.

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1781.79	347.5	348.5	—	2.44	— 1.0	—	1	Herschel
1804.74	11.4	23.9	—	2.08	—12.5	—	1	Herschel
1819.62	48.0	59.1	—	1.57	—11.1	—	—	Struve
1821.30	65.2	65.2	—	1.50	0.0	—	—	Herschel
1822.83	71.5	71.0	1.44	1.45	+ 0.5	—0.01	2-1	Herschel and South
1823.47	72.9	73.3	—	1.43	— 0.4	—	—	Herschel
1825.44	77.5	81.6	1.48	1.36	— 4.1	+0.12	6-3	South
1827.02	89.3	88.6	1.31	1.33	+ 0.7	—0.02	4	Struve
1828.50	92.1	95.3	—	1.31	— 3.2	—	6	Herschel
1830.20	105.0	103.8	1.22	1.30	+ 1.2	—0.08	12-8	Struve 3; Herschel 9-5
1831.36	108.8	109.1	1.38	1.30	— 0.3	+0.08	3-2	Herschel
1832.54	114.5	111.7	1.07	1.30	+ 2.8	—0.23	9-1	Herschel 6-1; Dawes 3-0
1833.31	120.3	118.7	1.30	1.31	+ 1.6	—0.01	7-6	Herschel 3-2; Dawes 4
1834.55	125.6	124.3	—	1.34	+ 1.3	—	3	Dawes
1835.50	130.5	128.5	1.31	1.36	+ 2.0	—0.05	5	Struve
1836.59	134.7	133.5	1.43	1.40	+ 1.2	+0.03	6	Struve
1837.51	138.3	137.0	1.42	1.43	+ 1.3	—0.01	6-5	Dawes 1-0; Struve 5
1838.45	143.4	140.7	1.48	1.47	+ 2.7	+0.01	7	Struve
1839.52	146.0	144.5	1.57	1.51	+ 1.5	+0.06	2+	Galle —; Dawes 1
1840.63	147.4	148.3	1.58	1.56	— 0.9	+0.02	8	Dawes 3; $O\Sigma$ 4; Struve 1
1841.55	151.3	151.5	1.60	1.61	— 0.2	—0.01	12+	Dawes 3; Kaiser —; Mädler 7; $O\Sigma$ 1
1842.47	155.7	154.1	1.83	1.66	+ 1.6	+0.17	9-8	Mädler 4; Dawes 1-0; Mädler 4

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1843.53	156.5	157.2	1.76	1.72	- 0.7	+0.04	8+	Dawes 1; Mädler 6; Kaiser —
1844.45	160.3	159.9	1.87	1.78	+ 0.4	+0.09	6+	Mädler 4; Greenwich 1; Mädler —
1846.45	164.6	164.7	2.02	1.90	- 0.1	+0.12	15+	Hind —; Jacob —; Mädler 11; $O\Sigma$. 2
1847.57	167.2	167.3	2.02	1.96	- 0.1	+0.06	18-16	Mädler 14; Dawes 2; $O\Sigma$. 1; Mitchell 1
1848.52	169.9	169.0	2.12	2.01	+ 0.9	+0.11	7-6	Mädler 2-1; Bond 1; Dawes 3; $O\Sigma$. 1
1849.60	171.2	170.5	2.03	2.05	+ 0.7	-0.02	4	Dawes 1; $O\Sigma$. 3
1850.61	171.0	173.0	2.11	2.14	- 2.0	-0.03	5	$O\Sigma$. 3; Mädler 2
1851.46	174.5	174.5	2.28	2.19	0.0	+0.09	24+	Flt. 43 obs.; Mä. 6; Da. 1; $O\Sigma$. 6; Mä. 9
1852.51	175.7	176.2	2.28	2.26	- 0.5	+0.02	18+	Miller 24-38 obs.; Mädler 11; $O\Sigma$. 4
1853.52	177.6	177.7	2.37	2.31	- 0.1	+0.06	18-17	Jacob 2; Mä. 6; Dawes 4-3; $O\Sigma$. 4; Mä. 2
1854.55	178.9	179.2	2.31	2.37	- 0.3	-0.06	20+	Ja. 3; Da. 3; $O\Sigma$. 2; Mo. 20 obs.; Mä. 5; Dem. 5
1855.53	180.5	180.7	2.42	2.43	- 0.2	-0.01	20-18	Dem. 3; Da. 1; Winn. 6-5; Sec. 4; $O\Sigma$. 4; Mä. 2-1
1856.50	181.6	182.0	2.52	2.49	- 0.4	+0.03	16	Winn. 1; Dem. 6; Sec. 2; $O\Sigma$. 4; Ja. 3
1857.58	182.5	183.3	2.49	2.55	- 0.8	-0.06	9	Mädler 2; Secchi 2; Jacob 3; Dembowski 2
1858.31	183.5	184.2	2.60	2.60	- 0.7	0.00	21	$O\Sigma$. 5; Jacob 3; Dembowski 6; Mädler 7
1859.59	185.6	185.9	2.67	2.68	- 0.3	-0.01	14-12+	Mo. 20 obs.; Mädler 8-6; $O\Sigma$. 4
1860.36	185.5	186.6	2.71	2.71	- 1.1	0.00	2	Dawes
1861.57	187.7	187.7	2.82	2.77	0.0	+0.05	10-8	Mädler 5-3; $O\Sigma$. 5
1862.73	189.8	189.0	2.89	2.84	+ 0.8	+0.05	8	Mädler 6; $O\Sigma$. 2
1863.34	189.2	189.7	2.77	2.89	- 0.5	-0.12	18	Dembowski 14; $O\Sigma$. 4
1864.70	190.8	190.9	2.94	2.95	- 0.1	-0.01	14	Englemann 2; Dembowski 12
1865.72	191.8	191.9	2.98	3.01	- 0.1	-0.03	13	$O\Sigma$. 3; Da. 1; En. 4; Ley. 1; Sec. 4; Dem. 4
1866.59	192.6	192.7	3.10	3.05	- 0.1	+0.05	26-25	En. 1; Ley. 2; Wk. 3; Sr. 3-2; $O\Sigma$. 6; Ka. —
1867.41	193.5	193.4	2.98	3.09	+ 0.1	-0.11	5+	Sr. 1; Wk. 1; Kn. —; Ma. 1; Dunér 1
1868.59	194.6	194.3	3.18	3.14	+ 0.3	+0.04	16	Ley. 1; $O\Sigma$. 2; Dunér 4; Dembowski 9
1869.60	195.2	195.1	3.05	3.19	+ 0.1	-0.14	6-5	Ley. 1-0; Dunér 5
1870.77	196.7	196.1	3.14	3.26	+ 0.6	-0.12	13	Dunér 1; Dembowski 12
1871.49	196.6	196.8	3.22	3.30	- 0.2	-0.08	7+	Pierce 2-3; Ley. —; Knott —; Dunér 3
1872.61	197.1	197.4	3.27	3.34	- 0.3	-0.07	16+	Ley. —; $O\Sigma$. 3; Dembowski 12
1873.55	198.3	198.0	3.33	3.38	+ 0.3	-0.05	5+	Ley. 1; $O\Sigma$. 2; Gledhill —
1874.60	199.4	198.9	3.23	3.44	+ 0.5	-0.21	17-18	Main 0-1; Ley. 2; $O\Sigma$. 4; Dembowski 11
1875.51	200.0	199.4	3.36	3.47	+ 0.6	-0.11	12-10+	Ley. 1-0; Sch. 4; W. & S. —; Du. 5; Nobile —
1876.47	200.2	200.1	3.39	3.52	+ 0.1	-0.13	9+	Dk. —; Hall 3; Gl. —; $O\Sigma$. 3; Ley. 1
1877.40	200.8	200.6	3.54	3.55	+ 0.2	-0.01	28+	Dem. 11; Dob. —; W. & S. —; Sch. 7; Jed. 5; $O\Sigma$. 3
1878.50	201.1	201.3	3.47	3.60	- 0.2	-0.13	19-17	β . 2-1; Dem 5; Dk. 3-2; Sch. 6; $O\Sigma$. 3
1879.49	202.3	201.9	3.67	3.64	+ 0.4	+0.03	10	Hall 4; Schiaparelli 6
1880.47	203.2	202.6	3.66	3.69	+ 0.6	-0.03	10	β . 1; Schiaparelli 9
1881.40	202.6	203.1	3.71	3.73	- 0.5	-0.02	14	Hough 5; Hall 3; Seabroke 6
1882.56	204.2	203.7	3.84	3.77	+ 0.5	+0.07	18-17	Hall 4; Sch. 6; $O\Sigma$. 3; Sea. 1-0 Jed. 4
1883.48	204.7	204.3	3.83	3.80	+ 0.4	+0.03	27	En. 6; Hall 3; Per. 4; Sch. 12; Jed. 2
1884.52	204.9	204.8	3.76	3.84	+ 0.1	-0.08	19	Hall 3; Perrotin 3; $O\Sigma$. 2; Schiaparelli 11
1885.56	206.0	205.4	3.93	3.89	+ 0.6	+0.04	27	de Ball 4; Hl. 3; Per. 2; Sch. 9; Jed. 3; En. 6
1886.48	206.8	205.9	4.02	3.92	+ 0.9	+0.10	15	Perrotin 5; Hall 6; Tarrant 4
1887.48	206.3	206.5	3.89	3.97	- 0.2	-0.08	11	Hall 4; Schiaparelli 7
1888.54	207.3	206.9	3.89	4.00	+ 0.4	-0.19	15-14	Hall 4; Schiaparelli 8-7; Maw 3
1889.39	208.3	207.4	4.02	4.03	+ 0.9	-0.01	5	$O\Sigma$. 2; Glasenapp 2; Schiaparelli 1
1890.51	207.5	208.0	4.04	4.07	- 0.5	-0.03	4	β . 3; Bigourdan 1
1891.49	208.5	208.5	—	4.11	0.0	—	1	Schiaparelli
1892.63	209.5	208.9	4.11	4.14	+ 0.6	-0.03	6	Comstock 3; Schiaparelli 2; Bigourdan 1
1893.60	209.6	209.4	4.26	4.17	+ 0.2	+0.09	6	Bigourdan 4; Maw 2
1894.56	209.8	209.8	4.09	4.19	0.0	-0.10	2	Glasenapp
1895.58	210.2	210.3	4.23	4.23	- 0.1	0.00	18	Comstock 3; Schiaparelli 10; Collins 2; See 3

ζ HERCULIS = Σ 2084.

$\alpha = 16^h 37^m.6$; $\delta = +31^\circ 47'$.
3, yellow ; 6, bluish.

Discovered by Sir William Herschel, July 18, 1782.

OBSERVATIONS.

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1783.55	69.3	—	—	Herschel	1847.45	104.4	1.23	18-17	Mädler
1826.63	23.4	0.91	5	Struve	1847.53	108.0	1.63	1	Dawes
1828.77	simplex	—	1	Struve	1847.68	111.3	1.42	2	O. Struve
1829.67	simplex	—	2	Struve	1848.40	98.8	1.08	3	Mädler
1831.65	simplex	—	1	Struve	1848.61	102.4	1.51	3	Dawes
1832.75	220.5	0.81	1	Struve	1848.76	104.2	1.53	2	O. Struve
1834.45	203.5	0.91	2	Struve	1849.48	99.2	1.71	1	Dawes
1835.45	196.9	1.09	5	Struve	1850.00	96.9	1.50	3	O. Struve
1836.57	188.0	—	3	Mädler	1850.54	91.7	1.4 $\pm$	2	Fletcher
1836.60	186.2	1.09	5	Struve	1850.55	91.3	1.27	3-1	Mädler
1838.70	168.5	1.35	3 $\pm$	Galle	1851.23	84.9	1.29	3	Mädler
1839.67	159.7	1.15	1	W. Struve	1851.51	89.3	1.3 $\pm$	6	Fletcher
1839.76	161.9	1.22	4	Dawes	1851.62	88.4	1.47	5	O. Struve
1840.58	161.7	1.49	1	W. Struve	1851.65	89.1	—	2	Miller
1840.66	157.1	1.25	5	O. Struve	1852.63	84.2	1.52	5	O. Struve
1840.66	161.9	1.22	4	Dawes	1852.63	82.8	1.21	8-7	Mädler
1841.44	149.3	1.12	9-8	Mädler	1852.64	84.0	1.24	5-2	Fletcher
1841.60	147.0	1.23	3	O. Struve	1852.77	84.1	—	2	Miller
1841.65	143.0	1.24	4-3	Dawes	1853.15	81.2	1.58	2	Jacob
1842.40	141.6	0.92	3	Mädler	1853.33	78.6	1.40	6-3	Miller
1842.58	138.5	1.07	3-1	Dawes	1853.39	77.3	1.23	8	Mädler
1842.64	146.0	1.21	3	O. Struve	1853.59	80.0	1.48	4	O. Struve
1843.60	130.5	0.90	8-7	Mädler	1853.83	74.7	1.19	3	Mädler
1843.64	129.9	1.30	3-2	Dawes	1854.06	78.0	1.52	3	Jacob
1843.71	130.0	0.94	9-8	Mädler	1854.66	76.8	1.56	3	O. Struve
1844.29	124.0	1.05	5-4	Mädler	1854.67	72.3	1.33	5	Mädler
1844.71	125.4	1.12	2	O. Struve	1855.05	69.6	1 $\pm$	13	Dembowski
1845.43	119.4	1.01	11	Mädler	1855.41	68.0	1.56	4-2	Winnecke
1845.64	121.3	1.24	3	O. Struve	1855.53	69.7	1.41	3	Secchi
1846.54	111.5	1.18	16	Mädler	1855.62	70.8	1.55	4	O. Struve
1846.69	110.5	1.31	2	O. Struve	1855.66	73.3	1.45	4	Morton
1846.89	112.2	—	5	Dawes	1856.25	66.2	1.60	3	Jacob
					1856.43	62.6	1.43	6-3	Winnecke
					1856.52	64.1	1.2	15	Dembowski
					1856.52	64.1	1.41	6	Secchi
					1856.62	64.7	1.49	3	O. Struve

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1857.38	60.0	1.07	4	Mädler	1870.49	190.3	1.10	11	Dembowski
1857.46	60.4	1.60	2	Morton	1870.59	193.6	1.21	6	Dunér
1857.59	59.5	1.29	6	Secchi	1871.42	181.1	1.27	14	Dembowski
1857.63	58.4	1.49	4	O. Struve	1871.52	179.6	1.34	1	O. Struve
1857.75	58.9	1.2	5	Dembowski	1871.54	183.3	1.02	5	Knott
1857.87	57.0	1.46	3	Jacob	1871.60	183.7	1.19	12	Dunér
1858.48	54.6	1.06	2	Secchi	1872.48	173.9	1.34	12	Dembowski
1858.56	49.9	1.0	8	Dembowski	1872.58	177.2	1.19	12	Dunér
1858.62	51.0	1.48	4	O. Struve	1872.60	168.8	1.14	3	O. Struve
1858.65	48.6	1.20	8-7	Mädler	1873.50	165.9	—	1	Mädler
1859.49	40.3	1.13	6	Mädler	1873.50	164.7	—	1	Romberg
1859.59	43.8	0.86	3	Secchi	1873.52	169.5	—	2	H. Bruhns
1859.61	45.8	1.34	6-5	Dawes	1873.46	166.7	0.98	3-2	W. & S.
1859.63	42.3	1.29	4	O. Struve	1873.52	162.4	1.39	11	Dembowski
1860.67	31.5	0.72	3	Secchi	1873.52	169.9	1.23	2	O. Struve
1860.74	32.5	1.38	1	O. Struve	1873.54	rotunda	—	1	Ferrari
1861.44	20.0	0.8 ±	2	Mädler	1873.70	166.3	1.40	4	Dunér
1861.57	17.1	1.05	4	O. Struve	1874.53	157.0	1.36	10	Dembowski
1862.54	361.8	cuneo	8	Dembowski	1874.57	155.5	0.78	2	Gledhill
1862.55	unsichtbar	—	1	Winnecke	1874.57	156.4	1.08	2	W. & S.
1862.74	341.	1.00	1	O. Struve	1874.62	162.9	1.40	4	O. Struve
1862.91	50.9	0.82	2	Mädler	1874.65	154.9	1.35	1	Dunér
1863.49	343.0	cuneo	4	Dembowski	1874.66	156.5	1.22	2	Newcomb
1864.43	—	semplice	3	Dembowski	1875.52	149.1	1.41	8	Dembowski
1865.32	—	semplice	2	Dembowski	1875.55	147.2	1.21	7	Schiaparelli
1865.54	rotunda	—	3	Secchi	1875.57	150.3	—	2	W. & S.
1865.55	250.0	<0.5	3	Englemann	1875.61	147.4	1.28	12	Dunér
1866.45	244.7	0.6	5	Dembowski	1876.52	143.1	1.32	2	Hall
1866.60	142.3	—	1	Searle	1876.54	138.1	1.17	7	Schiaparelli
1866.70	235.1	0.86	3	Dawes	1876.56	139.6	1.37	7	Dembowski
1866.74	228.6	0.97	2	O. Struve	1876.61	140.1	1.2 ±	1	Plummer
1866.81	229.2	0.83	2	Dawes	1876.62	148.8	1.24	4	O. Struve
1866.99	225.1	0.98	2	Dawes	1877.53	133.8	1.36	8	Dembowski
1867.52	225.6	0.80	7	Dembowski	1877.58	130.3	1.27	8	Schiaparelli
1867.59	227.6	—	1	Winlock	1877.58	141.2	1.60	1	Pritchett
1867.72	221.4	1.03	2	Dunér	1877.58	135.1	1.16	3	O. Struve
1868.44	210.1	0.94	6	Dembowski	1877.59	134.0	1.24	2	Hall
1868.48	206.1	0.99	4	Knott	1878.41	127.0	1.51	1	Burnham
1868.58	203.6	1.23	2	O. Struve	1878.53	124.0	1.43	4	Schiaparelli
1868.61	199.9	—	1	Zöllner	1878.53	127.0	1.29	2-1	Doberck
1868.67	213.3	1.05	5	Dunér	1878.58	126.7	1.38	7	Dembowski
1869.58	200.6	1.09	8	Dembowski	1878.59	128.7	1.23	3	O. Struve
1869.62	203.1	1.06	11	Dunér	1879.45	122.0	1.52	3	Burnham
1869.74	196.1	—	1	Peirce	1879.46	120.7	1.50	4	Hall
					1879.58	117.2	1.38	8	Schiaparelli
					1879.67	124.9	1.56	1	Pritchett

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1880.41	118.4	1.29	2-1	Doberek	1886.63	85.8	1.45	9	Schiaparelli
1880.48	115.0	—	3	Bigourdan	1886.73	88.0	1.42	9-7	Jedrzejewicz
1880.49	114.1	1.34	5	Burnham	1886.75	89.9	1.78	7	Englemann
1880.58	112.5	1.38	9	Schiaparelli	1887.55	83.6	1.59	6	Hall
1881.23	112.9	1.43	2	Doberek	1887.65	79.4	1.55	18	Schiaparelli
1881.45	110.6	1.53	5	Burnham	1888.51	78.7	1.52	6	Hall
1881.51	109.2	1.41	4	Hough	1888.57	74.3	1.88	3	Comstock
1881.51	110.6	1.43	5	Hall	1888.61	76.5	1.56	9-8	Schiaparelli
1881.64	101.8	1.41	2	O. Struve	1888.65	74.9	1.71	3	Maw
1881.74	108.9	1.47	1	Bigourdan	1888.69	70.9	1.74	1	O. Struve
1882.47	105.0	1.48	2-1	H. Struve	1889.45	77.0	1.00	1	Hodges
1882.47	104.3	1.67	2-1	Doberek	1889.52	72.6	1.67	3	Schiaparelli
1882.52	106.3	1.44	5	Hall	1889.52	76.2	1.2 $\pm$	2-1	Glasenapp
1882.52	98.7	1.49	4	O. Struve	1889.53	72.4	1.49	6	Hall
1882.60	101.5	1.47	11	Schiaparelli	1889.56	72.0	1.67	4	Comstock
1882.66	104.9	1.48	4-3	Jedrzejewicz	1889.66	70.2	1.73	3	Maw
1882.76	107.0	1.75	6	Englemann	1890.42	68.6	1.5	2	Glasenapp
1883.52	99.5	1.50	4	Perrotin	1890.51	68.5	1.49	6	Hall
1883.55	102.4	1.51	5	Hall	1890.70	65.8	1.68	3	Maw
1883.60	96.6	1.52	15	Schiaparelli	1890.77	64.2	1.46	5-4	Schiaparelli
1883.65	96.4	1.38	2	O. Struve	1891.52	64.3	1.35	6	Hall
1883.72	102.5	1.65	5	Englemann	1891.54	60.4	1.45	7-4	See
1884.45	94.9	—	2	Bigourdan	1891.55	63.3	1.50	2	Schiaparelli
1884.52	94.7	1.63	4	Hall	1891.62	62.7	1.45	5	Bigourdan
1884.55	94.1	1.47	3	Perrotin	1891.63	60.1	1.40	3	Maw
1884.55	90.9	1.32	1	Pritchett	1891.64	63.7	1.38	4	Tarrant
1884.58	90.8	1.64	9	Schiaparelli	1892.57	55.5	1.51	5	Comstock
1884.68	88.4	1.57	2	O. Struve	1892.63	56.0	1.37	8	Schiaparelli
1884.70	94.8	1.95	6-2	Seabroke	1893.68	47.6	1.42	3-2	Schiaparelli
1884.71	98.8	1.89	3	Englemann	1893.80	47.6	1.27	5	Bigourdan
1885.47	88.6	1.50	6	Perrotin	1894.51	43.8	1.24	3	Barnard
1885.52	89.4	1.70	4	Tarrant	1894.52	42.1	0.85	2	Glasenapp
1885.52	92.0	1.61	7	Hall	1894.54	40.4	1.23	2	Lewis
1885.62	86.3	1.57	5	Schiaparelli	1894.73	39.6	1.28	9-8	Collandreau
1885.64	92.1	1.59	4	Jedrzejewicz	1894.74	37.4	1.12	16-14	Bigourdan
1885.71	98.0	1.82	6-5	Englemann	1895.32	36.7	1.17	3	See
1885.69	90.5	—	3	Seabroke	1895.57	30.2	1.00	4	Comstock
1886.54	88.8	1.50	6	Hall					
1886.55	84.5	1.56	1	Perrotin					
1886.58	85.0	—	3	Seabroke					

SIR WILLIAM HERSCHEL made his first measure of this star, July 21, 1782, and found the position-angle to be $69^\circ.3$.*

In 1795 he again examined the object, and noted that the distance had

* *Astronomical Journal*, 357.

decreased, but that it was in the same quadrant as before; this appears, however, to be a mistake, as the companion at that time must have been in the opposite quadrant. It is remarkable that HERSCHEL could not separate the companion in 1802, as the angle was then $174^{\circ}.5$, and the distance $1''.24$.

Beginning with STRUVE's observation in 1826 the record is practically continuous, and we have measures for each year, except when the companion was so close as to be lost in the rays of the larger star.

The periastron is so near the central star, on account of the considerable eccentricity and the position of the node, that the companion has never been seen in this part of the orbit. According to the elements found below, the minimum distance is about $0''.45$. Therefore, in spite of the comparative faintness of the companion, whose magnitude is 6.5, while that of the central star is 3.0, this object ought to be constantly within the reach of our great refractors. In previous revolutions, however, the star has been lost, and it will therefore be a matter of great interest to follow it during the next periastron passage in 1899. Good observations in this part of the orbit are needed, and the rare phenomenon which will be presented by ζ *Herculis* about the end of this century will be worthy of the attention of observers with large telescopes.

Notwithstanding the three revolutions which have been completed since HERSCHEL's discovery in 1782, our knowledge of the orbit of this pair has remained somewhat unsatisfactory; the elements heretofore obtained are by no means accordant. This divergency of results may be attributed partly to errors of observation incident to the inequality of the components, and partly to a sensible mistake in the old position-angle of HERSCHEL, which ought to have been about 80° . Indeed, HERSCHEL's observation does not seem to lay claim to much accuracy, for on August 30, 1782, he says: "Saw it better than I ever did,"—implying that on the previous occasions the companion was not very well defined. The following table gives the elements published by previous investigators :

<i>P</i>	<i>T</i>	<i>e</i>	<i>a</i>	Ω	<i>i</i>	λ	Authority	Source
^{YRS.} 31.4678	1829.50	0.4545	1.189	[°] 39.43	[°] 50.9	[°] 262.1	Mädler, 1842	Dorp. Obs., IX, p. 192
30.216	1830.42	0.432	1.208	19.4	44.1	276.65	Mädler, 1847	Fixt. Syst., I, p. 269
36.357	1830.481	0.4482	—	214.35	43.7	284.9	Villardeau	A.N., vol. XXVI, p. 305
37.21	1830.56	0.4381	—	37.23	39.35	266.9	Fletcher, 1853	M.N., XIII, p. 258
36.715	1830.237	0.4831	1.350	41.9	49.1	290.6	Villardeau, 1854	C.R., XXXVIII, p. 871
34.221	1830.01	0.4239	1.223	45.93	34.87	209.5	Dunér, 1871	A.N., 1868
36.606	1829.635	0.5511	1.374	27.0	50.23	266.7	Plummer, 1871	M.N., XXXI, 195
34.58	1864.90	0.405	1.36	26.13	51.11	260.97	Flammarion, '74	Catal. Ét. Doub., p. 101
34.4	1864.8	0.463	1.284	41.73	43.23	252.75	Dobereck, 1880	A.N., 2332
34.411	1864.78	0.4666	1.345	44.1	44.53	251.8	Dobereck	

After an examination of all the observations we formed mean positions for each year, and from these mean places deduced the following elements:

$$\begin{array}{ll} P = 35.00 \text{ years} & \Omega = 37^\circ.5 \\ T = 1864.80 & i = 51^\circ.77 \\ e = 0.497 & \lambda = 101^\circ.7 \\ a = 1''.4321 & n = -10^\circ.2843 \end{array}$$

Apparent orbit:

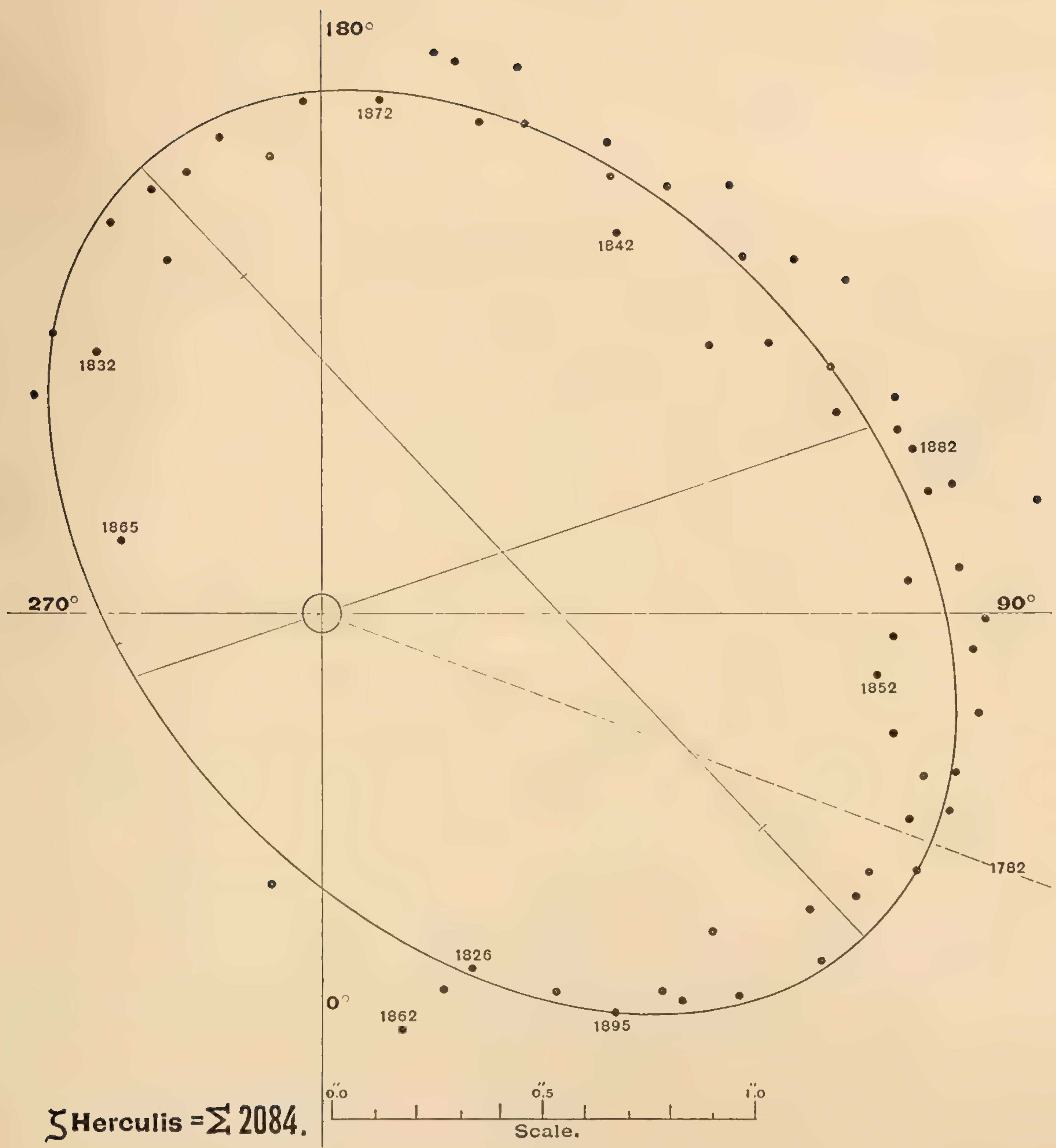
$$\begin{array}{ll} \text{Length of major axis} & = 2''.492 \\ \text{Length of minor axis} & = 1''.752 \\ \text{Angle of major axis} & = 43^\circ.1 \\ \text{Angle of periastron} & = 289^\circ.0 \\ \text{Distance of star from center} & = 0''.455 \end{array}$$

The following table of computed and observed places shows that the elements give a good representation of the observations, and render it probable that the present orbit is very near the truth. There are some errors in the position-angles which appear to be systematic, and we have not been able to improve the representation; for whatever would improve the agreement in one place would injure it in another, or in the same place during the next revolution.

It will be seen that this orbit is slightly more eccentric than most of those heretofore deduced, but it is not probable that the eccentricity will prove to be too large. If any change should be required in this element, it is likely to increase rather than diminish the value given above. The eccentricity of the orbit of ζ *Herculis* is near the mean value of this element among double stars.

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1782.55	69.3	80.5	—	1.51	-11.2	—	1	Herschel
1795.80	—	248.9	—	0.65	—	—	1	Herschel
1802.74	—	174.5	—	1.24	—	—	1	Herschel
1826.63	23.4	27.1	0.91	1.00	- 3.7	-0.09	5	Struve
1828.71	349.5	344.0	0.65	0.54	+ 5.5	+0.11	1	Struve
1832.72	220.5	216.0	0.81	0.97	+ 4.5	-0.16	1	Struve
1834.45	203.5	201.5	0.91	1.14	+ 2.0	-0.23	2	Struve
1835.45	196.9	191.6	1.09	1.20	+ 5.3	-0.11	5	Struve
1836.60	186.2	182.8	1.09	1.23	+ 3.4	-0.14	5	Struve
1838.70	168.5	167.5	1.35	1.24	+ 1.0	+0.11	3±	Galle
1839.76	161.9	159.9	1.22	1.25	+ 2.0	-0.03	4	Dawes
1840.66	157.1	153.7	1.25	1.25	+ 3.4	±0.00	5	O. Struve
1841.56	146.4	147.2	1.24	1.25	- 0.8	-0.01	16-6	Mädler 9-0; $O\Sigma$ 3; Dawes 4-3
1842.54	142.0	140.3	1.14	1.26	+ 1.7	-0.12	9-4	Mädler 3-0; Dawes 3-1; $O\Sigma$ 3
1843.65	130.1	132.1	1.30	1.28	- 2.0	+0.02	20-2	Mädler 8-0; Dawes 3-2; Mädler 9-0
1844.50	124.7	127.1	1.12	1.30	- 2.4	-0.18	7-2	Mädler 5-0; $O\Sigma$ 2
1845.64	121.3	119.3	1.24	1.32	+ 2.0	-0.08	3	O. Struve
1846.79	111.3	112.1	1.31	1.35	- 0.8	-0.04	7-2	$O\Sigma$ 2; Dawes 5-0



t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1847.55	107.9	107.4	1.43	1.38	+0.5	+0.05	21-20	Mädler 18-17; Dawes 1; $O\Sigma.2$
1848.59	101.8	101.4	1.52	1.41	+0.4	+0.11	8-5	Mädler 3-0; Dawes 3; $O\Sigma.2$
1849.48	99.2	96.5	1.71	1.44	+2.7	+0.27	1	Dawes
1850.36	93.3	91.7	1.39	1.46	+1.6	-0.07	8-6	$O\Sigma.3$; Fletcher 2; Mädler 3-1
1851.50	87.9	85.7	1.35	1.49	+2.2	-0.14	16-14	Mädler 3; Fletcher 6; $O\Sigma.5$; Miller 2-0
1852.67	83.8	79.8	1.32	1.52	+4.0	-0.20	20-14	$O\Sigma.5$; Mädler 8-7; Fletcher 5-2; Miller 2-0
1853.46	78.3	76.1	1.38	1.53	+2.2	-0.15	23-20	Jacob 2; Miller 6-3; Mä. 8; $O\Sigma.4$; Mä. 3
1854.46	75.0	71.2	1.47	1.53	+3.8	-0.06	11	Jacob 3; $O\Sigma.3$; Mädler 5
1855.46	70.8	66.4	1.47	1.53	+4.4	-0.06	24-11	Dem. 13-0; Secchi 3; $O\Sigma.4$; Morton 4
1856.48	64.8	62.2	1.43	1.51	+2.6	-0.08	27	Jacob 3; Dembowski 15; Secchi 6; $O\Sigma.3$
1857.61	59.0	55.5	1.35	1.46	+3.5	-0.11	24	Mä. 4; Mo. 2; Sec. 6; $O\Sigma.4$; Dem. 5; Ja. 3
1858.58	51.0	50.5	1.19	1.40	+0.5	-0.21	22-21	Secchi 2; Dembowski 8; $O\Sigma.4$; Mädler 8-7
1859.58	43.1	44.1	1.25	1.30	-1.0	-0.05	19-15	Mädler 6; Secchi 3-0; Dawes 6-5; $O\Sigma.4$
1860.70	32.0	36.5	1.05	1.16	-4.5	-0.11	4	Secchi 3; $O\Sigma.1$
1861.50	18.6	28.5	0.93	1.02	-9.9	-0.09	6	Mädler 2; $O\Sigma.4$
1862.73	11.2	12.9	1.00	0.78	-1.7	+0.22	9-1	Dembowski 8; $O\Sigma.1$; Mädler 2
1863.49	343.0	352.2	cuneo	0.59	-9.2	—	4	Dembowski
1865.55	250.0	256.6	<0.5	0.59	-6.6	-0.09	3	Englemann
1866.74	232.5	229.0	0.85	0.85	+3.5	± 0.00	14	Dem. 5; Dawes 3; $O\Sigma.2$; Dawes 2; Dawes 2
1867.62	223.5	217.2	0.91	0.99	+6.3	-0.08	9	Dembowski 7; Dunér 2
1868.54	208.3	207.7	1.05	1.09	+0.6	-0.04	17	Dembowski 6; Knott 4; $O\Sigma.2$; Dunér 5
1869.60	201.8	198.2	1.08	1.16	+3.6	-0.08	19	Dembowski 8; Dunér 11
1870.54	192.0	190.9	1.15	1.20	+1.1	-0.05	17	Dembowski 11; Dunér 6
1871.52	181.9	183.5	1.21	1.23	-1.6	-0.02	32	Dembowski 14; $O\Sigma.1$; Knott 5; Dunér 12
1872.55	173.3	176.0	1.22	1.24	-2.7	-0.02	27	Dembowski 12; Dunér 12; $O\Sigma.3$
1873.60	166.1	168.3	1.22	1.24	-2.2	-0.02	17	Dembowski 11; $O\Sigma.2$; Dunér 4
1874.60	160.0	161.1	1.37	1.24	-1.1	+0.13	14-15	Dembowski 10; $O\Sigma.4$; Dunér 0-1
1875.56	148.5	154.5	1.30	1.25	-6.0	+0.05	29-27	Dem. 8; Sch. 7; W. & S. 2-0; Dunér 12
1876.56	141.0	148.6	1.30	1.25	-7.6	+0.05	10	Hall 2; Dembowski 7; Plummer 1
1877.57	136.3	140.1	1.40	1.26	-3.8	+0.14	11	Dembowski 8; Pritchett 1; Hall 2
1878.51	126.9	133.6	1.40	1.28	-6.7	+0.12	10-13	$\beta.1$; Sch. 4; Doberck 2-1; Dembowski 7
1879.54	122.5	126.6	1.47	1.30	-4.1	+0.17	8-16	$\beta.3$; Hall 4; Schiaparelli 0-8; Pritchett 1
1880.49	115.8	120.4	1.34	1.32	-4.8	+0.02	10-15	Doberck 2-1; Big. 3-0; $\beta.5$; Sch. 0-9
1881.49	110.6	113.9	1.45	1.35	-3.3	+0.10	17	Doberck 2; $\beta.5$; Hough 4; Hall 5; Big. 1
1882.60	105.6	107.2	1.46	1.38	-1.6	+0.08	17-19	Dk. 2-0; Hl. 5; Sch. 0-11; Jed. 4-3; En. 6-0
1883.60	101.5	101.3	1.54	1.41	+0.2	+0.13	14-29	Per. 4; Hall 5; Sch. 0-15; En. 5 [En. 3-0
1884.58	94.1	96.6	1.51	1.44	-2.5	+0.07	28-17	Big. 2-0; Hl. 4; Per. 3; Prit. 1; Sch. 9; Sea. 6-0;
1885.58	89.8	90.5	1.57	1.47	-0.7	+0.10	29-22	Per. 6; Tar. 4-0; Hl. 7; Sch. 5; Jed. 4; Sea. 3-0
1886.63	87.0	85.0	1.54	1.50	+2.0	+0.04	35-32	Hl. 6; Per. 1; Sea. 3-0; Sch. 9; Jed. 9-7; En. 7
1887.60	81.5	80.2	1.57	1.52	+1.3	+0.05	24	Hall 6; Schiaparelli 18
1888.58	76.1	76.6	1.54	1.53	-0.5	+0.01	21-14	Hall 6; Comstock 3-0; Sch. 9-8; Maw 3-0
1889.56	72.7	70.6	1.55	1.54	+2.1	+0.01	18-17	Sch. 3; Glas. 2-1; Hall 6; Com. 4; Maw 3
1890.60	66.8	65.6	1.53	1.53	+0.2	± 0.00	16-15	Glas. 2; Hall 6; Maw 3; Schiaparelli 5-4
1891.57	62.2	60.9	1.43	1.50	+1.3	-0.07	23-20	Hall 6; See 7-4; Sch. 2; Big. 5; Maw 3
1892.60	55.3	55.5	1.44	1.46	-0.2	-0.02	13	Comstock 5; Schiaparelli 8
1893.74	47.6	48.2	1.34	1.37	-0.6	-0.03	8-7	Schiaparelli 3-2; Bigourdan 5
1894.58	42.1	44.1	1.20	1.31	-2.0	-0.11	7-19	Barnard 3; Glas. 2-0; Lewis 2; Big. 0-14
1895.32	36.7	38.8	1.17	1.21	-2.1	-0.04	3	See

The companion is worthy of regular attention in the part of the orbit now being described, but observation will become more urgent as the star approaches periastron in 1899. If good observations can be secured they will enable us to give the highest precision to the theory of the motion of this star; but if the measures in so delicate a case are affected by sensible systematic errors they

will prove to be of little value. The phenomena of the approaching appulse of ζ *Herculis* will therefore be difficult to observe, and results of importance can only be obtained by skillful treatment. It is hardly necessary to add that this phenomenon will not again be witnessed for more than a third of a century.

It seems worthy of remark that STRUVE, who devoted so much attention to the colors of double stars, noted the color of the companion as reddish, while it is now distinctly bluish, and although a change of color does not seem probable, this has been suspected as well as variability.

In order that astronomers may be able to compare the present theory with observations during the rapid motion of the companion in passing periastron, we give an ephemeris for the next ten years:

t	θ_c	ρ_c	t	θ_c	ρ_c
1896.50	28.5	1.02	1901.50	233.0	0.80
1897.50	15.5	0.82	1902.50	218.4	0.97
1898.50	351.9	0.56	1903.50	207.8	1.09
1899.50	289.7	0.47	1904.50	198.9	1.16
1900.50	258.4	0.58	1905.50	191.0	1.20

β 416 = LACAILLE 7215.

$\alpha = 17^h 12^m.1$; $\delta = -34^\circ 52'$.
6.4, yellowish ; 7.8, yellowish.

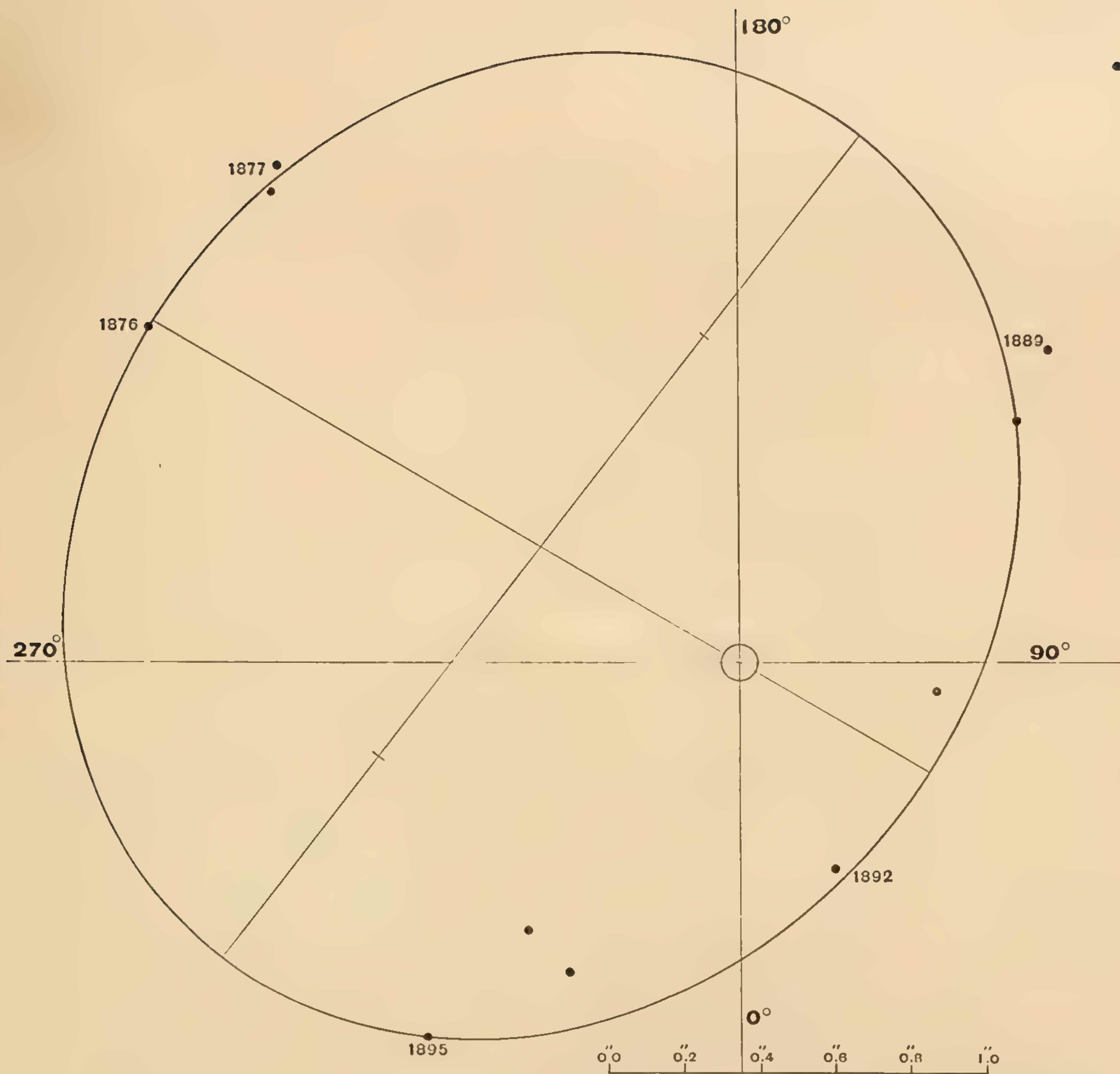
Discovered by Burnham in 1876.

OBSERVATIONS.

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1876.52	240 $\pm$	1.8 $\pm$	1	Burnham	1891.53	81.2	0.53	3-2	Burnham
1877.53	222.6	1.80	1	Cincinnati	1892.38	24.4	0.61	4-3	Burnham
1877.64	224.4	1.77	1	Russell	1894.57	330.8	0.94	7-2	Sellors
1888.72	147.5	1.88	1	Burnham	1894.63	334.7	1.27	3	Barnard
1889.43	135.2	1.17	2-1	Burnham	1895.60	321.7	0.91	2-1	Comstock
1889.63	131.9	0.97	1	Pollock	1895.74	320.0	1.30 $\pm$	1	See

Since the discovery of this rapid binary the companion has described an arc of 280° . The magnitudes of the components are 6.4 and 7.8 respectively, and as the pair is never closer than $0''.58$ the object is difficult only on account of its southern declination.* The period is surprisingly short for a system of

* *Astronomical Journal*, 372.



$\beta 416 = \text{Lac. } 7215.$

such considerable separation, and this circumstance lends decided probability to the view that the parallax is sensible. Provisional elements for this system have been computed by GLASENAPP, GORE and BURNHAM. Their results are as follows :

P	T	e	a	Ω	i	λ	Authority	Source
$\overset{\text{yrs.}}{34.85}$	1892.00	0.65	$\overset{''}{1.52}$	$\overset{\circ}{104.3}$	$\overset{\circ}{45.4}$	$\overset{\circ}{300.7}$	Glasenapp, 1893	Astron. and Astroph., May, 1893
34.48	1891.85	0.556	2.13	139.4	56.7	278.2	Gore, 1893	Monthly Notices, March, 1893
24.7	1892.26	0.56	1.46	122.0	44.4	93.5	Burnham, 1893	Publ. Lick Obs., vol. II, p. 247

The observations which I secured recently at the Washburn Observatory have enabled me to redetermine the orbit. Using all available measures, we find the following elements of $\beta 416$.

$$\begin{aligned}
 P &= 33.0 \text{ years} & \Omega &= 144^{\circ}.6 \\
 T &= 1891.85 & i &= 37^{\circ}.35 \\
 e &= 0.512 & \lambda &= 86^{\circ}.1 \\
 a &= 1''.2212 & n &= -9^{\circ}.0908
 \end{aligned}$$

Apparent orbit :

$$\begin{aligned}
 \text{Length of major axis} &= 2''.76 \\
 \text{Length of minor axis} &= 2''.38 \\
 \text{Angle of major axis} &= 142^{\circ}.5 \\
 \text{Angle of periastron} &= 59^{\circ}.5 \\
 \text{Distance of star from centre} &= 0''.61
 \end{aligned}$$

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1876.52	$\overset{\circ}{240. \pm}$	$\overset{\circ}{233.4}$	$\overset{''}{1.8 \pm}$	$\overset{''}{1.79}$	$\overset{\circ}{+6.6}$	$\overset{''}{+0.01}$	1	Burnham
1877.53	222.6	228.2	1.80	1.79	-5.4	+0.01	1	Cincinnati
1877.64	224.4	227.6	1.77	1.78	-3.2	-0.01	1	Russell
1888.72	147.5	147.7	1.88	1.19	-0.2	+0.69	1	Burnham
1889.43	135.2	136.7	1.17	1.04	-1.5	+0.13	2-1	Burnham
1889.63	131.9	133.1	0.97	1.00	-1.2	-0.03	1	Pollock
1891.53	81.2	75.2	0.53	0.60	+6.1	-0.07	3-2	Burnham
1892.38	24.4	34.0	0.61	0.61	-9.6	0.00	4-3	Burnham
1894.57	330.8	333.6	0.94	1.10	-2.8	-0.16	7-2	Sellers
1895.60	321.7	319.9	0.91	1.30	+1.8	-0.39	2-1	Comstock
1895.74	320.0	318.4	1.30 $\pm$	1.32	+1.6	-0.02	1	See

The angular motion during the last three years has not been very rapid, and the constancy of areas shows that the distances have been somewhat under-measured. It is now apparent that the period will be sensibly longer than BURNHAM supposed. The value found above is not likely to be in error by more than one year, while the correction of the eccentricity will hardly exceed ± 0.03 . Considering the small number of observations on which this orbit is based, the elements may be regarded as highly satisfactory. As this system is

visible in the United States, it is worthy of particular attention from American observers.

The following ephemeris gives the place of the companion for five years :

t	θ_c	ρ_c	t	θ_c	ρ_c
1896.50	310.6	1.43	1899.50	287.7	1.69
1897.50	302.1	1.54	1900.50	281.5	1.72
1898.50	294.6	1.62			

 $\Sigma 2173$.

$\alpha = 17^h 25^m.3$; $\delta = -0^\circ 59'$.
6, yellow ; 6, yellow.

Discovered by William Struve in July, 1829.

OBSERVATIONS.

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1829.56	147.2	0.62	2	Struve	1851.32	334.1	1.27	4	Madler
1831.69	141.5	0.62	3	Struve	1851.74	335.6	1.18	2	Madler
1836.69	single	—	4	Struve	1852.72	334.1	1.23	2	Madler
1837.70	353.	obl.?	1	Struve	1853.12	331.2	1.04	4	O. Struve
1840.47	347.1	0.5 $\pm$	1	Dawes	1854.66	330.5	1.37	3	Madler
1840.64	358.8	0.61	3	O. Struve	1856.53	153.2	0.9 $\pm$	1	Winnecke
1841.36	352.3	0.67	6-2	Madler	1856.53	329.1	1. $\pm$	4	Dembowski
1841.61	352.2	0.67	3	O. Struve	1856.53	329.8	0.97	1	Secchi
1841.64	347.4	0.71	2-1	Dawes	1856.90	326.0	0.94	4	O. Struve
1842.45	354.9	—	5	Kaiser	1857.43	326.9	0.88	1	Secchi
1842.51	349.9	0.75	3	Madler	1858.56	325.9	0.84	2	Secchi
1842.67	343.3	0.7 $\pm$	3	Dawes	1858.61	328.3	0.88	4-2	Madler
1843.30	343.1	0.74	3	O. Struve	1858.61	325.0	0.25 $\pm$	1	Morton
1843.50	346.2	0.78	8-5	Madler	1859.33	324.2	0.71	3	O. Struve
1843.54	341.2	0.9 $\pm$	6	Dawes	1861.57	324.0	—	3	Madler
1843.65	345.1	0.68	10-2	Kaiser	1861.63	315.2	0.48	1	O. Struve
1844.36	345.0	0.8 $\pm$	3	Madler	1864.45	160?	0.6?	2	Englemann
1845.55	342.1	0.97	1	Madler	1864.53	single		1	Dembowski
1846.46	339.4	1.07	6-5	Madler	1865.51	182.2	—	1	Leyton Obs.
1846.47	336.1	0.85	5	O. Struve	1866.32	360.7	0.47	3	O. Struve
1847.47	339.2	1.16	2	Madler	1866.43	181.3	—	1	Leyton Obs.
1848.44	339.2	1.15	1	Madler	1866.59	107.7	—	1	Winlock
1848.45	339.4	1.10	1	Dawes	1866.62	139.4	1.60	5-1	Searle
1848.58	340.4	1.23	1	Mitchell	1866.69	167.7	—	1	Winlock

t	θ_o	ρ_o	n	Observers
1867.79	174.5	0.68	1	Dunér
1868.18	161.3	0.65	3	O. Struve
1868.60	160.6	0.5 ±	2	Dembowski
1868.66	169.3	0.68	3	Dunér
1869.58	157.1	0.58	5-1	Dembowski
1869.93	161.1	0.68	6	Dunér
1870.35	156.4	0.8 ±	2	Gledhill
1870.45	156.8	0.81	6-4	Dembowski
1870.67	159.7	0.80	4	Dunér
1871.44	155.0	0.99	4-2	Dembowski
1871.64	156.5	0.87	6	Dunér
1872.15	155.7	0.89	5	O. Struve
1872.55	152.3	0.95	5-3	Dembowski
1873.50	154.1	1.00	2	W. & S.
1873.51	150.8	0.77	4-1	Dembowski
1873.67	152.6	1.10	1	Dunér
1874.46	150.0	0.91	4-3	Dembowski
1874.57	151.1	0.99	2-1	Gledhill
1874.59	151.2	0.90	2-1	W. & S.
1874.62	149.3	0.77	2	O. Struve
1874.66	148.8	1.09	2	Newcomb
1875.53	147.5	0.74	4	Dembowski
1875.57	146.5	0.83	7	Schiaparelli
1875.57	147.8	1. ±	1	W. & S.
1875.67	148.7	0.90	5	Dunér
1876.52	149.3	0.77	3	Hall
1876.55	144.8	0.69	5	Dembowski
1876.59	143.8	0.83	4	Schiaparelli
1876.65	144.0	0.61	2	O. Struve
1876.66	149.9	—	4	Doberck
1877.49	141.6	—	2	Cincinnati
1877.53	142.5	0.62	5-4	Dembowski
1877.59	141.4	0.65	2	O. Struve
1877.59	142.0	0.72	8	Schiaparelli
1877.68	153.5	0.67	2	Doberck
1878.40	142.5	0.52	1	Doberck
1878.48	139.4	0.60	4	Dembowski
1879.22	137.0	0.69	7-3	Cincinnati
1879.58	136.0	0.5 ±	8	Schiaparelli
1879.72	152.2	0.7 ±	3	Seabroke
1880.47	131.3	0.36	1	Burnham
1880.65	133.9	0.4 ±	9	Schiaparelli
1881.51	114.9	0.24	3	Burnham
1881.52	121.5?	0.27?	1	Hall

t	θ_o	ρ_o	n	Observers
1881.74	elong.?		1	Bigourdan
1882.57	109.9	0.3	7	Schiaparelli
1882.62	349.0	oblong	1	O. Struve
1883.50	elong. 20°-45°		4	Perrotin
1883.60	190.0	oblong	1	O. Struve
1883.60	single		7	Schiaparelli
1883.88	24.8	0.22	9	Englemann
1884.56	17.4	0.38	3-2	Perrotin
1884.59	99 ?	—	1	Bigourdan
1884.60	single		7	Schiaparelli
1884.61	42.7?	0.25 ± ?	3	Schiaparelli
1884.62	9.9	0.32	3	Hall
1885.66	21.9	0.30	8-6	Englemann
1886.55	356.6	0.56	3	Perrotin
1886.56	355.1	0.41	7	Schiaparelli
1886.56	353.0	0.42	3	Hall
1886.64	365.6	0.30	8	Englemann
1887.40	350.5	0.46	4	Tarrant
1887.56	348.5	0.53	7	Schiaparelli
1888.49	347.8	0.68	3	Leavenworth
1888.55	344.4	0.53	3	Hall
1888.60	346.9	0.58	8	Schiaparelli
1888.69	342.3	0.81	1	O. Struve
1889.46	345.0	0.66	5	Tarrant
1889.63	345.5	0.70	7	Schiaparelli
1890.26	341.5	in cont.	10	Giacomelli
1890.49	340.9	0.8 ±	2	Glasenapp
1890.69	343.1	0.84	3	Maw
1890.71	334.6	0.76	2	Bigourdan
1890.74	341.7	0.70	7-5	Schiaparelli
1891.51	340.1	0.97	3	Hall
1891.53	340.0	0.81	4	Schiaparelli
1891.58	339.7	0.93	3	Burnham
1891.69	340.3	0.91	3	Bigourdan
1892.54	341.8	0.90	4	Comstock
1892.61	339.1	1.10	1	Bigourdan
1892.62	339.3	0.88	7	Schiaparelli
1892.72	340.7	0.91	3	Maw
1893.68	338.0	1.08	3	Schiaparelli
1893.87	340.6	1.11	3	H. C. Wilson
1894.55	336.8	1.15	2	Lewis
1894.74	159.9	1.27	1	Callandreau
1895.30	337.3	1.19	3	See
1895.57	337.7	1.13	3	Comstock

When this interesting double star in the constellation *Ophiuchus* was discovered by WILLIAM STRUVE, the companion was measured on two nights,* and again observed in 1831; but in 1836 it had disappeared, so that under the best seeing the star appeared absolutely round. STRUVE therefore surmised (*Mensurae Micrometricae*, p. 294) that this is a case of occultation similar to those of γ *Coronae Borealis* and ω *Leonis*, "*summa attentione digna*." The companion came out on the opposite side in 1840, and was subsequently followed systematically by the best observers, so that at the present time a large amount of good material is available for the investigation of its orbit. The components are so nearly equal in brightness that the angles frequently require a correction of 180° , and for a time it remained uncertain whether the period would be 46 or 23 years. Prof. DUNÉR was the first astronomer who attempted to investigate the orbit of this pair; using measures up to 1876, the illustrious Director of the Observatory of Upsala arrived at the following results:

$$\begin{array}{ll} P = 45.43 \text{ years} & \Omega = 152^\circ.65 \\ T = 1872.91 & i = 80^\circ.53 \\ e = 0.1349 & \lambda = 7^\circ.26 \\ a = 1''.009 & \end{array}$$

From an investigation of all the observations, including the measures recently secured at the Leander McCormick Observatory in Virginia, we find the following elements of Σ2173:

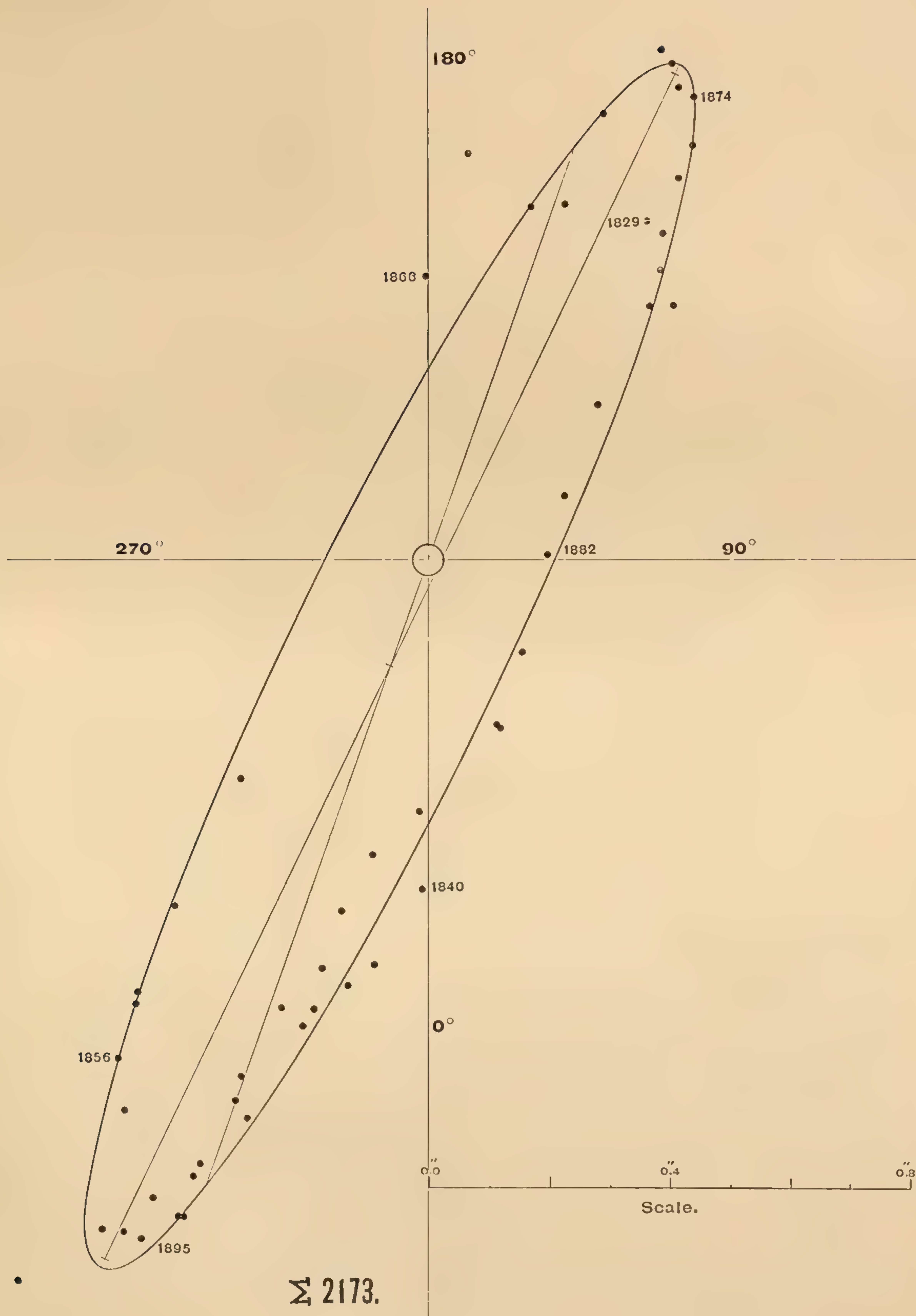
$$\begin{array}{ll} P = 46.0 \text{ years} & \Omega = 153^\circ.7 \\ T = 1869.50 & i = 80^\circ.75 \\ e = 0.20 & \lambda = 322^\circ.2 \\ a = 1''.1428 & n = -7^\circ.8261 \end{array}$$

Apparent orbit:

$$\begin{array}{ll} \text{Length of major axis} & = 2''.22 \\ \text{Length of minor axis} & = 0''.35 \\ \text{Angle of major axis} & = 154^\circ.5 \\ \text{Angle of periastron} & = 160^\circ.8 \\ \text{Distance of star from centre} & = 0''.18 \end{array}$$

The accompanying table of computed and observed places shows that these elements are very satisfactory.

* *Astronomische Nachrichten*, 3311.



COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1829.56	147.2	148.5	0.67	0.84	-1.3	-0.17	2-1	Struve
1831.69	141.5	143.4	0.62	0.69	-1.9	-0.07	3	Struve
1840.64	358.8	358.0	0.55	0.47	+0.8	+0.08	3-4	OΣ. 3; Dawes 0-1
1841.48	352.3	352.5	0.67	0.59	-0.2	+0.08	9-5	Mädler 6-2; OΣ. 3
1842.54	349.3	348.8	0.72	0.69	+0.5	+0.03	11-6	Kaiser 5-0; Mädler 3; Dawes 3
1843.57	345.7	345.9	0.77	0.82	-0.2	-0.05	18-16	Mä. 8-5; Ka. 10-2; OΣ. 0-3; Da. 6
1844.36	345.0	344.3	0.8±	0.90	+0.7	-0.10	3	Mädler
1845.55	342.1	343.2	0.97	0.95	-1.1	+0.02	1	Mädler
1846.46	339.4	340.8	1.07	1.06	-1.4	+0.01	6-5	Mädler
1847.47	339.2	339.6	1.16	1.14	-0.4	+0.02	2	Mädler
1848.49	339.6	338.4	1.16	1.21	+1.2	+0.05	3	Mädler 1; Dawes 1; Mitchell 1
1851.74	335.6	335.0	1.22	1.29	+0.6	-0.07	2-6	Mädler
1852.72	334.1	334.3	1.23	1.28	-0.2	-0.05	2	Mädler
1853.12	331.2	333.9	1.04	1.27	-2.7	-0.23	4	O. Struve
1854.66	330.5	332.7	1.37	1.24	-2.2	+0.13	3	Mädler
1856.65	328.3	330.2	0.97	1.10	-1.9	-0.13	9	Dem. 4; Se. 1; OΣ. 4
1857.43	326.9	329.3	0.88	1.04	-2.4	-0.16	1	Secchi
1858.59	326.4	327.5	0.86	0.93	-1.1	-0.07	7-4	Se. 2; Mä. 4-2; Mo. 1-0
1859.33	324.2	326.1	0.71	0.85	-1.9	-0.14	3	O. Struve
1861.60	319.6	319.7	0.48	0.57	-0.1	-0.09	4-1	Mädler 3-0; OΣ. 1
1866.32	180.7	184.8	0.47	0.28	-4.1	+0.19	3	O. Struve
1867.79	174.5	168.3	0.68	0.51	+6.2	+0.17	1	Dunér
1868.48	163.8	164.5	0.61	0.59	-0.7	+0.02	8	OΣ. 3; Dembowski 2; Dunér 3
1869.76	159.1	160.0	0.63	0.75	-0.9	-0.12	11-7	Dembowski 5-1; Dunér 6
1870.56	158.3	158.1	0.80	0.83	+0.2	-0.03	10-8	Dembowski 6-4; Dunér 4
1871.54	155.8	155.9	0.93	0.89	-0.1	+0.04	10-8	Dembowski 4-2; Dunér 6
1872.35	154.0	154.4	0.92	0.92	-0.4	0.00	10-8	OΣ. 5; Dembowski 5-3
1873.56	152.5	152.2	0.89	0.92	+0.3	-0.03	7-3	W. & S. 2; Dem. 4-1; Du. 1-0
1874.56	150.4	150.4	0.89	0.89	0.0	0.00	10-7	Dem. 4-3; Gl. 2-1; W. & S. 2-1; OΣ. 2
1875.58	147.6	148.4	0.82	0.84	-0.8	-0.02	17-16	Dem. 4; Sch. 7; W. & S. 1-0; Du. 5
1876.58	146.9	146.2	0.76	0.78	+0.7	-0.02	16-12	Hl. 3; Dem. 5; Sch. 4; Dk. 4-0
1877.57	144.4	143.7	0.67	0.70	+0.7	-0.03	17-14	Cin. 2-0; Dem. 5-4; Sch. 8; Dk. 2
1878.48	139.4	140.6	0.56	0.61	-1.2	-0.05	5	Doberck 1; Dembowski 4
1879.40	136.5	136.4	0.59	0.52	+0.1	+0.07	15-11	Cincinnati 7-3; Schiaparelli 8
1880.56	132.6	128.0	0.38	0.40	+4.6	-0.02	10	β. 1; Schiaparelli 9
1881.51	114.9	114.9	0.24	0.29	0.0	-0.05	3	Burnham
1882.61	91.6	90.5	0.2	0.21	+1.1	-0.01	1	Schiaparelli
1883.69	45.0	48.9	0.22	0.20	-3.9	+0.02	4-9	Perrotin 4-0; Englemann 0-9
1884.59	23.3	21.8	0.31	0.27	+1.5	+0.04	9-8	Perrotin 3-2; Sch. 3; Hall 3
1885.66	21.9	5.2	0.30	0.37	+16.7	-0.07	8-6	Englemann
1886.58	357.6	358.0	0.42	0.47	-0.4	-0.05	21	Per. 3; Sch. 7; Hall 3; En. 8
1887.48	349.5	352.7	0.50	0.59	-3.2	-0.09	11	Tarrant 4; Schiaparelli 7
1888.55	346.3	348.1	0.60	0.72	-1.8	-0.12	14	Lv. 3; Hall 3; Schiaparelli 8
1889.63	345.5	345.8	0.70	0.82	-0.3	-0.12	7	Schiaparelli [Big. 0-2; Sch. 7-5
1890.58	341.8	343.8	0.78	0.92	-2.0	-0.14	24-12	Gia. 10-0; Glasenapp 2; Maw 3;
1891.58	340.0	342.1	0.91	1.01	-2.1	-0.10	13	Hall 3; Sch. 4; β. 3; Big. 3
1892.62	340.2	340.6	0.95	1.09	-0.4	-0.14	15	Com. 4; Big. 1; Sch. 7; Maw 3
1893.77	339.3	338.9	1.09	1.19	+0.4	-0.10	6	Schiaparelli 3; H. C. W. 3
1894.55	336.8	338.3	1.15	1.22	-1.5	-0.07	2	Lewis
1895.30	337.3	337.9	1.22	1.24	-0.6	-0.02	3-1	See

Owing to the high inclination of the orbit, it is clear that a small error in angle would very sensibly alter the apparent radius vector of the companion, and for this reason good measures of distance are more trustworthy than

angles. Therefore, while the present orbit is based on both coordinates, unusual weight has been given to the observed distances.

The residuals in angle are very small, except in the case of ENGLEMAN'S measure of 1885, when the components were so close as to render all observations with a small telescope very uncertain. It should be remarked that the position for 1882 is based on a measure which was rejected by SCHIAPARELLI on account of its discordance; but as the other six measures by that distinguished astronomer give

$$\theta_o = 109^\circ.9 \quad \rho_o = 0''.30,$$

which cannot well be reconciled with the theory of the star's motion, it appears probable that the single outstanding observation is nearer the truth, and it is therefore adopted in the above table.

The most remarkable characteristic of γ 2173 is the relatively small eccentricity of its orbit. Although this element is not so well defined as might be desired, yet the value given above seems to be fairly indicated by the best observations, and is not likely to need any large correction. Good measures of distance about the time of maximum elongation, in 1898 and 1899, would fix the eccentricity more accurately, and accordingly for the next five years this system will deserve the particular attention of astronomers.

μ^1 HERCULIS BC = A.C. 7.

$$\alpha = 17^h 42^m.6 \quad ; \quad \delta = +27^\circ 47'.$$

9.4, bluish white ; 10, bluish.

Discovered by Alvan Clark in July, 1856.

OBSERVATIONS.

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1857.47	$63^\circ \pm$	—	1	Dawes	1865.43	80.5°	1.84	2-1	Knott
1857.50	59.3	1.82	2	Dawes	1865.44	82.0	1.27	5	Dembowski
1857.85	71.7	1.74	1	Secchi	1866.59	86.3	—	1	Winlock
1859.70	60.4	2.05	3	Dawes	1866.56	86.3	—	1	Searle
1860.30	67.7	1.64	1	O. Struve	1866.68	89.5	1.10	2	O. Struve
1862.83	78.5	1.50	1	O. Struve	1867.58	97.9	—	3	Searle
1864.43	77.6	1.81	1	Dawes	1867.59	93.0	—	1	Winlock
1864.49	67.5	1.70	1	Englemann	1868.50	97.7	0.88	1	O. Struve
1864.76	78.8	1.76	1	Winnecke	1868.61	106.4	—	1	Winlock

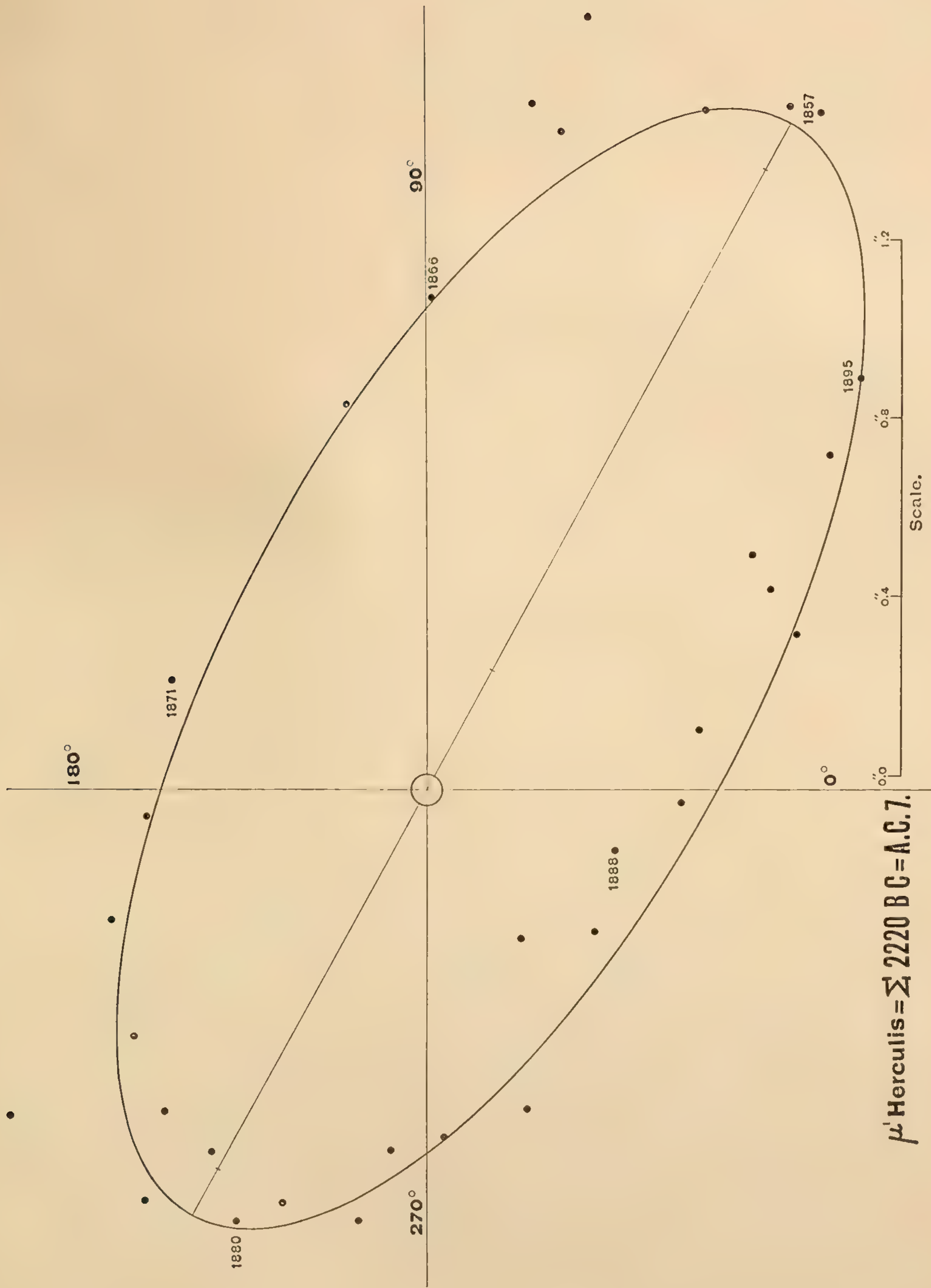
t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1869.73	130.9	—	1	Winlock	1883.53	262.1	0.74	3	Burnham
1869.73	111.7	—	2	Peirce	1883.57	261.1	0.84	3	Hough
					1883.58	262.9	0.66	3	Hall
1871.51	100 $\pm$	0.6 $\pm$	2	W. & S.	1883.58	261.4	0.86	2	Frisby
1871.52	156.8	0.62	1	O. Struve	1883.63	274.8	—	5	Schiaparelli
					1883.96	80.6	0.62	8-6	Englemann
1873.50	180.5	—	1	Romberg					
1873.50	174.5	—	1	H. Bruns	1884.64	273.4	0.65	3	Hall
1873.50	175.4	—	1	Müller	1884.68	272.7	0.77	1	O. Struve
1873.50	185.5	0.63	1	O. Struve					
1873.50	90 $\pm$	0.6 $\pm$	1	W. & S.	1885.51	268.1	1.15	2-1	Holetschek
1873.67	semplíce	—	1	Dembowski	1885.56	288.0	0.61	3	Hall
					1885.62	245.2	—	2	Smith
1874.48	202.4	0.76	4-2	Newcomb					
1874.65	100.5	0.4 $\pm$	2	Gledhill	1886.60	302.1	0.39	5	Hall
1875.58	215.2	—	6	Schiaparelli	1887.54	318.3	0.49	6-5	Schiaparelli
1875.69	225.9	—	1	Newcomb	1887.58	321.5	0.42	3	Hall
1875.69	220.6	1.18	5-3	Hall					
1875.70	217.6	—	1	Holden	1888.47	330.7	0.45	3-2	Tarrant
					1888.62	343.1	0.43	11-9	Schiaparelli
1876.59	223.4	0.72	4	Hall	1888.63	341.4	0.39	4	Hall
1876.60	228.7	1.01	4	O. Struve					
1876.68	216.0	0.83	4	Dembowski	1889.51	357.9	0.55	4	Burnham
					1889.58	354.4	0.58	3	Schiaparelli
1877.47	236.0	—	1	Seabroke	1889.65	0.6	0.34	4	Hall
1877.56	234.3	1.10	2	O. Struve					
1877.59	227.9	0.8?	5	Schiaparelli	1890.38	9.4	0.66	4	Burnham
1877.59	232.8	0.85	2	Hall	1890.55	13.2	0.51	4	Hall
1877.62	229.9	0.92	4	Dembowski	1890.78	15.0	0.57	3	Schiaparelli
1878.45	234.9	1.05	6	Burnham	1891.55	21.4	0.6	2	Schiaparelli
1878.50	233.8	0.88	2	Hall	1891.57	24.8	0.54	4	Hall
1878.64	238.2	1.17	1	O. Struve	1891.60	23.6	0.90	3	Bigourdan
1879.45	242.7	0.90	5	Burnham	1892.58	29.1	0.83	4	Comstock
1879.55	239.5	0.97	3	Hall	1892.62	30.3	0.87	5-4	Schiaparelli
1879.75	234.8	—	11	Seabroke	1892.63	30.5	0.90	1	Bigourdan
					1892.65	31.6	0.84	4	Hall
1880.46	230.2	0.7?	5	Schiaparelli					
1880.47	245.9	0.96	7	Burnham	1893.62	36.0	0.90	1	Bigourdan
1880.65	246.3	1.00	4	Hall					
1880.78	246.5	1.18	3	Frisby	1894.43	41.1	1.19	7	Barnard
					1894.46	38.0	0.95	4	Hough
1881.41	252.1	0.92	5	Burnham	1894.54	38.7	1.17	3	Stone
1881.52	254.2	0.87	3	Hough	1894.77	41.6	1.16	3	Comstock
1881.55	249.1	1.01	5	Hall					
					1895.34	41.2	0.86	1	See
1882.52	259.1	0.70	4	Hall	1895.54	44.0	1.3?	2-1	Schiaparelli
1882.53	255.4	—	1	H. Struve	1895.60	44.4	1.16	3	Comstock
1882.53	261.7	0.90	3	Hough	1895.73	43.7	1.13	2	See
1882.56	263.2	1.03	3	O. Struve	1895.73	43.4	1.34	1	See
1882.60	266.8	—	7	Schiaparelli	1895.73	44.8	1.10	2-1	Moulton

In July, 1856, ALVAN CLARK discovered that the bluish companion of μ *Herculis* = γ 2220 is a close double star; he estimated the magnitudes of the component to be 10 and 11. The object was first measured by DAWES who predicted the binary character of the system; by repeating his observations in 1859 and 1864, he was able to announce a decided orbital motion. The object has since received considerable attention from the best observers, and the material now available for an orbit is sufficient to define the elements in a very satisfactory manner. Owing to the faintness and difficulty of the pair, the measures must be carefully combined in order to get a satisfactory set of mean places; the distances of some observers are notably too small, and hence they are omitted in forming the yearly means. Most of the early observations of DAWES seem to be affected by sensible errors, and hence we give his work in full.

t	θ_0	ρ_0	
1857.472	58.97	1.853	
1857.562	60.08	1.75 $\pm$	
1859.650	58.91	2.304	distance indifferent
1859.691	59.51	1.422	observation very poor
1859.757	62.02	2.040	difficult in distance
1864.431	77.59	1.806	undoubtedly binary

While measuring the wide pair in 1857, he observed that "the stars *B* and *C* certainly point rather to the north of μ ." He gives the angle of μ *Herculis* relative to *BC* as $242^\circ.2$; and hence we gather that the angle of the pair *BC* must have been at least $63^\circ.0$. Since the allineation of the two faint stars with μ *Herculis* would probably be more exact than even micrometer settings, it seems certain that most of DAWES' measured angles are too small; we have therefore chosen certain nights only in making up the means, and have selected the distances with some regard to the subsequent motion of the star. This selection of DAWES' material is necessary in order to represent satisfactorily the whole series of observations by an orbit based on both angles and distances. The following list gives the elements published by previous computers:

P	T	e	a	Ω	i	λ	Authority	Source
$+54.25$ ^{YRS.}	1877.13	0.3023	1.46	57.95	60.72	156.35	Doberek, 1879	A.N., 2287
$+45.39$	1880.142	0.2139	1.369	62.11	67.01	181.97	Leuschner, 1889	Pub. A.S.P., p. 46
$+48.65$	1839.585	0.14853	1.2807	63.38	65.18	182.05	Celoria, 1890	A.N., 2949
42.09	1880.43	0.16922	1.356	62.65	63.82	183.87	Hall, 1894	A.J., No. 324



We find the following elements of μ^1 *Herculis* BC:

$$\begin{array}{ll} P = 45.0 \text{ years} & \Omega = 61^\circ.4 \\ T = 1879.80 & i = 64^\circ.28 \\ e = 0.219 & \lambda = 180^\circ.0 \\ a = 1''.390 & u = +8^\circ.0 \end{array}$$

Apparent orbit:

$$\begin{array}{ll} \text{Length of major axis} & = 2''.78 \\ \text{Length of minor axis} & = 1''.148 \\ \text{Angle of major axis} & = 61^\circ.4 \\ \text{Angle of periastron} & = 241^\circ.4 \\ \text{Distance of star from centre} & = 0''.304 \end{array}$$

The period here given can hardly be in error by more than one year, while the uncertainty of the eccentricity probably does not surpass ± 0.02 . The elements are therefore well defined, and may indeed be regarded as extraordinarily good for an object of such difficulty.

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1857.47	63.0	61.8	—	1.69	+1.2	—	1	Dawes
1857.56	60.1	62.0	1.75 $\pm$	1.69	-1.9	+0.06	1	Dawes
1859.70	62.0	66.9	1.73	1.65	-4.9	+0.08	1-2	Dawes
1860.30	67.7	68.4	1.64	1.63	-0.7	+0.01	1	O. Struve
1862.83	78.5	75.3	1.50	1.46	+3.2	+0.04	1	O. Struve
1864.56	78.2	81.1	1.76	1.30	-2.9	+0.46	2-3	Dawes 1; Englemann 0-1; Winn. 1
1865.44	81.3	84.9	1.55	1.20	-3.6	+0.35	7-6	Knott 2-1; Dembowski 5
1866.68	89.5	91.2	1.10	1.05	-1.7	+0.05	2	O. Struve
1867.58	95.4	97.2	—	0.94	-1.8	—	4	Searle 3; Winlock 1
1868.56	102.0	105.1	0.88	0.82	-3.1	+0.06	2-1	O. Struve 1; Winlock 1-0
1869.73	121.3	118.0	—	0.69	+3.3	—	3	Winlock 1; Peirce 2
1871.52	156.8	148.5	0.62	0.57	+8.3	+0.05	1	O. Struve
1873.50	185.5	182.3	0.63	0.60	+3.2	+0.03	1	O. Struve
1874.48	202.4	200.5	0.76	0.70	+1.9	+0.06	4-2	Newcomb
1875.66	217.8	213.6	1.18	0.82	+4.2	+0.36	12-3	Sch. 6-0; Hall 5-3; Holden 1
1876.62	219.7	221.6	0.85	0.92	-1.9	-0.07	8-12	Hall 4; O Σ . 0-4; Dembowski 4
1877.59	231.0	228.4	0.96	1.00	+2.6	-0.04	13-8	O Σ . 2; Sch. 5-0; Hall 2; Dem. 4
1878.50	235.6	234.0	1.11	1.06	+1.6	+0.05	8-7	β . 6; Hall 2-0; O Σ . 0-1
1879.50	239.0	239.7	0.94	1.08	-0.7	-0.14	19-8	β . 5; Hall 3; Seabroke 11-0
1880.63	246.2	245.9	1.05	1.07	+0.3	-0.02	14	β . 7; Hall 4; Frisby 3
1881.49	250.6	251.0	0.97	1.03	-0.4	-0.06	10	β . 5; Hall 5
1882.55	261.2	258.0	0.97	0.96	+3.2	+0.01	18-6	Hl. 4-0; H Σ . 1-0; Ho. 3; O Σ . 3; Sch. 7-0
1883.64	264.5	266.5	0.80	0.85	-2.0	-0.05	16-8	β . 3; Ho. 3; Hl. 3-0; Frisby 2; Sch. 5-0
1884.65	273.0	276.6	0.77	0.74	-3.6	+0.03	4-1	Hall 3-0; O Σ . 1
1885.56	288.0	288.5	0.88	0.65	-0.5	+0.23	5-4	Holetschek 2-1; Hall 3
1886.60	302.1	305.5	0.39	0.58	-3.4	-0.19	5	Hall
1887.56	319.9	324.4	0.49	0.55	-4.5	-0.06	9-5	Schiaparelli 6-5; Hall 3-0
1888.57	342.3	343.8	0.44	0.58	-1.5	-0.14	15-11	Tarrant 0-2; Sch. 11-9; Hall 4-0
1889.58	359.3	0.7	0.57	0.66	-1.4	-0.09	8-7	β . 4; Schiaparelli 0-3; Hall 4-0
1890.57	12.5	12.0	0.62	0.75	+0.5	-0.13	11-7	β . 4; Hall 4-0; Schiaparelli 3
1891.57	23.3	22.5	0.90	0.87	+0.8	+0.03	9-3	Schiaparelli 2-0; Hall 4-0; Big. 3
1892.62	30.4	29.9	0.89	1.00	+0.5	-0.11	14-5	Com. 4-0; Sch. 5-4; Big. 1; Hall 4-0
1893.62	36.0	35.4	0.90	1.12	+0.6	-0.22	1	Bigourdan
1894.55	39.9	39.7	1.17	1.23	+0.2	-0.06	17-13	Bar. 7; Ho. 4-0; Stone 3; Com. 3 [See 1
1895.55	43.3	43.5	1.34	1.33	-0.2	+0.01	9-1	See 1-0; Sch. 2-0; Com. 3-0; See 2-0;

We remark the star is now wider than most observers have indicated by their recent measures. The distance for 1895 is based upon two nights' work, one of the observations being taken by SCHIAPARELLI, the other by the writer at Madison and accidentally omitted in *Astronomical Journal*, No. 359. This observation is:

1895.732 43°.2 1".34 1 n See

The images are noted as "good but faint." There is no doubt that the distance is now at least 1".3, and it will increase for some years. Observers should follow this system carefully. The following is an ephemeris:

t	θ_c	ρ_c	t	θ_c	ρ_c
1896.60	47.0	1.43	1899.60	55.1	1.63
1897.60	49.9	1.51	1900.60	57.5	1.67
1898.60	52.6	1.58			

τ OPHIUCHI = Σ 2262.

$\alpha = 17^h 57^m.6$; $\delta = -8^\circ 11'$.
5, yellowish ; 6, yellowish.

Discovered by Sir William Herschel, April 28, 1783.

OBSERVATIONS.

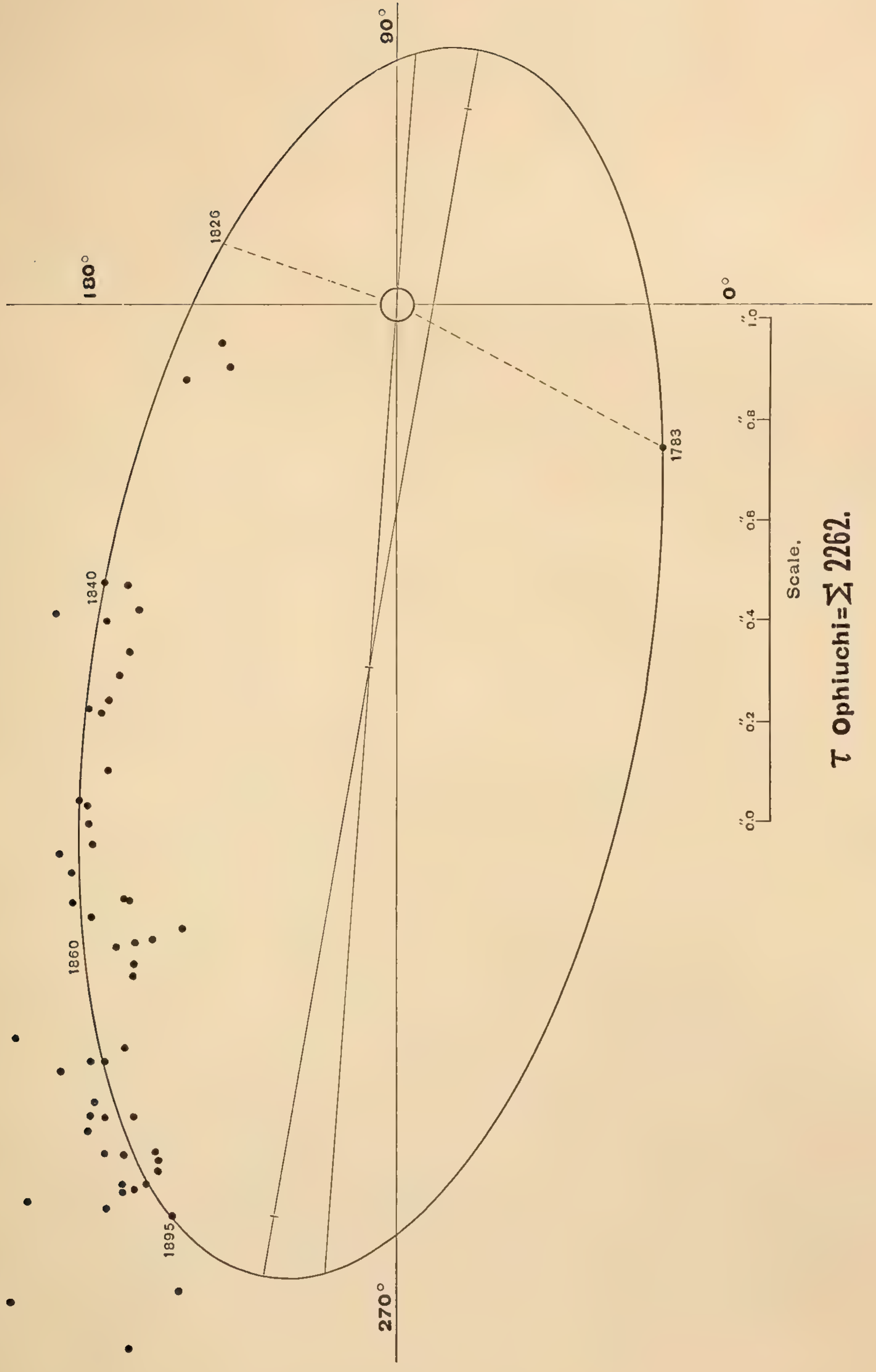
t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1783.34	331.6	elong.	1	Herschel	1843.11	224.6	0.80	—	Kaiser
1802.74	360 $\pm$	elong.	1	Herschel	1843.54	228.8	0.80	11	Mädler
1804.44	360 $\pm$	elong.	1	Herschel	1843.61	229.0	0.95 $\pm$	2	Dawes
1825.71	176.0	cuneata	1	Struve	1844.34	229.8	0.79	2	Mädler
1827.28	146.0	oblonga	1	Struve	1844.74	218.7	0.79	1	Challis
1835.68	192.9	0.35	6-2	Struve	1845.65	232.4	0.87	1	O. Struve
1836.62	199.9	0.44	5	Struve	1846.22	239.5	1.00	—	Jacob
1837.70	200.8	0.35	1	Struve	1846.51	229.4	0.78	8	Mitchell
1840.51	223.1	0.94	1	O. Struve	1846.69	230.7	0.97	2	O. Struve
1840.68	221.5	0.88	4-1	Dawes	1847.82	233.9	0.97	1	O. Struve
1841.53	217.3	0.75	8	Mädler	1848.10	229.7	1.18	2	Mitchell
1841.60	228.1	0.87	3-2	O. Struve	1848.66	232.7	1.01	1	Dawes
1841.66	225.7	0.79	5-1	Dawes	1850.77	234.0	1.0	21	Jacob
1842.57	225.6	0.77	5	Mädler	1851.66	239.4	1.0	—	Fletcher
1842.64	226.9	—	1	Dawes	1851.67	238.2	1.19	1	O. Struve

t	θ_0 °	ρ_0 "	n	Observers	t	θ_0 °	ρ_0 "	n	Observers
1852.65	239.5	1.10	2	Jacob	1870.04	247.3	1.43	8	Dembowski
1852.65	239.7	1.23	2	O. Struve	1870.71	250.7	1.26	1	Dunér
1852.66	238.6	1.27	4-3	Mädler	1871.66	251.0	1.31	2	Dunér
1853.79	238.3	1.17	4	Mädler	1872.01	247.8	1.55	8	Dembowski
1854.67	238.0	1.22	1	Dawes	1872.58	248.1	1.69	1	O. Struve
1854.70	236.1	1.20	1	O. Struve	1873.54	248.9	2.12	1	Leyton Obs.
1854.74	238.2	1.09	3	Mädler	1874.08	248.5	1.60	8	Dembowski
1855.49	238.1	1.30	3	Dembowski	1874.57	250.7	1.48	1	Leyton Obs.
1855.55	236.9	1.27	2	Secchi	1874.67	251.1	1.63	1	O. Struve
1855.67	240.4	1.31	2	O. Struve	1875.61	248.9	1.61	8	Schiaparelli
1856.24	240.7	1.20	4	Secchi	1876.02	249.3	1.67	10	Dembowski
1856.58	240.5	1.20	6	Dembowski	1876.60	247.6	1.73	3	Schiaparelli
1856.62	242.6	1.29	1	Winnecke	1876.62	250.4	2.05	1	Stone
1857.55	239.6	1.26	3	Secchi	1876.64	251.1	1.72	3	Hall
1857.63	241.4	1.20	4	Dembowski	1876.67	248.2	1.78	1	Waldo
1857.67	240.2	1.44	2	O. Struve	1876.70	246.5	1.58	1	O. Struve
1858.20	243.6	1.41	—	Jacob	1877.55	249.0	1.53	4	Hall
1858.52	241.8	1.20	6	Dembowski	1877.61	250.5	1.90	8	Cincinnati
1858.64	240.7	1.33	3	Mädler	1877.66	248.6	1.64	7	Schiaparelli
1858.71	240.9	1.47	1	O. Struve	1878.02	250.4	1.72	8	Dembowski
1859.63	242.7	1.64	1	O. Struve	1878.52	254.1	1.69	2	Doberek
1860.77	245.8	1.30	1	Secchi	1879.35	247.9	1.63	2	Burnham
1861.60	244.4	1.29	3	Mädler	1879.41	250.1	1.78	26-25	Cincinnati
1861.63	242.9	1.43	1	O. Struve	1879.72	250.3	1.74	5	Schiaparelli
1863.05	244.6	1.40	13	Dembowski	1880.07	249.7	1.78	3	Cincinnati
1863.57	246.5	1.20	4	Knott	1880.65	251.6	1.80	6	Schiaparelli
1864.47	247.8	1.92	2	Englemann	1880.66	251.1	1.64	2	Hall
1865.52	249.4	1.40	—	Kaiser	1880.67	252.2	1.89	3	Jedrzejewicz
1865.60	243.1	1.23	1-2	Leyton Obs.	1881.55	251.3	1.71	3	Hall
1865.72	244.1	1.51	1	O. Struve	1881.79	252.7	1.67	2	Smith
1865.89	245.9	1.42	13	Dembowski	1882.49	252.0	2.05	3	H. C. Wilson
1866.43	246.3	1.66	3-2	Leyton Obs.	1882.54	253.3	1.73	3	Hall
1866.58	247.5	2.48	3-2	Winlock	1882.60	252.1	1.86	7	Schiaparelli
1866.59	247.7	1.65	2-3	Searle	1882.62	250.8	2.13	1	O. Struve
1866.62	243.3	1.75	1	O. Struve	1883.38	254.5	1.84	9	Englemann
1866.72	247.6	1.60	2	Secchi	1883.51	252.1	1.66	3	Perrotin
1867.56	251.5	2.49	2-1	Winlock	1883.53	253.0	2.37	2-1	H. C. Wilson
1867.98	246.0	1.43	9	Dembowski	1883.55	253.8	1.60	1	Seabroke
1868.57	247.6	1.29	3	C. S. Peirce	1883.58	253.4	1.78	5	Hall
1868.58	246.4	—	1	Leyton Obs.	1883.61	252.0	1.83	6	Schiaparelli
1868.61	249.5	1.44	1	Winlock	1883.66	254.8	1.79	3	Jedrzejewicz
1869.56	248.4	—	1	Leyton Obs.	1884.41	253.5	1.94	1	H. C. Wilson
1869.64	248.2	1.41	6	Dunér	1884.60	253.0	1.82	3	Hall
1869.73	245.0	1.41	1	C. S. Peirce	1884.78	251.6	1.74	6	Schiaparelli

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1885.48	258.1	1.79	3	Tarrant	1890.57	254.6	1.78	1	Hayn
1885.56	253.5	1.66	4	Hall	1891.48	257.6	2.0 $\pm$	1	See
1885.57	251.2	1.76	4	de Ball	1892.65	255.2	1.75	4	Schiaparelli
1885.58	256.0	2.01	5	Jedrzejewicz	1892.58	254.6	1.78	4	Comstock
1886.22	254.8	1.98	7	Englemann	1893.50	254.1	1.81	3	Maw
1886.54	254.0	1.67	4	Hall	1893.70	254.7	1.83	1	Bigourdan
1886.62	256.2	1.85	6	Jedrzejewicz	1894.59	254.4	1.88	2	Glasenapp
1887.09	252.0	1.72	4	Schiaparelli	1894.77	254.7	1.64	3	Comstock
1887.57	252.5	1.81	4	Hall	1894.78	253.2	1.91	1	Bigourdan
1888.56	253.1	1.70	5	Hall	1895.56	256.1	1.78	3	Schiaparelli
1888.61	254.4	1.71	4	Schiaparelli	1895.58	255.4	1.98	2	Collins
1888.71	255.2	1.80	3	Maw	1895.59	253.4	1.94	5	Schwarzschild
1889.57	255.6	2.23	2	Glasenapp	1895.72	254.7	1.86	4	See
1889.68	253.5	1.69	1	Schiaparelli	1895.72	257.8	1.90 $\pm$	2	Moulton

Since the discovery of this double star in 1783, the radius vector of the companion has swept over an arc of 285° . But while the length of the arc would ordinarily be sufficient to fix the character of the orbit, it happens unfortunately in this case that the observations are neither very consistent nor very well distributed over the arc; and since by far the greater number of observed positions lie in the sixty degrees described since 1836, a satisfactory determination of the elements is embarrassed by difficulties of a somewhat formidable character. But when we examine HERSCHEL's angle of 1783 in the light of his remarks, there seems to be every reason to regard it as fairly correct. In his notes on the observation of τ *Ophiuchi*, he says: "The closest of all my double stars can only be suspected with 460, but 932 confirms it to be a double star. It is wedge-formed with 460; with 932 one-half of the small star, if not three-quarters, seems to be behind the large star. The morning is so fine that I can hardly doubt the reality; but according to custom, I shall put it down as a phenomenon that may be a deception." If we depend on the approximate accuracy of HERSCHEL's earliest measure, and deduce the areal velocity from the most recent observations, where both angles and distances can be relied upon, we are led to an orbit which will not differ greatly from the truth. The following orbits have been published by previous investigators:

P	T	e	a	Ω	i	λ	Authority	Source
^{yrs.} 87.036	1840.07	0.03746	0.8178	55° 5'	51° 47'	145° 40'	Mädler, 1847	Fixt. Syst., I, 255
120.0	1824.8	0.575	—	130 0	48 30	146 6	Hind, 1849	M.N., IX, p. 145
185.2	1820.63	0.5818	1.111	69 31	53 5	28 35	Dobereck, 1877	A.N., 2037
217.87	1818.50	0.6055	1.193	67 1	46 8	36 26	Dobereck, 1877	A.N., 2041



We find the following elements of τ *Ophiuchi*:

$$\begin{array}{ll} P = 230.0 \text{ years} & \Omega = 76^\circ.4 \\ T = 1815.0 & i = 57^\circ.6 \\ e = 0.592 & \lambda = 18^\circ.05 \\ a = 1''.2495 & n = +1^\circ.5652 \end{array}$$

Apparent orbit:

$$\begin{array}{ll} \text{Length of major axis} & = 2''.46 \\ \text{Length of minor axis} & = 1''.09 \\ \text{Angle of major axis} & = 80^\circ.0 \\ \text{Angle of periastron} & = 85^\circ.8 \\ \text{Distance of star from centre} & = 0''.712 \end{array}$$

The accompanying table shows that this orbit gives a very satisfactory representation of both angles and distances.

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1783.34	331.6	313.7	elongated	0.75	+17.9	—	1	Herschel
1802.74	360 $\pm$	31.8	elongated	0.49	—31.8	—	1	Herschel
1804.44	360 $\pm$	45.5	elongated	0.51	—45.5	—	1	Herschel
1826.50	161.0	164.6	oblonga	0.37	— 3.6	—	2	Struve
1835.68	192.9	211.2		0.35	—18.3	—0.26	6-2	Struve
1836.62	199.9	213.8		0.44	—13.9	—0.20	5	Struve
1837.70	200.8	216.6		0.35	—15.8	—0.33	1	Struve
1840.60	222.3	222.3		0.91	$\pm$ 0.0	+0.13	5-2	O. Struve; Dawes 4-1
1841.60	223.7	224.2		0.80	— 0.5	—0.02	16-11	Mädler 8; $O\Sigma$ 3-2; Dawes 5-1
1842.60	226.2	225.7		0.77	+ 0.5	—0.07	6-5	Mädler 5; Dawes 1-0
1843.41	227.5	227.0		0.85	+ 0.5	—0.03	12+	Kaiser —; Mädler 11; Dawes 2
1844.54	229.8	228.3		0.79	+ 1.5	—0.12	2-3	Mädler 2; Challis 0-1
1845.65	232.4	229.9		0.87	+ 2.5	—0.08	1	O. Struve
1846.47	233.2	230.8		0.92	+ 2.4	—0.06	10+	Jacob —; Mitchell 8; $O\Sigma$ 2
1847.82	233.9	232.4		0.97	+ 1.5	—0.05	1	O. Struve
1848.66	232.7	233.2		1.01	— 0.5	—0.03	1	Dawes
1850.77	234.0	235.2		1.00	— 1.2	—0.10	21obs	Jacob
1851.66	238.2	236.0		1.09	+ 2.2	—0.04	1+	Fletcher —; $O\Sigma$ 1
1852.65	239.3	236.8		1.20	+ 2.5	+0.04	8-7	Jacob 2; $O\Sigma$ 2; Mädler 4-3
1853.79	238.3	237.6		1.17	+ 0.7	—0.02	4	Mädler
1854.70	237.4	238.5		1.17	— 1.1	—0.05	5	Dawes 1; $O\Sigma$ 1; Mädler 3
1855.57	238.5	239.1		1.28	— 0.6	+0.04	7	Dembowski 3; Secchi 2; $O\Sigma$ 2
1856.48	240.6	239.7		1.23	+ 0.9	—0.03	11-10	Secchi 4; Dembowski 6; Winn. 1
1857.62	240.4	240.5		1.30	— 0.1	$\pm$ 0.00	9	Secchi 3; Dembowski 4; $O\Sigma$ 2
1858.52	241.7	241.1		1.35	+ 0.6	+0.03	10+	Jacob —; Dem. 6; Mädler 3; $O\Sigma$ 1
1859.63	242.7	241.8		1.64	+ 0.9	+0.30	1	O. Struve
1860.77	245.8	242.6		1.30	+ 3.2	—0.07	1	Secchi
1861.62	243.7	243.3		1.36	+ 0.4	—0.03	4	Mädler 3; $O\Sigma$ 1
1863.31	245.5	243.9		1.30	+ 1.6	—0.12	17	Dembowski 13; Knott 4
1864.47	247.8	244.6		1.92	+ 3.2	+0.47	2	Englemann
1865.68	246.5	245.2		1.39	+ 1.3	—0.08	14-16	Kaiser —; Ley. 1-2; $O\Sigma$ 1; Dem. 13
1866.59	246.5	245.6		1.66	+ 0.9	+0.17	8	Ley. 3-2; Wk. 3-0; Sr. 2-3; $O\Sigma$ 1;
1867.77	248.7	246.2		1.43	+ 2.5	—0.08	11-9	Winlock 2-0; Dembowski 9 [Sec. 2
1868.59	247.8	246.6		1.37	+ 1.2	—0.16	4-3	Peirce 3; Leyton 1-0; Winlock 1
1869.64	248.3	247.0		1.41	+ 1.3	—0.14	7	Leyton 1-0; Dunér 6; Peirce 1
1870.37	249.0	247.3		1.35	+ 1.7	—0.21	9	Dembowski 8; Dunér 1

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1871.66	251.0	248.0	1.31	1.59	+3.0	-0.28	2	Dunér
1872.30	248.0	248.3	1.62	1.60	-0.3	+0.02	9	Dembowski 8; $O\Sigma$ 1
1873.54	248.9	248.8	2.12	1.62	+0.1	+0.50	1	Leyton Observers.
1874.44	250.1	249.1	1.57	1.63	+1.0	-0.06	10	Dembowski 8; Leyton 1; $O\Sigma$ 1
1875.61	248.9	249.6	1.61	1.65	-0.7	-0.04	8	Schiaparelli [$O\Sigma$ 1
1876.54	249.6	250.0	1.75	1.67	-0.4	+0.08	17-19	Dem.10; Sch. 3; St.1; Hl. 3; Wdo. 1;
1877.61	249.4	250.4	1.69	1.68	-1.0	+0.01	19	Hall 4; Cincinnati 8; Schiaparelli 7
1878.27	250.4	250.6	1.71	1.69	-0.2	+0.02	8-10	Dembowski 8; Doberck 2
1879.49	249.4	251.1	1.72	1.71	-1.7	+0.01	33-32	β . 2; Cincinnati 26-25; Sch. 5
1880.51	251.1	251.5	1.78	1.72	-0.4	+0.06	14	Cin. 3; Sch. 6; Hall 2; Jed. 3
1881.67	252.0	251.9	1.69	1.74	+0.1	-0.05	5	Hall 3; Smith 2
1882.56	252.1	252.2	1.88	1.75	-0.1	+0.13	14-13	H.C.W. 3; Hl. 3; Sch. 7; $O\Sigma$ 1-0
1883.53	252.8	252.6	1.84	1.76	+0.2	+0.08	17-28	En. 9; Per. 3; H.C.W. 2-1; Sea. 1; Hl. 5;
1884.60	252.7	252.9	1.83	1.77	-0.2	+0.06	10	H.C.W. 1; Hl. 3; Sch. 6 [Sch. 6; Jed. 3
1885.55	253.5	253.2	1.81	1.78	+0.3	+0.03	13-16	Tar. 3; Hall 4; deBall 4; Jed. 5
1886.46	254.4	253.6	1.83	1.80	+0.8	+0.03	11-17	Englemann 7; Hall 4; Jed. 6
1887.33	252.3	253.9	1.77	1.81	-1.6	-0.04	8	Schiaparelli 4; Hall 4
1888.64	254.2	254.3	1.75	1.82	-0.1	-0.07	8	Hall 5; Maw 3
1889.57	255.6	254.6	2.13	1.83	+1.0	+0.30	2-1	Glasenapp
1890.57	254.6	254.9	1.78	1.84	-0.3	-0.06	1	Hayn
1891.48	257.6	255.2	2. $\pm$	1.85	2.4	+0.15 $\pm$	1	See
1892.58	254.6	255.5	1.78	1.85	-0.9	-0.07	4	Comstock
1893.50	254.1	255.8	1.81	1.86	-1.7	-0.05	3	Maw
1894.68	254.5	256.2	1.76	1.87	-1.7	-0.11	5	Glasenapp 2; Comstock 3
1895.72	256.2	256.5	1.86	1.88	-0.3	-0.02	6-4	See 4; Moulton 2-0

The following is an ephemeris for the next five years:

t	θ_c	ρ_c	t	θ_c	ρ_c
	$^{\circ}$	$''$		$^{\circ}$	$''$
1896.50	256.7	1.88	1899.50	257.6	1.90
1897.50	257.0	1.89	1900.50	257.9	1.91
1898.50	257.3	1.90			

It will be evident from what has been said that this orbit is still open to some uncertainty, but a material improvement in the elements will not be possible for many years. Since the companion is at present nearing the apastron of the apparent ellipse, the motion will continue to be very slow; yet the pair will still be worthy of occasional attention from observers. While the period found above is perhaps uncertain to the extent of 15 years, it does not seem probable that the eccentricity can be in error by more than ± 0.05 . Accordingly there is no probability that even the lapse of ages will *radically* change these elements of τ Ophiuchi.

70 OPHIUCHI = $\Sigma 2272$. $\alpha = 18^h 0^m.4$; $\delta = +2^\circ 33'$.

4.5, yellow ; 6, purplish.

Discovered by Sir William Herschel, August 7, 1779.

OBSERVATIONS.

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1779.77	90	—	1	Herschel	1836.42	128.9	6.38	8	Mädler
1781.74	80.8	4.45	1-2	Herschel	1836.51	127.7	6.48	4	Encke
1802.34	336.1	—	1	Herschel	1836.52	129.5	6.34	5	Bessel
1804.42	318.8	—	2	Herschel	1836.66	129.5	6.15	8	Struve
1819.64	168.5	—	5	Struve	1837.13	127.7	6.47	3	Dawes
1820.77	160.2	—	2	Struve	1837.46	128.3	6.74	4	Encke
1821.74	157.6	—	5	Struve	1837.60	127.5	6.46	16	Bessel
1822.42	154.8	4.27	2	H. and So.	1837.72	128.0	6.15	4	Struve
1822.64	153.9	—	3	Struve	1838.57	126.6	6.64	7	Galle
1825.56	148.1	4.76	14	South	1839.52	125.2	6.78	2	Galle
1825.57	148.2	3.98	14	Struve	1839.65	125.9	6.55	2	Dawes
1827.02	145.1	4.37	2	Struve	1840.35	128.0	6.00	—	Kaiser
1828.58	140.6	5.37	1	Herschel	1840.48	126.6	6.52	10	O. Struve
1828.71	140.2	4.78	4	Struve	1840.59	124.9	6.63	4	Dawes
1829.59	138.1	5.08	6	Struve	1841.50	125.4	6.40	8	Mädler
1829.60	140.6	—	1	Herschel	1841.65	123.4	6.54	5	Kaiser
1830.39	138.2	6.01	9	Herschel	1841.68	123.4	6.63	4	Dawes
1830.50	135.8	5.47	10	Bessel	1841.74	123.8	6.85	7	Be. and Sel.
1830.57	137.3	5.53	6	Dawes	1842.31	125.1	6.63	8	O. Struve
1830.84	135.7	5.31	2	Struve	1842.53	124.6	6.25	3	Mädler
1831.53	136.5	5.94	8-6	Herschel	1842.53	123.3	6.72	2	Dawes
1831.53	134.0	5.68	7	Bessel	1842.59	122.6	6.48	22	Kaiser
1831.68	134.7	5.41	5	Struve	1842.60	123.5	6.79	18	Schlüter
1832.55	133.8	5.71	3	Dawes	1843.47	122.0	—	1	Dawes
1832.57	135.4	5.35	4-3	Herschel	1843.52	121.1	6.70	3	Encke
1832.69	133.0	5.79	5	Bessel	1843.58	123.1	6.44	16	Mädler
1833.42	132.8	6.14	1	Dawes	1844.36	120.7	6.84	5	Encke
1834.47	131.1	5.85	4	Struve	1844.52	122.0	6.48	5	Mädler
1834.57	130.6	6.13	7	Dawes	1845.43	120.8	6.77	9	Hind
1834.61	130.8	6.13	7	Bessel	1845.48	121.0	6.56	5	O. Struve
1835.60	130.7	6.11	5	Struve	1845.54	120.8	6.58	16	Mädler
					1846.25	120.2	6.83	1	Jacob
					1846.46	120.1	6.14	7	Hind
					1846.56	117.1	7.43	—	Durham obs.
					1846.58	119.8	6.65	10	Mädler

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1847.25	120.5	6.56	4	O. Struve	1857.13	110.6	6.45	3	Jacob
1847.45	117.2	7.19	—	Durham obs.	1857.41	112.5	6.19	1	Winnecke
1847.59	120.3	—	1	Mitchell	1857.51	110.4	6.20	4	Secchi
1847.60	118.5	6.79	8	Mädler	1857.58	110.3	6.52	2	Dawes
1848.12	118.8	6.80	3	Dawes	1857.64	109.5	6.25	4	Dembowski
1848.49	118.4	6.84	4	Mädler	1857.67	110.2	6.15	2	Morton
1848.52	118.0	6.8	2	Bond	1857.69	110.1	6.40	4	O. Struve
1849.39	118.1	6.64	5	O. Struve	1858.12	109.7	6.10	3	Jacob
1850.42	116.8	6.88	8	Radcliffe	1858.39	108.6	6.08	2	Morton
1850.49	115.2	6.86	2	Worster & Ja.	1858.44	109.3	6.04	4	Dembowski
1850.64	116.7	6.94	4	Mädler	1858.63	108.9	5.83	9	Mädler
1850.66	117.0	6.46	4	Fletcher	1859.30	109.0	6.20	5	O. Struve
1851.20	115.2	6.65	4	Mädler	1859.72	109.3	6.24	4	Dawes
1851.58	115.8	6.38	8	Fletcher	1859.75	109.0	6.44	5	Auwers
1851.67	115.4	6.34	5	O. Struve	1859.76	107.8	6.10	5	Powell
1851.73	115.5	6.67	7	Mädler	1859.80	107.0	6.25	1	Mädler
1852.63	116.0	6.36	6	Fletcher	1860.61	106.3	6.07	3	Secchi
1852.67	115.0	6.55	5	O. Struve	1860.74	109.0	6.41	—	Luther
1852.71	114.7	6.56	11	Mädler	1860.76	106.7	6.52	5	Auwers
1852.74	114.0	6.73	15	Jacob	1861.46	107.0	5.89	1	Radcliffe
1853.55	113.6	—	9	Powell	1861.69	106.6	5.92	7	Mädler
1853.55	116.5	6.36	6	Dembowski	1861.74	106.0	6.21	6	Auwers
1853.62	114.6	6.49	6	Dawes	1861.81	105.4	5.8	3	Powell
1854.08	113.6	6.36	21	Jacob	1862.40	105.6	5.86	3	O. Struve
1854.24	113.0	6.51	2	Jacob	1862.55	106.0	6.05	1	Winnecke
1854.24	113.3	6.51	6	O. Struve	1862.62	105.5	5.72	9	Dembowski
1854.64	113.4	6.23	12	Dembowski	1862.72	105.2	5.69	6	Mädler
1854.67	113.0	6.27	10	Mädler	1863.47	104.0	6.07	11	Adolph
1854.73	113.7	6.34	3	Dawes	1863.51	104.1	5.28	2	Secchi
1854.78	112.9	—	3	Powell	1863.51	104.2	5.60	9	Dembowski
1855.03	115.3	6.86	2	Luther	1863.55	104.5	5.76	1	Talmage
1855.45	111.6	6.25	3	Searle	1863.58	106.2	5.19	1	Ferguson
1855.56	114.2	6.34	1	Winnecke	1863.64	105.8	5.82	5	Hall
1855.63	112.7	6.33	5	Mädler	1864.48	104.8	5.42	2	Englemann
1855.69	113.3	6.47	2	Dawes	1864.60	103.5	5.45	11	Dembowski
1855.75	112.4	—	7	Powell	1865.30	102.6	5.27	8	Englemann
1855.82	—	7.23	1	Schmidt	1865.51	102.7	5.43	4	Secchi
1856.09	111.8	6.44	5	O. Struve	1865.51	102.3	5.35	9	Dembowski
1856.33	111.5	6.40	7	Jacob	1865.56	103.9	5.24	2	Talmage
1856.50	111.5	6.32	3	Mädler	1865.62	100.6	5.31	20	Kaiser
1856.50	112.6	6.40	8	Winnecke	1866.13	101.6	5.26	8	Dembowski
1856.55	111.2	6.12	3	Secchi	1866.29	101.0	5.29	5	O. Struve
1856.63	111.8	6.38	6	Dembowski	1866.49	101.8	5.26	5	Talmage
					1866.54	100.8	5.50	4	Harvard
					1866.69	101.1	5.27	3	Secchi

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1867.41	98.1	5.33	1	Radcliffe	1875.52	83.7	3.48	9	Dembowski
1867.44	99.8	5.22	2	Knott	1875.62	84.1	3.44	8	Schiaparelli
1867.52	100.5	—	1	Talmage	1875.68	84.8	3.84	4	Radcliffe
1867.57	100.4	5.06	7	Dembowski	1876.48	82.1	3.55	5	Schur
1867.57	99.2	5.10	3	Harvard	1876.52	78.9	3.46	2	Doberck
1868.47	98.4	4.85	7	Dembowski	1876.52	80.9	3.32	7	Dembowski
1868.56	98.5	4.98	2	Knott	1876.54	80.2	3.55	3	Plummer
1868.57	99.9	5.11	2	Radcliffe	1876.59	81.3	3.39	6	Schiaparelli
1868.64	101.1	5.41	1	Talmage	1876.64	80.9	3.56	3	Hall
1868.72	97.6	4.84	4	Dunér	1876.64	81.5	3.27	4	Jedrzejewicz
1868.72	99.1	4.69	2	O. Struve	1876.66	79.7	3.72	1	Waldo
1868.90	98.0	4.92	5	Brünnow	1877.51	77.6	3.08	8	Dembowski
1869.68	100.2	5.31	1	Talmage	1877.52	77.6	3.47	2	Doberck
1869.69	96.9	4.59	3	Dunér	1877.55	75.8	3.36	4	Hall
1869.73	98.1	5.12	1	Peirce	1877.58	79.4	3.18	10	Jedrzejewicz
1869.91	96.5	4.70	8	Dembowski	1877.65	78.5	3.39	8	Plummer
1870.51	94.0	4.4	2	Gledhill	1877.66	77.3	3.12	10	Schiaparelli
1870.51	94.1	4.55	8	Dembowski	1877.68	78.5	3.12	4	Cincinnati
1870.52	94.4	4.62	2	Talmage	1877.68	79.5	3.15	4	Schur
1871.48	92.6	4.30	2	W. & S.	1878.51	74.5	2.96	7	Dembowski
1871.49	94.9	4.42	2	Radcliffe	1878.54	75.3	3.04	3	Seabroke
1871.51	90.8	4.61	2	Peirce	1878.54	75.5	3.03	4	Doberck
1871.53	92.6	4.27	8	Dembowski	1878.72	71.9	3.13	4	Goldney
1871.55	96.7	4.36	1	Talmage	1879.41	69.2	2.84	18	Cincinnati
1871.59	94.9	4.30	3	Knott	1879.50	69.8	2.84	10	Schiaparelli
1871.64	92.7	4.29	3	Gledhill	1879.59	71.3	2.93	5	Hall
1871.72	92.6	4.20	1	Dunér	1879.64	67.9	2.94	5	Cincinnati
1872.47	91.8	4.19	2	Brünnow	1879.65	70.3	3.04	4	Seabroke
1872.49	91.5	4.30	3	Ferrari	1879.66	68.6	3.01	5	Jedrzejewicz
1872.49	90.8	4.28	2	Radcliffe	1880.47	65.8	2.44	3	Doberck
1872.49	90.7	4.04	9	Dembowski	1880.49	62.1	2.69	6	Franz
1872.51	91.5	4.29	3	W. & S.	1880.57	65.5	2.75	6	Hall
1872.60	93.6	4.08	4	O. Struve	1880.66	64.9	2.69	10	Schiaparelli
1873.51	89.5	3.90	1	Gledhill	1880.66	62.8	2.75	6	Jedrzejewicz
1873.51	88.8	3.89	8	Dembowski	1880.74	62.7	2.55	2	Seabroke
1873.51	88.8	4.10	1	W. & S.	1881.23	61.7	2.80	2	Doberck
1873.55	84.7	3.95	1	Talmage	1881.53	60.6	2.49	5	Hall
1873.71	88.8	4.22	3	Radcliffe	1881.72	56.3	2.45	2	Bigourdan
1874.48	88.8	4.01	4	Radcliffe	1881.77	62.7	2.45	2	Seabroke
1874.57	86.1	3.66	8	Dembowski	1882.49	52.3	2.92	1	Wilson
1874.58	88.6	3.67	1	Talmage	1882.52	55.7	2.29	2	Doberck
1874.69	87.5	3.79	3	O. Struve	1882.57	56.1	2.31	7	Hall
1874.73	87.5	3.92	1	Gledhill	1882.61	51.8	2.33	9	Schiaparelli
					1882.62	48.8	2.25	4	Jedrzejewicz
					1882.69	51.2	2.96	3	Seabroke
					1882.72	51.6	2.31	4	Englemann

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1883.49	45.6	2.28	4	Perrotin	1890.42	338.5	2.40	2	Glasenapp
1883.58	40.0	2.36	8	Seagrave	1890.49	338.3	2.42	8	Giacomelli
1883.62	43.7	2.21	15	Schiaparelli	1890.56	335.8	2.13	7	Hall
1883.64	42.2	2.22	6	Jedrzejewicz	1890.61	336.5	2.01	3	Maw
1883.68	45.2	2.51	3	Küstner	1890.61	336.6	2.16	1	Wellmann
1883.68	44.0	2.30	3	Seabroke	1890.70	334.8	2.02	6	Schur
1883.72	43.6	2.25	6	Englemann	1890.70	334.9	2.22	16	Bigourdan
					1890.73	336.1	2.15	9	Schiaparelli
1884.41	37.6	2.30	1	Wilson					
1884.53	35.9	2.18	1	Pritchett	1891.54	328.3	2.11	4	Maw
1884.56	34.5	2.09	6	Perrotin	1891.56	327.5	2.23	6	Hall
1884.59	37.6	2.16	7	Hall	1891.58	329.1	2.16	6	Schur
1884.62	35.3	2.07	8	Schiaparelli	1891.59	326.0	2.33	6	Knorre
1884.69	35.2	2.20	5	Englemann	1891.60	328.5	2.15	6	Schiaparelli
1884.70	34.8	2.45	3-1	Seabroke	1891.63	327.2	2.37	2	See
					1891.65	326.7	2.21	9	Bigourdan
1885.50	26.0	2.08	4	Perrotin					
1885.55	25.1	1.97	4-2	Sea. & Sm.	1892.37	321.9	2.28	4	Burnham
1885.57	29.5	1.88	7	Hall	1892.41	320.5	2.36	1	Collins
1885.64	24.3	2.07	8	Englemann	1892.49	321.7	2.26	3	Maw
1885.65	26.5	2.07	2	Schiaparelli	1892.57	321.3	2.19	4	Comstock
1885.71	23.4	2.19	5	Jedrzejewicz	1892.62	319.3	2.25	5	Bigourdan
					1892.64	321.0	2.24	6	Schur
1886.53	13.8	1.98	7	Hall	1892.65	320.3	2.22	17	Schiaparelli
1886.56	15.3	1.97	7	Perrotin					
1886.66	13.7	2.01	7	Jedrzejewicz	1893.47	313.8	2.22	3	Maw
1886.66	14.1	1.81	14	Schiaparelli	1893.58	313.4	2.41	3	Tucker
1886.67	14.8	1.88	7	Englemann	1893.62	313.6	2.27	4	Schur
1886.67	15.6	2.01	4-2	Smith	1893.62	312.5	2.34	5	Comstock
					1893.69	309.2	2.22	1	H. C. Wilson
1887.55	359.6	—	1	Smith	1893.70	312.3	2.21	11	Schiaparelli
1887.61	3.6	1.92	6	Hall					
1887.63	4.3	1.87	18	Schiaparelli	1894.50	309.8	2.47	8	Ebell
1887.81	3.5	1.91	4	Tarrant	1894.54	307.4	2.29	3	Maw
					1894.59	304.6	2.38	12-11	Knorre
1888.41	352.7	2.07	3	Comstock	1894.60	306.3	2.26	4	Schur
1888.55	354.5	2.17	4	Maw	1894.75	302.5	2.30	4	Comstock
1888.57	353.4	2.02	6	Hall	1894.77	301.3	2.45	5-6	Callandreaux
1888.62	355.4	2.00	3	Giacomelli	1894.77	303.2	2.21	6	Schiaparelli
1888.64	355.1	1.88	10-9	Schiaparelli	1894.79	302.5	2.33	5	Bigourdan
1888.65	352.4	2.14	1	Leavenworth					
1888.66	354.7	2.66	3	Copeland	1895.32	298.6	2.22	3	See
1888.85	353.1	1.92	6	Tarrant	1895.50	298.2	2.53	2	Glasenapp
					1895.51	301.6	2.31	5	Schur
1889.30	348.7	2.16	2	Burnham	1895.55	298.7	2.14	9	Schiaparelli
1889.48	344.9	1.60	2	Hodges	1895.58	296.9	2.26	4	Maw
1889.50	345.7	2.18	5	Comstock	1895.60	297.0	2.35	4	Schwarzschild
1889.57	344.5	2.10	6	Hall	1895.62	295.0	2.24	5	Hough
1889.64	346.4	1.96	5	Maw	1895.70	296.0	2.01	5	See
1889.70	344.9	1.99	17-16	Schiaparelli	1895.72	296.3	2.01	3-1	Moulton
1889.77	343.6	1.84	4	Schur					

*Researches on the Orbit of 70 Ophiuchi, and on a Periodic Perturbation in the Motion of the System Arising from the Action of an Unseen Body.**

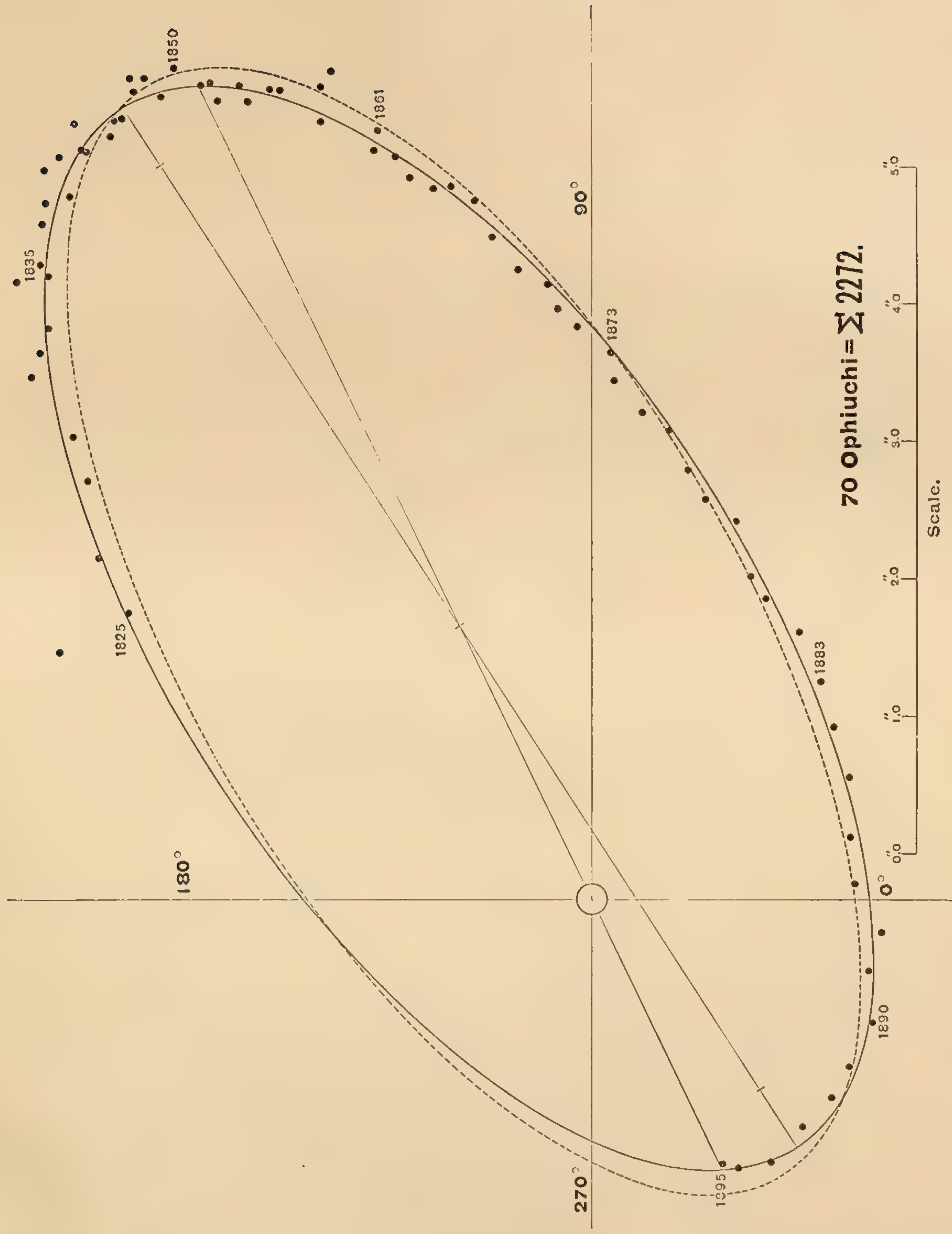
While engaged recently in the observation of double stars at the Leander McCormick Observatory of the University of Virginia, I took occasion to measure 70 *Ophiuchi* on three good nights (*A. J.* 349). On comparing the results with SCHUR's ephemeris, four months later, I noticed with surprise that the observed angle was over four degrees in advance of the theoretical place. As the Virginia measures had been made under favorable conditions and with extreme care, it became evident that even the orbit to which PROFESSOR SCHUR had devoted so much attention would need revision. Accordingly, after all the observations had been collected from original sources and tabulated in chronological order, I proceeded to investigate the orbit in the usual manner, and obtained a set of elements very similar to those which BURNHAM has given in *Astronomy and Astrophysics* for June, 1893. On comparing the computed with the observed places there appeared to be a sensible irregularity in the angular motion; and as the observed places were admittedly exact to a very high degree, it was impossible to attribute such large and continued deviations to errors of observation. It was also observed that the sign of $\theta_o - \theta_c$ showed a peculiar periodicity; the residuals being for many years steadily of one sign, and then as uniformly of the other. After making some unsuccessful efforts to correct the apparent orbit, from which the elements had been derived by the method of KLINKERFUES, I decided to project the orbit found by SCHUR, so as to compare his apparent ellipse directly with the places given by the mean observations for each year. Though I was aware that SCHUR's orbit had been based wholly on angles of position, I was not a little surprised to find that the distances had been vitiated in the remarkable periodic manner indicated by the pointed ellipse in the accompanying diagram. And since I had uniformly adhered to the use of both angles and distances in deriving the orbits of double stars, it was not allowable to violate the distances as PROFESSOR SCHUR had done, nor could we pass over such remarkable periodic errors in the residuals of the angles. We were thus confronted with a case in which it was apparently impossible to satisfy both angles and distances. A closer examination of the diagram suggested the idea of a periodic perturbation, alternately in angle and then in distance; and the drawing, in conjunction with the computations, enabled me to see that the case is one worthy of special attention. After some delay (*A. J.* 358) the additional observations

* *Astronomical Journal*, 363.

placed at my disposal by PROFESSORS HOUGH and COMSTOCK, in conjunction with the independent measures made at Madison by Mr. MOULTON and myself (*A. J.* 359) confirmed the correctness of the Virginia measures, and left no doubt of the rapid deviation of the companion from SCHUR's orbit. Before considering the physical cause of this unexpected phenomenon, I desire to remark that, in the preparation of this paper, my friend Mr. ERIC DOOLITTLE, C. E., has rendered valuable assistance. He has carried out the calculations entrusted to him not only with care and accuracy, but also with zeal and enthusiasm, and has, therefore, contributed in no small degree to the early completion of this investigation.

Since SIR WILLIAM HERSCHEL's discovery of this beautiful system the companion has described considerably more than one revolution. More orbits have been computed for this binary than for any other in the northern sky, but, in spite of the immense labor which astronomers have bestowed upon this star, the motion has proved to be so refractory and so anomalous that the companion has departed from every orbit heretofore obtained. It follows from the phenomena disclosed in this paper that the system contains a dark body, and that no satisfactory orbit can be obtained until this disturbing cause is taken into account. The following list of the orbits found by previous investigators will be of interest to astronomers; in most cases the data have been taken from original sources, but in a few instances we have relied upon the table of elements given by GORE in his useful "Catalogue of Binary Stars for which Orbits have been Computed."

<i>P</i>	<i>T</i>	<i>e</i>	<i>a</i>	Ω	<i>i</i>	λ	Authority	Source
^{yrs.} 73.862	1806.88	0.430	4.3284	147.2	46.42	283.1	Eneke, 1829	B.J., 1832
79.091	1814.155	0.34737	5.554	128.15	64.2	259.4	Eneke, 1830	B.J., 1832, p. 295
80.34	1807.06	0.4667	4.392	137.03	48.1	145.77	Herschel, 1833	Mem. R.A.S., vol. V, p. 217
80.61	1806.746	0.47715	4.3159	133.8	42.87	287.23	Mädler, 1835	A.N., 289
92.869	1812.73	0.4438	5.316	126.9	64.86	279.8	Mädler, 1842	A.N., 444; Dorp.Obs., IX, 185
87.52	1807.60	0.482	4.675	128.55	51.5	293.3	Jacob	
88.48	1807.48	0.4973	—	122.23	47.33	294.1	Hind, 1849	M.N., IX, p. 145
92.338	1810.671	0.4445	4.966	127.35	61.05	212.97	Villargeau, 1851	C.R., XXXII, p. 51
98.146	1806.92	0.546	4.48	111.7	49.93	187.5	Powell, 1855	M.N., XV, p. 42
93.10	1808.12	0.4894	—	124.53	55.27	159.53	Jacob, 1857	A.N., 1082
95.966	1808.27	0.4935	4.731	123.13	57.35	160.53	Klinkerf., 1858	A.N., 1135
94.37	1808.79	0.49149	4.704	125.4	57.9	155.7	Schur, 1868	A.N., 1682
92.77	1807.9	0.3859	4.88	122.0	62.0	163.0	Flammarion 1874	C.R., LXXXIX, p. 1248
94.93	1809.64	0.47286	4.770	127.37	60.0	149.72	Tisserand, 1876	Flam. Cat. Ét. Doub., p. 166
94.44	1808.90	0.4672	4.790	127.38	58.08	151.92	Pritchard, 1878	Oxf. Obs., I, p. 63
87.84	1807.65	0.4912	4.50	120.08	58.47	171.75	Gore, 1888	M.N., XLVIII, No. 5
88.04	1895.28	0.4994	4.45	120.8	57.0	174.92	Mann, 1890	Sid. Mes., Nov., 1890
88.3954	1808.0707	0.4751	4.60	121.31	60.08	168.3	Schur, 1893	A.N., 3220-21 [1893
87.75	1895.6	0.50	4.56	123.5	58.3	190.8	Burnham 1893	Astron. and Astroph., June,
87.70	1895.58	0.500	4.548	125.7	58.42	198.25	See 1895	A.J., 363



An inspection of this table discloses the fact that the early investigations, so far as they are reliable, led to periods sensibly less than 90 years, while the determinations made between 1845 and 1880, or, when the companion was describing the apastron of the real ellipse, favored a period of at least 94 years. Thus TISSERAND and PRITCHARD, so lately as 1876 and 1878, find periods of 94.93 and 94.44 years, respectively. In 1868 SCHUR obtained a period of 94.37 years, and similar periods before and since have been deduced by other trustworthy computers.

There is thus unmistakable evidence of a retardation in the motion of the companion near apastron; more recently this inequality has become an acceleration. It was observed by GORE in 1888 that the old orbits did not represent recent measures satisfactorily, and, accordingly, he derived a new set of elements with a period of 87.84 years, which was substantially confirmed by subsequent work of MANN and BURNHAM. Finally PROFESSOR SCHUR made an exhaustive investigation of all the observations up to 1893, and adjusted his orbit by the method of least squares to about 400 mean observations of position-angle. He says that in this work he could not advantageously employ the measures of distance, owing to the differences of the individual observers. The angles, however, were admitted to be admirably adapted to a fine determination of the elements, and, accordingly, PROFESSOR SCHUR's able discussion of 400 observations inspired the belief that his orbit would give good places of the companion for a great many years, if not for an almost indefinite period. But this just expectation has not been realized, owing to the action of an unseen body which disturbs the elliptical motion of the companion. To establish the existence and general character of the perturbations thus disclosed we submit the following considerations:

(1) A reference to PROFESSOR SCHUR's able and exhaustive paper in the *Astronomische Nachrichten*, No. 3220, 21, will enable the reader to judge of the improbability of an orbit based on such a multitude of good measures proving to be defective within two years of its completion, unless disturbing causes were at work to produce the sudden acceleration in angular motion. It is inconceivable that this rapid deviation could take place without a true physical cause. The error in the angle now amounts to about five degrees.

(2) In regard to the older observations we may remark, as PROFESSOR SCHUR and others before him have done, that SIR WILLIAM HERSCHEL's angles are open to some uncertainty, owing to a possible error in the reading or in the records; so that his observations do not give an exact or trustworthy criterion for the period. HERSCHEL says, however, explicitly, that on "Oct. 7,

1779, the stars were exactly in the parallel, the following star being the largest;" and, as it does not seem that any sensible error could affect the angle which he has thus recorded, we see from the measures in 1872-3 that the resulting period would be approximately 92 years. This is an additional indication that the period of this star is not constant. A careful examination of the other early measures shows that the first really good position is that of STRUVE in 1825. These measures are so uniform and consistent, and appear in every way so worthy of entire confidence, that I quote the record from the *Mensurae Micrometricae* in full:

t	θ_o	ρ_o		t	θ_o	ρ_o	
1825.42	150.1	3.89	4, 6	1825.61	149.3	4.05	
1825.43	147.0	4.05		1825.62	146.8	3.92	
1825.44	149.1	3.94		1825.63	147.3	3.85	
1825.48	148.8	4.05		1825.63	148.4	3.99	
1825.50	146.4	4.21		1825.64	147.0	4.01	
1825.60	148.1	3.90		1825.66	148.5	4.01	4, 6
1825.60	149.5	3.85		1825.71	148.8	4.02	
				Mean	1825.56	148.2	3.98

An examination of these separate measures clearly indicates that the error in the mean result does not surpass $0^\circ.5$ in angle, and $0''.1$ in distance. By SCHUR's orbit the angle is corrected two degrees, and when the radius vector is thus thrown forward to $146^\circ.2$ the computed and observed distances are nearly identical. As STRUVE took special pains to secure good measures on a large number of nights, and obtained the foregoing beautiful and consistent results, we may regard his mean position as one of the highest precision. The probable error of such measures would evidently be very small.

(3) We see from the diagram illustrating the apparent ellipse that SCHUR's orbit falls within the positions given by the measures prior to 1845; so that nearly all the observations of STRUVE, BESSEL, DAWES, MÄDLER, etc., require a sensible negative correction in distance. In figure *B* the differences $\rho_o - \rho_e$ of the individual measures used by SCHUR are plotted to scale, and a glance at the figure will show the improbability of such classic observers as STRUVE, BESSEL and DAWES making the constant errors here indicated. It would be still more remarkable if the observers between 1845 and 1870 have as uniformly erred in the opposite direction. How has it happened that from 1825 to 1845 the distances were steadily over-measured by the best observers, while during the next period the distances were constantly under-measured? Individual observers have what may be called a personal equation (though this is far from constant and is difficult to determine with any certainty) but it

could not happen that all the best observers would err alike, although in opposite directions, during the two periods. PROFESSOR SCHUR's corrections are evidently inadmissible.

(4) The peculiar periodic manner in which SCHUR's apparent ellipse crosses and re-crosses the general path which best represents the mean positions, first suggested to my mind the hypothesis of a disturbing body. Figure *C* is based upon these mean positions, and a comparison with the curve in *B* shows that the mean positions are typical of all the observations for any given year. Since I was desirous of avoiding any possible prejudice of the material used, I have retained, without alteration, the mean positions which had been formed in August before suspecting the existence of a disturbing influence.

(5) We suggest that the companion of 70 *Ophiuchi* is attended by a dark satellite, and that the visible companion, therefore, moves in a sinuous curve about the common centre of gravity of the new system, with a period somewhat less than 40 years, and in a retrograde direction. As SCHUR's orbit is based on a least-square adjustment of all the observations extending over two entire revolutions of the invisible body, it may reasonably be inferred that his apparent ellipse will represent very nearly the true motion of the centre of gravity, while the apparent ellipse which best represents the observed distances will give a general outline of the path of the visible star in its sinuous motion. Let us recur to the diagram of the apparent ellipse and imagine that the visible companion and the centre of gravity are in the tangent to the ellipse at the epoch of intersection in 1818. Then, the motion of the visible star being retrograde, we perceive that it will gain steadily on the centre of gravity, and, in 1836, the two will be in line with the original position, after half a sidereal revolution; from 1836 to 1845 the satellite will make another quarter revolution, and again the bright companion will be in the tangent to the apparent ellipse and in advance of the common centre of gravity. As the visible star will now steadily fall behind in its retrograde motion about the centre of gravity, it is clear that from 1845 to 1872, which is three-fourths of a revolution, the motion of the bright body *will appear to be abnormally slow*. This is the apparent retardation previously mentioned as giving rise to the long periods found by computers who used observations extending over the apastron portion of the real orbit. Assuming that the motion is undisturbed, and hence that the areas are constant, PROFESSOR SCHUR was compelled to run his ellipse further out in this part of the orbit in order to represent the observed angles. From the diagram we see that the retrograde motion of the visible star continues after 1872, and, as this apparently accelerates the visible

motion of the companion relative to the central star, SCHUR's ellipse is drawn inside of most of the observations of this period. The falling of the measured distances beyond SCHUR's orbit shows plainly the periodic motion of the visible star in accordance with the above theory. From this sketch of the effects of the disturbing body it is evident that, at the time SCHUR completed his orbit, the visible star and the unseen body were nearly in line with the central star. And since the visible companion in 1825, according to STRUVE, had an angle of $148^{\circ}.2$, whereas SCHUR makes it $146^{\circ}.2$, or, substantially the same as the centre of gravity at that epoch, it follows that our hypothesis, making SCHUR's orbit represent the motion of the centre of gravity, is indeed very nearly correct. Any slight correction that may be required for the periastron of SCHUR's ellipse in order to make it represent the true path of the centre of gravity, had better be deferred until additional observations disclose more clearly the nature and extent of the perturbations.

(6) We may fix the approximate elements of the visible companion about the centre of gravity as follows: From 1818 to 1890, or 72 years, is the time required for two revolutions, as explained in the preceding paragraph, and hence we see that *the period is approximately thirty-six years*. The motion is retrograde, and from the diagram of the apparent orbit, we may conclude that the distance of the visible star from the common centre of gravity is about $0''.3$. It is natural to suppose that the plane of the orbit is not greatly inclined to that found by SCHUR, but existing data will not fix all the elements with the desired precision. Perhaps until the path of the centre of gravity is known with great accuracy, the simple hypothesis of a circular orbit, with node and inclination identical with the similar elements of the visible pair, will be sufficient to explain phenomena, and it follows that *both angles and distances are comparatively well represented by this hypothesis*.

It is found, however, on more detailed examination that the representation can be somewhat improved by the adoption of the following elements:

$P' = 36$ years	$\Omega' = 151^{\circ}.0$
$T' = 1822.0$	$i' = 60^{\circ}.1$
$e' = 0.475$	$\lambda' = 191^{\circ}.7$
$a' = 0''.30$	$n' = 10^{\circ}.0$

While this orbit gives a good representation of the motion of the bright body about the common centre of gravity, the data are so rough that the determination of such delicate elements must be regarded as provisional only.

In the following table we have compared SCHUR's elements with the mean

positions for each year; the residuals are given in the columns headed $\theta_0 - \theta_1$ and $\rho_0 - \rho_1$. It is at once evident that the angles are beautifully represented down to 1893, after which the error in angle rapidly accumulates until it now amounts to nearly *five degrees*! The errors in distance are illustrated in diagram *C*, which shows the same general features as diagram *B*, where the points represent the individual measures employed by SCHUR.

The elements of the orbit which best represents the observed distances are as follows:

$$\left. \begin{aligned} P &= 88.3954 \text{ years} \\ T &= 1808.0707 \\ e &= 0.500 \\ a &= 4''.548 \end{aligned} \right\} \text{SCHUR's values} \quad \begin{aligned} \Omega &= 125^\circ.7 \\ i &= 58^\circ.42 \\ \lambda &= 198^\circ.25 \\ n &= -4^\circ.0728 \end{aligned}$$

Apparent orbit:

$$\begin{aligned} \text{Length of major axis} &= 9''.00 \\ \text{Length of minor axis} &= 4''.17 \\ \text{Angle of major axis} &= 122^\circ.9 \\ \text{Angle of periastron} &= 295^\circ.8 \\ \text{Distance of star from centre} &= 2''.198 \end{aligned}$$

COMPARISON OF COMPUTED WITH OBSERVED PLACES ACCORDING TO THE TWO SETS OF ELEMENTS.

t	θ_0	ρ_0	$\theta_0 - \theta_1$	$\rho_0 - \rho_1$	$\theta_0 - \theta_2$	$\rho_0 - \rho_2$	$d\theta''$	n	Observers.
1779.77	90.0	—	—8.8	—	—8.11	—	—0.708	1	Herschel
1781.74	99.2	4.49	+1.6	—0.27	+4.40	—0.11	+0.359	1-2	Herschel
1802.34	336.1	—	+1.4	—	+0.71	—	+0.027	1	Herschel
1804.42	318.8	—	—0.3	—	—3.15	—	—0.128	2	Herschel
1819.64	168.5	—	—0.9	—	+5.08	—	—0.244	5	Struve
1820.77	160.2	—	—2.7	—	+0.78	—	+0.042	2	Struve
1821.74	157.6	—	—0.8	—	+2.65	—	+0.154	5	Struve
1822.42	154.8	4.27	—0.8	+1.12	+2.04	+0.72	+0.126	2	Herschel and South
1822.64	153.9	—	—1.0	—	+1.75	—	+0.109	3	Struve
1825.56	148.2	3.98	+2.0	+0.03	+3.17	—0.37	+0.238	28-14	South 14-0; Struve 14
1827.02	145.1	4.37	+2.1	+0.07	+2.82	—0.28	+0.227	2	Struve
1828.71	140.2	4.78	+0.3	+0.10	+0.72	—0.22	+0.062	4	Struve
1829.59	138.1	5.08	—0.3	+0.23	+0.39	—0.15	+0.035	6	Struve
1830.57	136.8	5.58	0.0	+0.54	—0.02	+0.24	—0.002	27	H. 9; Bessel 10; Dawes 6; W. Struve 2
1831.58	135.1	5.68	—0.3	+0.45	—0.38	+0.36	+0.036	20-18	H. 8-6; Bessel 7; W. Struve 5
1832.62	133.4	5.75	—0.6	+0.35	—0.84	+0.08	—0.083	8	Dawes 3; Bessel 5
1833.42	132.8	6.14	—0.2	+0.61	—0.34	+0.34	—0.034	1	Dawes
1834.55	130.8	6.04	—0.8	+0.34	—1.25	+0.12	—0.128	18	W. Struve 4; Dawes 7; Bessel 7
1835.60	130.7	6.11	+0.2	+0.26	—0.22	+0.07	—0.023	5	Struve
1836.52	128.9	6.34	—0.5	+0.38	—1.08	+0.20	—0.115	25	Mädler 8; Encke 4; Bessel 5; W. Struve 8
1837.64	127.9	6.45	—0.4	+0.35	—0.96	+0.21	—0.104	27	Dawes 3; Encke 4; Bessel 16; W. Struve 4
1838.59	126.6	6.64	—0.7	+0.43	—1.38	+0.32	—0.152	7	Galle
1839.58	125.5	6.66	—0.9	+0.36	—1.51	+0.25	—0.169	4	Galle 2; Dawes 2
1840.47	126.5	6.38	+1.0	0.00	+0.33	—0.09	+0.037	14+	Kaiser—; O. Struve 10; Dawes 4
1841.64	124.0	6.60	—0.5	+0.14	—1.13	+0.10	—0.129	26	Mädler 8; Kaiser 5; Dawes 4; Be. and Schl. 7
1842.57	123.8	6.57	+0.1	+0.04	—0.50	—0.01	—0.057	35	O. Struve 8; Mädler 3; Dawes 2; Kaiser 22
1842.60	123.5	6.79	—0.1	+0.26	—0.78	+0.20	—0.090	20	Schlüter
1843.55	122.1	6.57	—0.7	—0.02	—1.26	—0.06	—0.146	20-19	Dawes 1-0; Encke 3; Mädler 16
1844.44	121.3	6.66	—0.7	+0.03	—1.36	+0.01	—0.158	10	Encke 5; Mädler 5

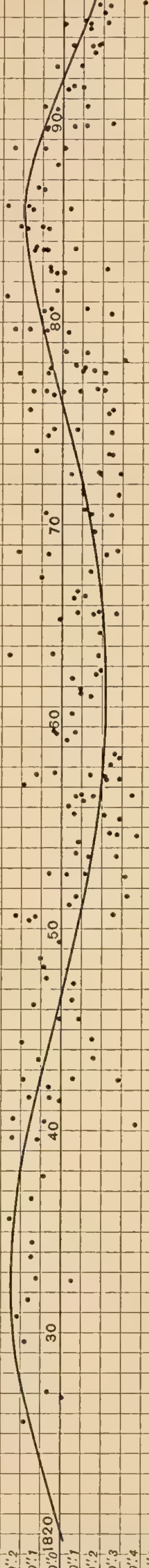
t	θ_0	ρ_0	$\theta_0 - \theta_1$	$\rho_0 - \rho_1$	$\theta_0 - \theta_2$	$\rho_0 - \rho_2$	$d\theta''$	n	Observers
1845.48	120.9	6.64	-0.3	-0.03	-0.86	-0.03	-0.101	30	Hind 9; O. Struve 5; Mädler 16
1846.46	119.3	6.76	-0.7	+0.06	-1.63	+0.07	-0.190	18+	Jacob 1; Hind 7; Dur. Obs. —; Mädler 10
1847.47	119.1	6.85	-0.3	+0.14	-0.96	+0.16	-0.112	13+	O. Struve 4; Dur. Obs. —; Mitchell 1; Mä. 8
1848.38	118.4	6.81	-0.3	+0.09	-0.96	+0.13	-0.112	9	Dawes 3; Mädler 4; Bond 2
1849.39	118.1	6.64	+0.3	-0.09	-0.31	-0.03	-0.036	5	O. Struve
1850.55	116.4	6.78	-0.5	+0.07	-1.01	+0.13	-0.118	18	Rad. 8; W. & J. 2; Mädler 4; Fletcher 4
1851.54	115.5	6.57	-0.5	-0.13	-1.04	-0.04	-0.121	24	Mädler 4; Fletcher 8; O. Struve 5; Mädler 7
1852.69	114.9	6.56	-0.2	-0.11	-0.63	0.00	-0.073	37	Fletcher 6; O. Struve 5; Mädler 11; Jacob 15
1853.57	114.9	6.42	+0.6	-0.22	+0.12	-0.11	+0.014	21-12	Powell 9-0; Dem. 6; Dawes 6 [Po. 3-0
1854.48	113.2	6.37	-0.3	-0.22	-0.71	-0.11	-0.081	57-54	Ja. 21; Ja. 2; OΣ. 6; Dem. 12; Mä. 10; Da. 3;
1855.52	113.3	6.45	+0.6	-0.09	+0.35	+0.04	+0.039	20-13	Lu. 2; Sr. 3; Winn. 1; Mä. 5; Da. 2; Po. 7-0
1856.43	111.7	6.34	+0.1	-0.14	-0.41	0.00	-0.046	32	OΣ. 5; Ja. 7; Mä. 3; Winn. 8; Sec. 3; Dem. 6
1857.52	111.0	6.31	+0.2	-0.10	-0.04	+0.05	-0.005	20	Ja. 3; Winn. 1; Sec. 4; Da. 2; Dem. 4; Mä. 2;
1858.39	109.1	6.01	-0.9	-0.32	-1.07	-0.16	-0.116	18	Ja. 3; Mo. 2; Dem. 4; Mä. 9 [OΣ. 4
1859.66	108.4	6.24	-0.4	+0.02	-0.45	+0.18	-0.048	20	OΣ. 5; Dawes 4; Auwers 5; Powell 5; Mä. 1
1860.70	107.3	6.33	-0.4	+0.21	-0.29	+0.40	-0.030	8+	Secchi 3; Luther —; Auwers 5
1861.67	106.2	5.70	-0.5	-0.31	-0.50	-0.14	-0.052	17	Rad. 1; Mädler 7; Auwers 6; Powell 3
1862.59	105.6	5.83	-0.2	-0.07	-0.02	+0.09	-0.002	19	O. Struve 3; Winnecke 1; Dem. 9; Mädler 6
1863.54	104.8	5.62	0.0	-0.17	+0.19	+0.01	+0.019	29	Adh. 11; Sec. 2; Dem. 9; Ta. 1; Fer. 1; Hl. 5
1864.54	104.1	5.43	+0.5	-0.23	+0.84	-0.06	+0.082	13	Englemann 2; Dembowski 11
1865.50	102.4	5.32	0.0	-0.20	+0.22	-0.03	+0.021	43	En. 8; Secchi 4; Dem. 9; Ta. 2; Kaiser 20
1866.43	101.2	5.31	0.0	-0.07	+0.48	+0.11	+0.044	25	Dem. 8; OΣ. 5; Ta. 5; Hv. 4; Secchi 3
1867.50	99.6	5.18	-0.2	-0.03	+0.43	+0.14	+0.038	14-13	Rad. 1; Kn. 2; Ta. 1-0; Dem. 7; Hv. 3
1868.65	98.6	4.90	+0.5	-0.13	+1.26	+0.05	+0.101	22	Dem. 7; Kn. 2; Rad. 2; Du. 4; OΣ. 2; Brw. 5
1869.80	96.7	4.64	+0.4	-0.19	+1.32	-0.03	+0.109	11	Dunér 3; Dembowski 8
1870.51	94.2	4.52	-1.0	-0.18	-0.40	-0.08	-0.032	12	Gledhill 2; Dem. 8; Ta. 2; [Gl. 3; Du. 1
1871.56	93.4	4.34	+0.1	-0.16	+1.27	-0.03	+0.099	22	W. & S. 2; Rad. 2; Pei. 2; Dem. 8; Ta. 1; Kn. 3;
1872.51	91.6	4.20	+0.2	-0.13	+1.41	-0.01	+0.105	23	Brw. 2; Fer. 3; Rad. 2; Dem. 9; W. & S. 3; OΣ. 4
1873.56	88.1	4.01	-0.1	-0.09	+0.43	0.00	+0.031	14	Gl. 1; Dem. 8; W. & S. 1; Ta. 1; Rad. 3
1874.61	87.7	3.81	+1.1	-0.09	+2.71	+0.01	+0.183	17	Rad. 4; Dem. 8; Ta. 1; OΣ. 3; Gledhill 1
1875.61	84.2	3.59	+0.1	-0.10	+1.74	-0.04	+0.113	21	Dem. 9; Sch. 8; Rad. 4 [Jed. 4; Wdo. 1
1876.57	80.7	3.48	-0.6	0.00	+1.53	+0.07	+0.093	31	Sh. 5; Dk. 2; Dem. 7; Pl. 3; Sch. 6; Hall 3;
1877.60	77.4	3.23	-0.5	-0.05	+1.83	+0.02	+0.104	50	Dem. 8; Dk. 2; Hl. 4; Jed. 10; Pl. 8; Sch. 10;
1878.58	74.3	3.05	+0.1	-0.01	+2.49	+0.03	+0.134	18	Dem. 7; Sea. 3; Dk. 4; Gold. 4 [Cin. 4; Sh. 4
1879.57	69.5	2.95	-0.4	+0.09	+2.28	+0.12	+0.115	47	Cin. 18; Sch. 10; Hl. 5; Cin. 5; Sea. 4; Jed. 5
1880.59	64.0	2.64	-0.9	-0.01	+1.98	-0.01	+0.093	33	Dk. 3; Fr. 6; Hl. 6; Sch. 10; Jed. 6; Sea. 2
1881.56	60.3	2.55	+1.0	+0.08	+3.91	+0.06	+0.172	11	Doberck 2; Hall 5; Big. 2; Sea. 2 [En. 4
1882.60	52.5	2.48	+0.2	+0.20	+3.35	+0.16	+0.137	30	H.C.W. 1; Dk. 2; Hl. 7; Sch. 9; Jed. 4; Sea. 3;
1883.62	44.0	2.31	-0.3	+0.18	+2.42	+0.11	+0.094	45	Per. 4; Seag. 8; Sch. 15; Jed. 6; Kü. 3; Sea. 3; En. 6
1884.56	36.0	2.17	+0.3	+0.16	+2.01	+0.07	+0.077	31-29	H.C.W. 1; Pr. 1; Per. 6; Hl. 7; Sch. 8; En. 5; Sea. 3-1
1885.61	25.9	2.06	+0.1	+0.13	+0.72	+0.02	+0.026	30-28	Per. 4; Sea. 4-2; Hl. 7; En. 8; Sch. 2; Jed. 5
1886.61	14.3	1.93	-0.9	+0.04	-0.89	-0.07	-0.031	46-44	Hl. 7; Per. 7; Jed. 7; Sch. 14; En. 7; Sm. 4-2
1887.68	3.8	1.91	-0.1	+0.01	-1.03	-0.09	-0.036	29-28	Sm. 1-0; Hl. 6; Sch. 18; Tar. 4 [Cop. 3; Tar. 6
1888.62	353.9	2.11	-0.4	+0.15	-2.10	+0.07	-0.075	36-35	Com. 3; Maw 4; Hl. 6; Giac. 3; Sch. 10-9; Lv. 1;
1889.53	345.9	2.08	+0.6	+0.06	-2.26	-0.01	-0.082	37-34	β. 2; Hod. 2-0; Com. 5; Hl. 6; Maw 5; Sch. 17-16
1890.57	336.7	2.21	+0.6	+0.08	-2.38	+0.04	-0.090	46	Glas. 2; Giac. 8; Hl. 7; Maw 3; Well. 1; Big. 16;
1891.59	327.4	2.23	-0.6	0.00	-3.58	-0.02	-0.141	33	Maw 4; Hl. 6; Knr. 6; Sch. 6; See 2; Big. 9 [Sch. 9
1892.52	320.8	2.26	-0.4	-0.04	-3.52	-0.05	-0.142	34	β. 4; Col. 1; Maw 3; Com. 4; Big. 5; Sch. 17
1893.62	312.9	2.25	-0.8	-0.15	-2.30	-0.10	-0.094	19-20	Maw 3; Com. 5; H.C.W. 0-1; Sch. 11
1894.69	304.2	2.30	-2.7	-0.14	-4.80	-0.03	-0.195	30-29	Maw 3; Knr. 12-11; Com. 4; Sch. 6; Big. 5
1895.32	298.6	2.22	-4.3	-0.21	-6.98	-0.09	-0.280	3	See
1895.64	296.1	2.14	-4.8	-0.28	-5.62	-0.12	-0.221	20-18	Maw 4; Com. 3; Ho. 5; See 5; Moulton 3 1

The values of P and T are taken from SCHUR's orbit, because the values of these elements derived from so many observations may be regarded as very nearly the mean of all the periods and epochs which result from the observa-

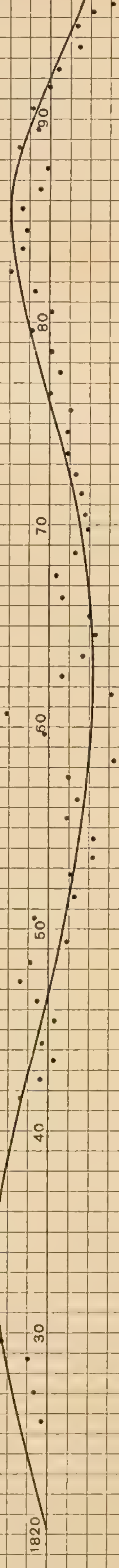


A - Perturbations in Angle

ρ_c, ρ_e
 Scale 0''0 1820
 0''5
 0''4
 0''3
 0''2
 0''1



B - Perturbations in Distance



C - Perturbations in Distance

tions prior to 1893. The residuals which follow from the use of these elements are given in the columns marked $\theta_0 - \theta_2$ and $\rho_0 - \rho_2$. In the case of the second elements the periodic errors in angle are very noticeable, but, as the simple differences $\theta_0 - \theta_2$ would not be strictly comparable at different distances, we have reduced all these angular displacements to seconds of the arc of a great circle by the formula

$$\frac{r''(\theta_0 - \theta_2)^\circ}{57^\circ.3} = d\theta''$$

where r'' denotes the apparent length of the radius vector in seconds of arc, and $(\theta_0 - \theta_2)^\circ$ the residuals of position-angle expressed in degrees. The displacement $d\theta''$ is tabulated and also illustrated graphically in diagram *A*. It will be seen that the maximum or minimum displacement in angle is practically identical in time with the zero of the curves of distance in *B* and *C*; and that the zero of the curve of angles corresponds to the maximum or minimum of the curve of distances. This displacement of phase would be a necessary consequence of the orbital motion of the visible companion about the common centre of gravity, and may be said to establish completely the reality of that phenomenon. The present theory does not require the several phases of the curves to be of equal length, since the tangent to the ellipse itself revolves very unequally in different parts of the orbit, and the zero of the curve of distance, for example, depends on the coincidence of this tangent with the line connecting the bright with the dark body.

(8) The problem here presented of finding the elements of the orbit of the visible companion from irregularities in the elliptical motion is very much more difficult than those arising from the irregular proper motions of perturbed stars, such as *Sirius* and *Procyon*. In the case of the phenomena first investigated by BESSEL, the centre of gravity of the system moves uniformly on the arc of a great circle; but in this case the centre of gravity moves on the arc of a very small ellipse and with a velocity which follows a very complex law. Indeed the velocity at any point of the orbit is inversely as the perpendicular from the central star to the tangent to the ellipse at the point in question; and, as the central star may in general occupy any point whatever of the apparent ellipse, we see that the velocity varies in an extremely complicated manner. In view of these facts it seems best, especially from the point of view of practical double-star work, to determine first of all the path of the centre of gravity and the elements of its orbit. Suppose we designate the rectangular coordinates of this centre, relative to the principal star, by x' , y' ; and the coordinates of the visible companion referred to the same origin by

x, y ; then if α and β denote the differences of these coordinates, the observations will furnish a series of equations of the form:

$$\begin{array}{ll} \alpha_1 = x_1' - x_1 & \beta_1 = y_1' - y_1 \\ \alpha_2 = x_2' - x_2 & \beta_2 = y_2' - y_2 \\ \alpha_3 = x_3' - x_3 & \beta_3 = y_3' - y_3 \\ \alpha_4 = x_4' - x_4 & \beta_4 = y_4' - y_4 \\ \alpha_5 = x_5' - x_5 & \beta_5 = y_5' - y_5 \\ \hline \alpha_n = x_n' - x_n & \beta_n = y_n' - y_n \end{array}$$

Five points, each determined by two such equations, are theoretically sufficient to fix the elements of the orbit of the visible star about the common centre of gravity; a larger number of equations, when combined in an advantageous manner, so as to render the errors of observation a minimum, will make the determination more exact, and define the elements with the desired precision. In the case of 70 *Ophiuchi*, SCHUR's orbit is to all appearances a good first approximation to the path of the centre of gravity, but it does not seem worth while to enter upon the more refined analysis here indicated until additional measures of the visible companion have confirmed the accuracy of this hypothesis. Apart from these theoretical difficulties, the sensible perturbations of the central star upon the motion of its attendant system will give rise to obstacles which are scarcely less formidable.

(9) While we have spoken of the dark body as attending the companion, it is clear that similar phenomena would result from the action of a body revolving round the central star. In this case, however, the considerable distance which would result from a period of 36 years might render the stability of the system somewhat precarious, especially if the orbit be eccentric like that of the visible companion. And as there is every reason to suppose that the system is the outgrowth of nebular condensation, and is, therefore, adjusted to conditions of stability and permanence, it is more natural to regard the companion as the binary. In this case the small mass might give rise to a period of 36 years even if the pair be very close. The separation of the new system is not likely to be less than $0''.4$, and it may be more than twice that distance. If we adopt the parallax of $0''.162$ found by KRUEGER it will follow that the major semi-axis of the orbit of the visible companion is 28.07 astronomical units, and the combined mass is 2.83 that of the sun; and hence we conclude that the orbit of the visible companion about the common centre of gravity has a major semi-axis of 1.84 astronomical units. Therefore, while the bright companion describes an eccentric orbit with a major axis which is slightly less than that of *Neptune*, the action of the dark body causes it to

describe another ellipse, which in size considerably surpasses that of the planet *Mars*.

(10) With regard to the position of the dark body we remark that an exact prediction is difficult, but the general indications are that at the epoch 1896.50 it lies approximately in the direction of 260° *. As the companion is now near periastron, the present is a favorable opportunity for searching for the dark body, since in this position the orbit will be expanded owing to the perturbations of the central star. In case it should be imagined that the unseen body attends the central star, it would be natural to locate it in the direction of 160° .

(11) Many years ago a disturbing body in the system of 70 *Ophiuchi* was suspected by MÄDLER, JACOB and SIR JOHN HERSCHEL, and on two occasions, more recently, BURNHAM has searched for it without success. After examining both stars with the Dearborn 18-inch refractor in 1878 he adds: "Both stars round;" while a still more critical search with the Lick 36-inch refractor led him to remark: "I could not see any third component and both stars appeared to be round, with all powers." In spite of this negative evidence, observers with great telescopes will find this system worthy of special examination. Whatever be the result of optical search for the unseen body, it will now become a matter of great interest to measure the visible companion with the most scrupulous care until the nature and extent of its perturbations are fully established.

99 HERCULIS = A.C. 15.

$\alpha = 18^h 3^m.2$; $\delta = +30^\circ 33'$.
6.0, yellow ; 11.7, purple.

Discovered by Alvan Clark, July 10, 1859.

OBSERVATIONS.									
t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1859.61	347.4	1.61	1	Dawes	1872.56	6.0	1.46	1	O. Struve
1859.65	347.0	1.80	1	Dawes					
1860.30	342.3	2.28	1	O. Struve	1877.56	22.0	1.19	1	O. Struve
1866.68	360.8	1.73	1	O. Struve	1878.46	24.4	1.09	3-1	Burnham
1868.50	358.6	1.69	1	O. Struve	1879.47	26.5	1.13	1	Burnham

* The estimated position given in *A. J.* 363 for 1895 was 330° ; the retrograde motion would diminish the angle considerably, but the principal change in the theoretical position results from the elements above referred to.

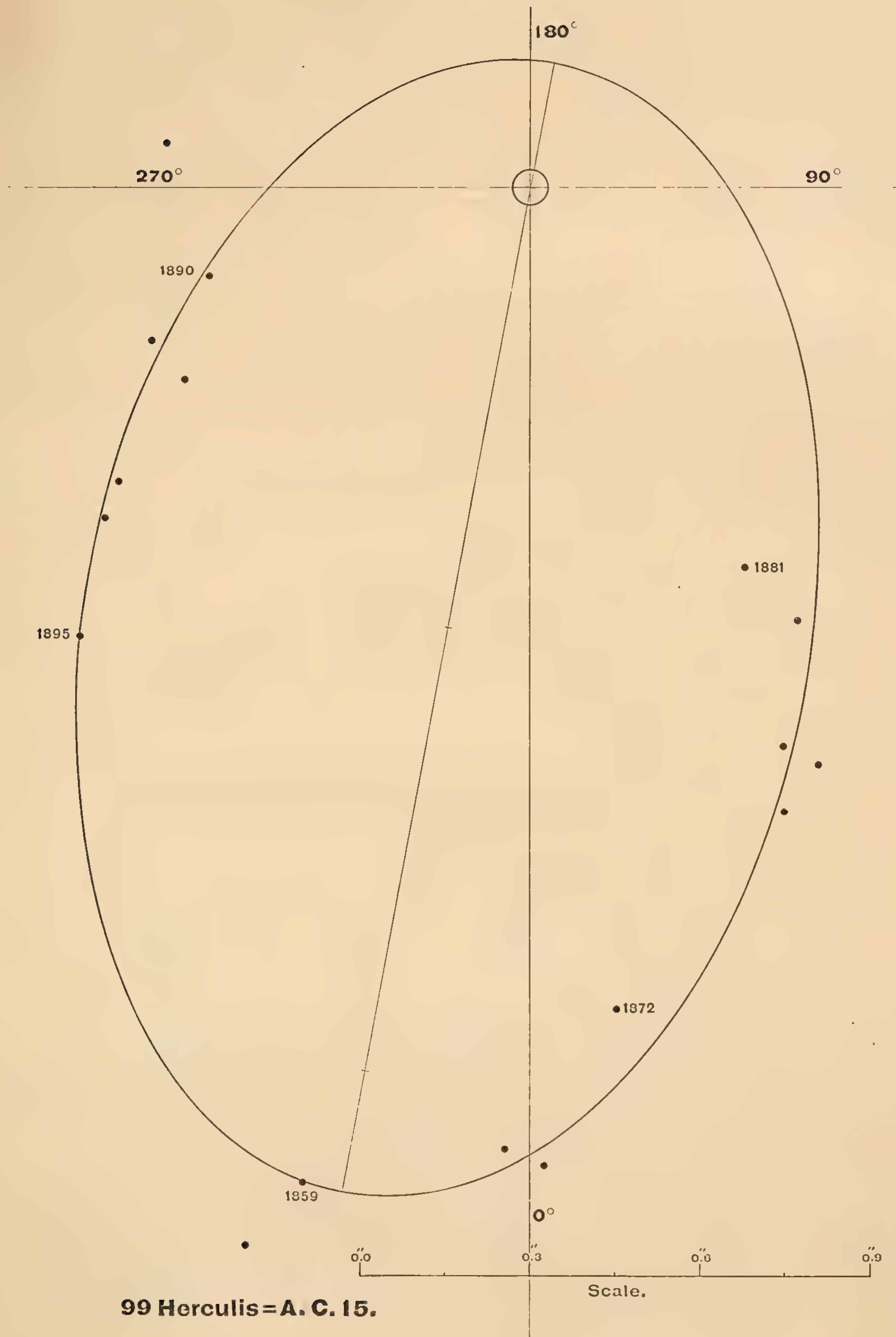
t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1880.53	31.6	0.90	2-1	Burnham	1891.56	292.0	0.72	2-3	Burnham
1881.43	29.4	0.51	1	Burnham	1892.40	299.2	0.70	3	Burnham
1883.60	72.9	1.30	1	O. Struve	1894.74	305.7	0.88	1	Comstock
1883.70	82.4	1.04	1	O. Struve	1895.47	309.5	1.04	6	Barnard
1888.54	77.4	1.05	1	O. Struve	1895.50	308.0	0.95	2	See
1889.50	281.2	0.65	1	Burnham	1895.73	315.2	1.12	3	See
1890.45	285.1	0.59	3-2	Burnham	1895.73	313.4	1.00	2-1	Moulton

This difficult double star was discovered by CLARK while testing the telescope he had just made for DAWES, at the latter's private observatory.* The physical connection of the pair was suspected, and during the same year two sets of good measures were obtained by DAWES. OTTO STRUVE began to give his attention to the pair the following year, and continued his measures from time to time until 1888. His first observations are very satisfactory, and of the highest value in fixing the elements of the orbit; but the later measures are less trustworthy, owing to the great inequality and closeness of the components. The series of measures begun by BURNHAM in 1878, and continued until the close of his work in California, is of great importance, and in conjunction with STRUVE's observations and those recently made by the writer at Madison, enables us to fix the elements with a relatively high degree of precision.

In order to obtain a good orbit from such measures, the means must be formed in a judicious manner, regard being had to the known motion of the companion. After careful study of all the observations, we have formed a suitable set of mean places, and deduced the corresponding elements. The orbits previously found for this system are:

GORE, 1890 <i>M.N.</i> , Nov. 1893	SEE, 1895 unpublished
$P = 53.55$ years	57.5 years
$T = 1885.58$	1887.30
$e = 0.7928$	0.806
$a = 1''.12$	1''.163
$\Omega = 50^\circ.1$	77°.0
$i = 38^\circ.6$	35°.5
$\lambda = 110^\circ.73$	90°.0

* *Astronomical Journal*, 366.



The adopted elements of 99 *Herculis* are as follows:

$$\begin{array}{ll}
 P = 54.5 \text{ years} & \Omega = \text{indeterminate} \\
 T = 1887.70 & i = 0^\circ.0 \\
 e = 0.781 & \text{Angle of periastron} = 169^\circ.5 \\
 a = 1''.014 & n = +6^\circ.6055
 \end{array}$$

The apparent is the same as the real orbit.

$$\begin{array}{ll}
 \text{Length of major axis} & = 2''.028 \\
 \text{Length of minor axis} & = 1''.278 \\
 \text{Angle of major axis and periastron} & = 169^\circ.5
 \end{array}$$

TABLE OF COMPUTED AND OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1859.65	347.0	348.4	1.80	1.81	- 1.4	-0.01	1	Dawes
1859.96	344.8	348.9	1.94	1.81	- 4.1	+0.13	2	Dawes 1; O. Struve 1
1866.68	360.8	357.8	1.73	1.74	+ 3.0	-0.01	1	O. Struve
1868.50	358.6	360.5	1.69	1.70	- 1.9	-0.01	1	O. Struve
1872.56	6.0	7.0	1.46	1.56	- 1.0	-0.10	1	O. Struve
1877.56	22.0	17.4	1.19	1.29	+ 4.6	-0.10	1	O. Struve
1878.46	24.4	20.1	1.04	1.21	+ 4.3	-0.17	3-1	Burnham
1879.47	26.5	23.2	1.13	1.14	+ 3.3	-0.01	1	Burnham
1880.53	31.6	27.3	0.90	1.04	+ 4.3	-0.14	2-1	Burnham
1881.43	29.4	31.0	0.77	0.96	- 1.6	-0.19	1-2	Burnham 1; O. Struve 0-1
1889.50	257.4	262.7	0.65	0.42	- 5.3	+0.23	1-1	Burnham 0-1; O. Struve 1-0
1890.45	285.1	280.5	0.59	0.56	+ 4.6	+0.03	3-2	Burnham
1891.56	292.0	292.9	0.72	0.71	- 0.9	+0.01	2-3	Burnham
1892.40	299.2	299.2	0.70	0.80	0.0	-0.10	3	Burnham
1894.74	305.7	311.3	0.88	1.04	- 5.6	-0.16	1	Comstock
1895.50	308.0	314.2	0.95	1.11	- 6.2	-0.16	2	See
1895.73	315.2	315.1	1.12	1.13	+ 0.1	-0.01	3	See

EPIHEMERIS.

t	θ_c	ρ_c	t	θ_c	ρ_c
1896.50	317.5	1.18	1899.50	325.3	1.39
1897.50	320.4	1.26	1900.50	327.6	1.45
1898.50	323.0	1.33			

While this orbit may need slight modification in the course of time, it does not seem probable that a sensible improvement can be effected for a good many years, as the motion is now very slow, and chiefly in the direction of the radius vector. The orbit is remarkable for its high eccentricity, and for having no sensible inclination. This circumstance enables us to contemplate directly the real orbit, and renders 99 *Herculis* an object of the highest interest. The pair is always rather difficult, owing to the inequality of the components, and exact measurement is seldom possible. But at present the star is relatively easy, and ought to be given some attention by observers.

ζ SAGITTARII.

$\alpha = 18^h 56^m.3$; $\delta = -30^\circ 1$.
3.9, yellow ; 4.4, yellow.

Discovered by Winlock in July, 1867.

OBSERVATIONS.

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1867.59	257.7	0.86	1	Winlock	1888.66	259.3	0.67	7	β . & Lv.
1867.80	260.8	0.48	1	Newcomb	1889.41	255.1	0.81	5	Burnham
1878.70	84.2	0.42	1	Burnham	1890.49	251.1	0.76	3	Burnham
1879.71	54.8	0.3 $\pm$	1	Burnham	1891.53	246.5	0.61	3	Burnham
1880.62	62.1	0.55	2	Burnham	1892.39	245.1	0.60	3	Burnham
1881.61	36.1	0.31	2	Burnham	1895.32	194.7	0.35	3	See
1886.62	271.3	0.65	4	Hall	1895.62	193.6	0.13	2	Barnard
1886.74	271.1	—	1-0	Pollock	1895.74	193.1	0.20 $\pm$	1	See
1887.64	265.3	—	5-0	Pollock					

Owing to the great southern declination of ζ *Sagittarii*, which renders it inaccessible to European observers, and makes observations difficult even in the United States, the object was comparatively neglected for a number of years. The first observations were made by WINLOCK and NEWCOMB in the year of its discovery. The pair was not again observed until 1878, when BURNHAM began to give it regular attention.* His series of measures now show that ζ *Sagittarii* belongs to the class of bright, close binaries with short periods. This object has therefore become one of particular interest to American observers.

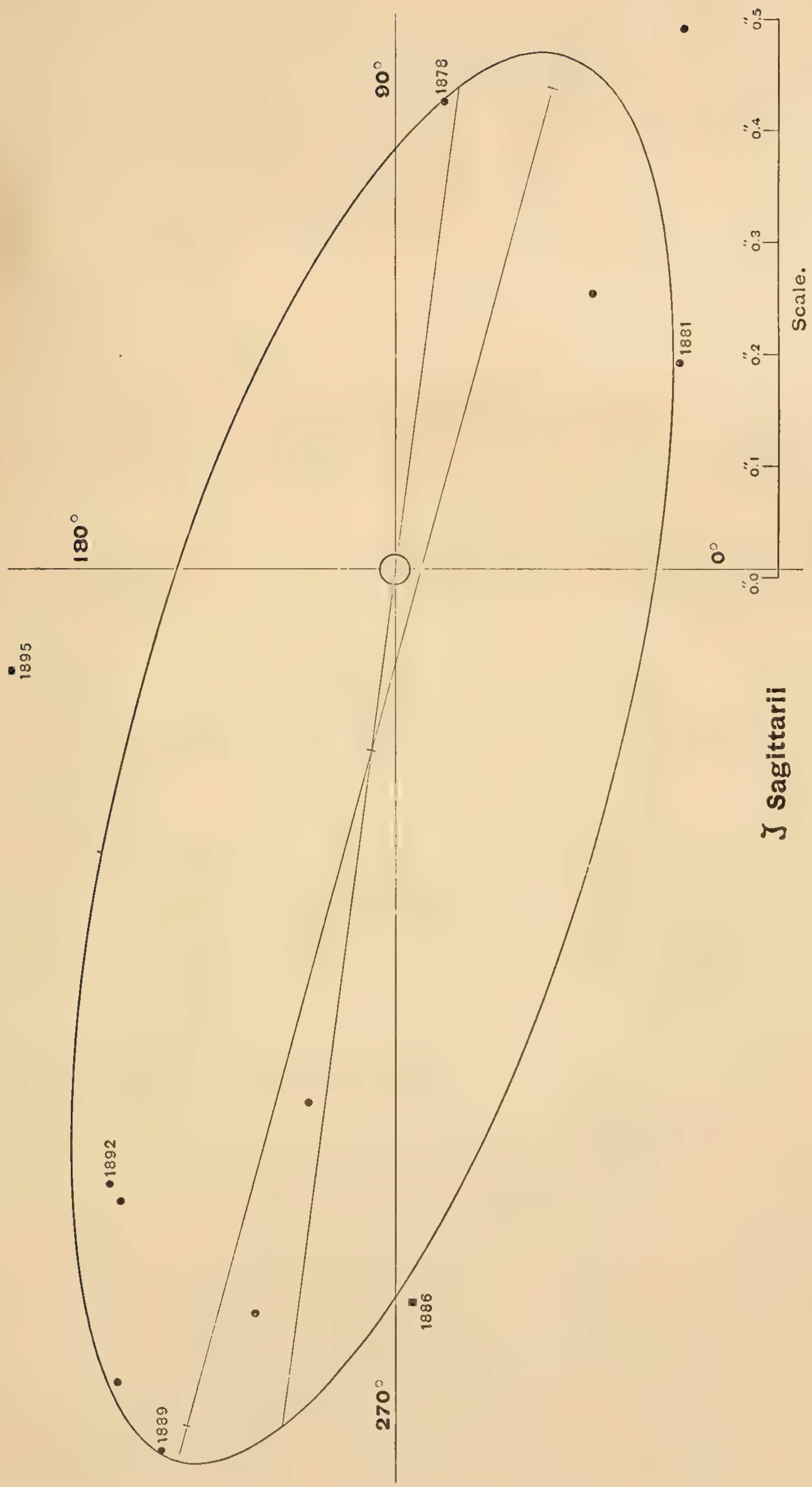
The first investigation of the orbit was made by MR. J. E. GORE, who published the following elements (*Monthly Notices*, R.A.S., 1886, p. 444):

$$\begin{aligned} P &= 18.69 \text{ years} & i &= 58^\circ.8 \\ T &= 1882.86 & \Omega &= 83^\circ.37 \\ e &= 0.1698 & \lambda &= 263^\circ.35 \\ a &= 0''.53 \end{aligned}$$

MR. J. W. FROLEY has more recently examined this orbit (*Astronomy and Astrophysics*, June, 1893), and obtained a set of elements which do not require any large corrections:

$$\begin{aligned} P &= 17.715 \text{ years} & \Omega &= 75^\circ.35 \\ T &= 1878.62 & i &= 73^\circ.95 \\ e &= 0.30 & \lambda &= 327^\circ.35 \\ a &= 0''.68 \end{aligned}$$

* *Astronomical Journal*, 355.



While in Virginia recently, I took occasion to measure this star, and, although the object was seen with difficulty, owing to its low altitude, I could discover a distinct elongation in the direction $194^{\circ}.7$; the distance could not be fixed with much confidence, but my settings of the micrometer gave $0''.35$. The estimates of distance were substantially the same, but I am now convinced, from my distinct recollection of the appearance of the object, that both the measure and the estimate were too large. The star could not be separated, although it was sharply elongated with a power of 1300; the distance was probably less than $0''.25$.

From an examination of all the measures of this pair, we have derived the following elements:

$$\begin{array}{ll} P = 18.85 \text{ years} & \Omega = 69^{\circ}.3 \\ T = 1878.80 & i = 67^{\circ}.32 \\ e = 0.279 & \lambda = 328^{\circ}.1 \\ a = 0''.686 & n = -19^{\circ}.098 \end{array}$$

Apparent orbit:

$$\begin{array}{ll} \text{Length of major axis} & = 1''.300 \\ \text{Length of minor axis} & = 0''.423 \\ \text{Angle of major axis} & = 74^{\circ}.8 \\ \text{Angle of periastron} & = 82^{\circ}.8 \\ \text{Distance of star from centre} & = 0''.168 \end{array}$$

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1867.80	260.8	254.8	0.48	0.81	+ 6.0	-0.33	1	Newcomb
1878.70	84.2	85.6	0.42	0.41	- 1.4	+0.01	1	Burnham
1879.71	54.8	69.4	0.3±	0.48	-14.6	-0.18	1	Burnham
1880.62	62.1	57.7	0.55	0.42	+ 4.4	+0.13	2	Burnham
1881.61	36.1	42.2	0.31	0.33	- 6.1	-0.02	2	Burnham
1886.62	271.3	273.7	0.65	0.59	- 2.4	+0.06	4	Hall
1888.66	259.3	260.9	0.67	0.79	- 1.6	-0.12	7	Burnham 6; Leavenworth 1
1889.41	255.1	256.7	0.81	0.81	- 1.6	±0.00	5	Burnham
1890.49	251.1	251.7	0.76	0.78	- 0.6	-0.02	3	Burnham
1891.53	246.5	246.5	0.61	0.70	± 0.0	-0.09	3	Burnham
1892.39	245.1	241.8	0.60	0.60	+ 3.3	±0.00	3	Burnham
1895.32	194.7	194.3	0.35	0.22	+ 0.4	+0.13	3	See

EPIHEMERIS.

t	θ_c	ρ_c	t	θ_c	ρ_c
1896.50	118.1	0.24	1899.50	49.8	0.37
1897.50	76.5	0.47	1900.50	29.9	0.28
1898.50	63.5	0.46			

When we consider the small number of observations, and the discordant character of some of them, we must regard these elements as highly

satisfactory. It is not likely that they will be materially changed by future observations, but for some time this rapid binary will deserve careful attention. The eccentricity of the orbit appears to be fairly well defined, and is rather smaller than usual; good observations during the next five years will enable us to fix this element with the desired precision. The star is now very difficult, and will remain so for several years, but it is constantly within reach of our large refractors.

γ CORONAE AUSTRALIS = H₂ 5084.

$\alpha = 18^{\text{h}} 59^{\text{m}}.6$; $\delta = -37^{\circ} 12'$.
5.5, yellowish ; 5.5, yellowish.

Discovered by Sir John Herschel, June 20, 1834.

OBSERVATIONS.

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1834.47	37.1	3 $\pm$	1	Herschel	1859.72	338.1	1.5 $\pm$	4-2	Powell
1835.43	37.0	—	1	Herschel	1861.69	328.8	1.5 $\pm$	4-1	Powell
1835.56	36.7	—	1	Herschel	1862.27	325.3	1.5 $\pm$	5-1	Powell
1836.43	34.5	3.67	1	Herschel	1863.84	318.1	—	4	Powell
1837.35	32.0	2.63	1	Herschel	1870.19	286.9	—	2	Powell
1837.44	33.9	2.76	1	Herschel	1871.22	281.9	—	1	Powell
1837.45	32.2	2.04	1	Herschel	1875.65	257.4	1.45	4	Schiaparelli
1837.46	32.7	2.40	1	Herschel	1876.64	253.1	1.67	—	Stone
1847.32	14.1	2.30	1	Jacob	1877.43	248.4	1.49	5	Schiaparelli
1850.51	5.9	2.29	4	Jacob	1877.63	246.6	1.44	4-3	Stone
1851.48	4.4	2.26	6	Jacob	1878.49	242.6	1.36	2	Stone
1852.27	3.4	1.89	3	Jacob	1880.46	233.1	1.15	1	Russell
1853.52	359.1	1.83	—	Jacob	1880.67	232.4	1.32	1	Hargrave
1853.71	358.6	2 $\pm$	4-1	Powell	1881.72	225.5	1.42	3-2	H. C. Wilson
1854.26	356.2	1.71	3	Jacob	1883.62	217.7	1.66	4-1	H. C. Wilson
1854.78	355.6	—	3	Powell	1886.58	200.3	1.37	6	Pollock
1855.77	352.9	—	5	Powell	1886.70	203.5	1.52	1	Russell
1856.22	350.8	1.68	8-7	Jacob	1887.69	196.6	1.16	4	Pollock
1856.67	348.1	1.66	3	Jacob	1887.73	196.2	1.68	4-1	Tebbutt
1857.21	348.4	1.67	5	Jacob	1888.61	189.3	1.71	6-3	Tebbutt
1857.66	346.3	1.55	3	Jacob	1888.71	188.0	1.2	1	Leavenworth
1858.20	343.4	1.53	3	Jacob					

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1889.41	185.4	1.70	4-3	Burnham	1891.75	176.7	1.54	9-4	Tebbutt
1889.84	185.4	2.30	4-1	Tebbutt	1892.64	172.9	1.65	5-2	Tebbutt
1890.59	182.9	1.61	4	Tebbutt	1894.80	165.5	1.62	5-6	Tebbutt
1890.65	180.3	0.99	6-4	Sellors	1895.73	161.9	1.59	1-2	See
1891.53	176.9	1.68	3	Burnham	1895.73	164.1	1.55	2-1	Moulton
1891.70	177.6	1.33	3	Sellors					

During his sojourn at Feldhausen HERSCHEL made careful measures of this object with the seven-foot equatorial, and on two occasions swept over it with the twenty-foot reflector.* In sweep 461 he saw the pair under specially favorable conditions, and estimated the distance of the components at 3". This value is therefore adopted in the table of observations instead of the distance (1".23?) indicated by the micrometer, which was vitiated by troublesome hitching of the threads, and had to be rejected as worthless. HERSCHEL showed from his observations that the system had a considerable retrograde motion, and hence it was subsequently followed by JACOB, POWELL, RUSSELL, TEBBUTT and other southern observers. At the present time the arc described amounts to 238°, and even if the observations are not very numerous, they are sufficient, both in point of quantity and quality, to give an orbit which will undoubtedly prove to be substantially correct.

The components are nearly equal in magnitude, and, as they are never closer than 1".42, the pair is always comparatively easy; and even if difficulties arise in the measurement of distance, there will be practically no difficulty, as HERSCHEL remarks, in determining the angle with the necessary accuracy. In dealing with the orbit of a bright pair with equal components, it is clear that unusual weight should be given to the position angles, and especially when the stars are fairly wide, but the measured distances are affected by relatively large errors. The orbit of this star is therefore based mainly on the angles, but the distances have been of no small service in the final definition of the elements. Some of the orbits which have been published by previous investigators are as follows:

P	T	e	a	Ω	i	λ	Authority	Source
100.8 ^{hrs.}	1863.08	0.602	2.549	352.2	53.6	266.4	Jacob, 1858	M.N., XV, p. 208
55.58	1882.77	0.6989	2.400	229.15	111.45	—	Schiaparelli, 1876	A.N., 2073
54.98	1883.20	0.6974	2.44	227.4	69.3	283.95	Downing, 1883	M.N., XLIII, p. 368
81.78	1886.53	0.322	1.885	45.4	47.43	141.0	Gore, 1885	M.N., XLVI, p. 103
78.80	1887.40	0.324	1.85	41.0	50.5	—	Wilson, 1886	Gore's Catalogue, p.H
93.34	1885.19	0.303	2.034	49.3	48.8	153.4	Powell, 1890	Gore's Catalogue, p.H
121.24	1879.33	0.331	2.191	57.95	35.62	181.1	Sellors, 1892	M.N., LIII, p. 45
154.41	1876.84	0.4244	2.55	77.23	35.6	175.3	Gore, 1892	M.N., LII, p. 503

* *Astronomische Nachrichten*, 3323.

An investigation of all the observations has led to the following elements of γ *Coronae Australis*:

$P = 152.7 \text{ years}$
 $T = 1876.80$
 $e = 0.420$
 $a = 2''.453$

$\Omega = 72^\circ.3$
 $i = 34^\circ.0$
 $\lambda = 180^\circ.2$
 $n = -2^\circ.3575$

Apparent orbit:

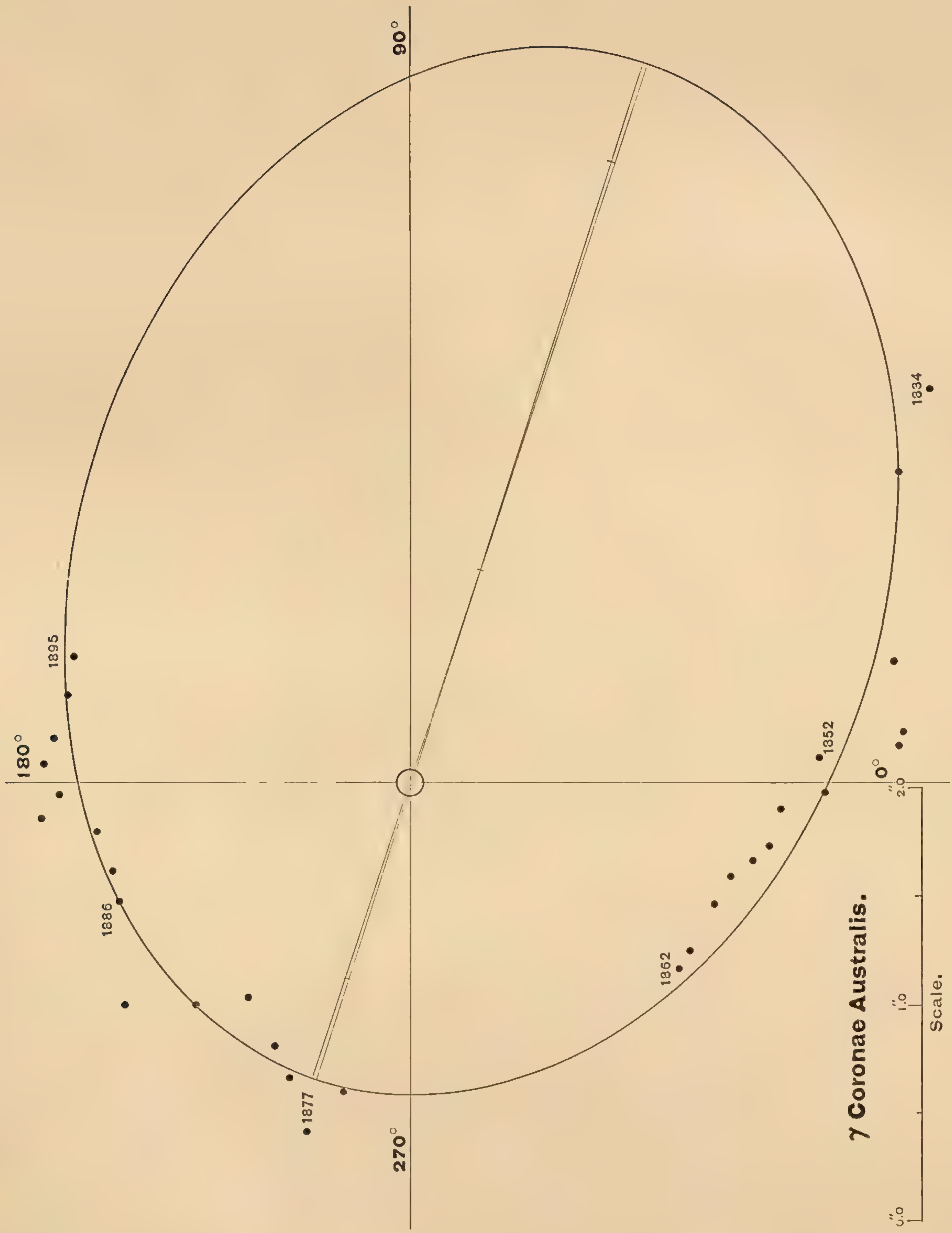
Length of major axis = $4''.906$
Length of minor axis = $3''.661$
Angle of major axis = $72^\circ.2$
Angle of periastron = $252^\circ.1$
Distance of star from centre = $1''.033$

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
	$^\circ$	$^\circ$	$''$	$''$	$^\circ$	$''$		
1834.47	37.1	37.1	3 $\pm$	2.80	± 0.0	+0.20	1	Herschel
1837.42	32.7	32.7	2.66	2.66	± 0.0	± 0.00	4	Herschel
1847.32	14.1	14.6	2.30	2.20	-0.5	+0.10	1	Jacob
1850.51	5.9	5.9	2.29	2.03	± 0.0	+0.26	4	Jacob
1851.48	4.4	4.4	2.26	2.00	± 0.0	+0.26	6	Jacob
1852.27	3.4	2.3	1.89	1.96	+1.1	-0.07	3	Jacob
1853.61	358.8	358.2	1.91	1.90	+0.6	+0.01	6 $\pm$	Jacob; Powell 4-1
1854.52	355.9	355.5	1.71	1.86	+0.4	-0.15	6-3	Jacob 3; Powell 3-0
1856.44	349.5	349.3	1.67	1.78	+0.2	-0.11	11-10	Jacob 8-7; Jacob 3
1857.43	347.3	345.5	1.61	1.73	+1.8	-0.12	8	Jacob 5; Jacob 3
1858.20	343.4	342.6	1.53	1.70	+0.8	-0.17	3	Jacob
1859.72	338.1	336.8	1.5 $\pm$	1.64	+1.3	-0.14	4-2	Powell
1861.69	328.8	328.5	1.5 $\pm$	1.58	+0.3	-0.08	4-1	Powell
1862.27	325.3	325.8	1.5 $\pm$	1.56	-0.5	-0.06	5-1	Powell
1863.84	318.1	319.0	—	1.52	-0.9	—	4	Powell
1870.19	286.1	287.0	—	1.44	-0.9	—	2	Powell
1871.22	281.9	281.3	—	1.43	+0.6	—	1	Powell
1875.65	257.4	258.1	1.45	1.43	-0.7	+0.02	4	Schiaparelli
1876.64	253.1	253.0	1.67	1.43	+0.1	+0.24	—	Stone
1877.53	247.5	247.9	1.47	1.42	-0.4	+0.05	9-7	Schiaparelli 5; Stone 4-3
1878.49	242.6	243.0	1.36	1.43	-0.4	-0.07	2	Stone
1880.57	232.7	232.0	1.24	1.43	+0.7	-0.19	2	Russell 1; Hargrave 1
1881.72	225.5	226.1	1.42	1.43	-0.6	-0.01	3-2	H. C. Wilson
1883.62	217.7	216.3	1.66	1.43	+1.4	+0.23	4-1	H. C. Wilson
1886.64	201.9	200.6	1.44	1.44	+1.3	± 0.00	7	Pollock 6; Russell 1
1887.71	196.4	195.2	1.42	1.46	+1.2	-0.04	8-5	Pollock 4; Tebbutt 4-1
1888.66	188.6	190.6	1.46	1.47	-2.0	-0.01	7-3	Tebbutt 6-3; Leavenworth 1
1889.62	185.4	186.1	1.70	1.49	-0.7	+0.21	8-3	Burnham 4-3; Tebbutt 4-0
1890.62	181.6	181.6	1.61	1.51	± 0.0	+0.10	10-4	Tebbutt 4; Sellors 6-0
1891.53	176.9	177.0	1.68	1.54	-0.1	+0.14	3	Burnham
1892.64	172.9	172.3	1.65	1.57	+0.6	+0.08	5-2	Tebbutt
1894.80	165.5	163.5	1.62	1.65	+2.0	-0.03	5-6	Tebbutt
1895.73	159.2	159.9	1.59	1.69	-0.7	-0.10	2	See

EPIHEMERIS.

t	θ_c	ρ_c	t	θ_c	ρ_c
	$^\circ$	$''$		$^\circ$	$''$
1896.50	157.4	1.71	1899.50	147.2	1.85
1897.50	154.0	1.76	1900.50	143.8	1.90
1898.50	150.6	1.80			



It will be seen that my orbit is quite similar to that found by GORE. Though the period is not defined with the greatest accuracy, it does not seem probable that the value given above can be uncertain by more than five years. The eccentricity will certainly be in the immediate neighborhood of the value here assigned, and an error exceeding ± 0.02 is very improbable. The orbit of γ *Coronae Australis* is therefore comparatively well determined, and yet as great accuracy in the orbits of double stars is ultimately desirable, southern observers will find this system worthy of constant attention.

 β DELPHINI = β 151.

 $\alpha = 20^{\text{h}} 32^{\text{m}}.9$; $\delta = +14^{\circ} 15'$.

4, yellow ; 6, yellowish.

Discovered by Burnham with his celebrated six-inch Clark Refractor in August, 1873.

OBSERVATIONS.									
t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1873.60	$355. \pm$	0.7	1	Burnham	1885.61	222.9	<0.4	1	H. Struve
1874.66	15.5	0.65	5	Dembowski	1885.95	216.6	0.38	8	Englemann
1874.70	13.6	0.49	3-1	Newcomb	1886.78	257.8	obl.	1	H. Struve
1874.73	8.0	0.69	1	O. Struve	1886.88	238.1	$0.22 \pm$	7	Schiaparelli
1875.61	14.7	0.42	4	Schiaparelli	1886.91	219.5	0.39	4	Englemann
1875.65	20.1	0.54	4	Dembowski	1887.55	278.5	0.36	5	Tarrant
1876.65	25.8	0.48	4	Dembowski	1887.66	272.0	0.39	5	H. Struve
1877.27	17.7	0.35	2	Schiaparelli	1887.75	308.1	$0.3 \pm$	1	Hough
1877.71	29.7	0.51	5	Dembowski	1887.85	287.8	$0.2 \pm$	8	Schiaparelli
1877.79	40.8	0.32	2	Burnham	1888.65	304.0	0.30	5	Burnham
1878.65	53.7	0.24	4	Burnham	1888.76	300.9	0.35	3	H. Struve
1878.75	59.2	—	1	Dembowski	1888.84	311.5	0.25	17	Schiaparelli
1879.56	$90 \pm$	elong. doubtful	2	Burnham	1889.50	314.2	0.31	5	Burnham
1880.68	133.6	0.26	3	Burnham	1889.78	318.5	0.43	6	H. Struve
1880.75	214.5	$0.2 \pm$	2	Hall	1889.86	319.2	$0.37 \pm$	11	Schiaparelli
1881.54	149.2	0.26	5	Burnham	1890.49	324.2	0.45	4	Burnham
1881.88	154.7	—	1	Bigourdan	1890.89	326.5	0.43	12	Schiaparelli
1882.60	167.5	0.26	3	Burnham	1891.45	331.6	0.38	4	Burnham
1883.25	183.9	0.19	7	Englemann	1891.64	330.1	0.39	3	Hall
1883.55	182.5	0.23	3	Burnham	1891.76	334.0	0.48	5	H. Struve
1884.69	195.9	0.32	3	Hall	1891.85	158.2	—	1	Bigourdan
1884.71	197.7	0.32	4	Englemann	1891.87	333.7	0.43	9	Schiaparelli
1884.77	199.2	0.29	5	Burnham	1892.39	338.7	0.50	4	Burnham
					1892.88	337.6	0.49	2	Barnard
					1892.93	340.7	0.52	5	Schiaparelli

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1893.52	339.2	0.58	2	Leavenworth	1894.79	348.6	—	1	H.C. Wilson
1893.53	338.8	0.73	2	H.C. Wilson	1894.83	347.2	0.48	13	Schiaparelli
1893.62	335.3	0.57	3	Hough					
1893.70	342.2	0.56	5	Barnard	1895.31	351.8	0.50	1	See
1893.79	346.8	0.51	3	Comstock	1895.42	349.8	0.73	6	Barnard
1893.87	344.2	0.49	13	Schiaparelli	1895.61	352.1	0.80	1	See
1893.95	345.8	—	1	Bigourdan	1895.61	352.1	0.64	1	See
1894.51	346.3	0.56	8	Barnard	1895.66	350.8	0.58	3	Comstock

When discovered in 1873 the companion was near its maximum elongation, and was easily measured by DEMBOWSKI in 1874. The measures of the next few years showed that the pair had a rapid direct motion.* In 1879–80 the distance of the components became so small (about $0''.20$) that the object could be elongated only by the most powerful telescopes. The measures at this time are therefore few in number, and necessarily of doubtful accuracy.

Since the epoch of DEMBOWSKI's measures in 1874, the radius-vector of the companion has swept over 335 degrees of position-angle, and the intervening observations enable us to determine the orbit with a comparatively high degree of precision. The following table gives the orbits hitherto published for this star:

P	T	e	a	Ω	i	λ	Authority	Source
^{yr.} 26.07	1882.19	0.357	0.55	163.6	54.9	354.6	Dubiago, 1884	A.N., 2602
30.91	1882.25	0.337	0.517	2.67	59.33	327.8	Gore, 1885	Proc. R.I.A., IV; no.5
16.95	1885.80	0.096	0.460	10.9	61.6	220.9	Celoria, 1888	A.N., 2824
22.97	1882.37	0.260	0.501	174.2	64.1	343.9	Glasenapp, 1893	A.N., 3177
24.16	1882.38	0.284	0.51	174.4	64.64	344.2	Glasenapp, 1893	A.N., 3177

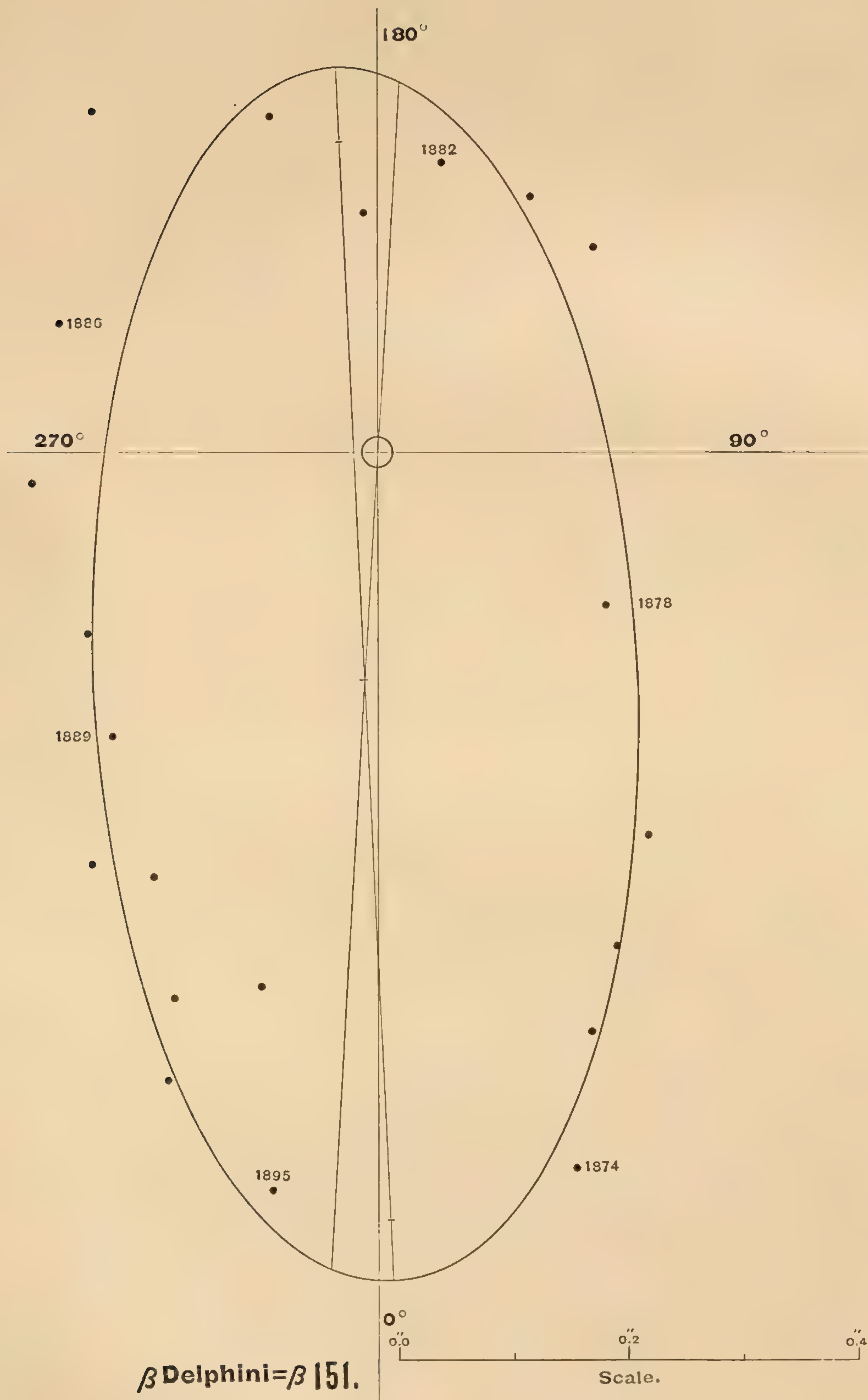
From an investigation of all the observations we find the following elements for β Delphini:

$$\begin{aligned}
 P &= 27.66 \text{ years} & \Omega &= 3^\circ.9 \\
 T &= 1883.05 & i &= 61^\circ.35 \\
 e &= 0.373 & \lambda &= 164^\circ.93 \\
 a &= 0''.6724 & n &= +13^\circ.015
 \end{aligned}$$

Apparent orbit:

$$\begin{aligned}
 \text{Length of major axis} &= 1''.060 \\
 \text{Length of minor axis} &= 0''.477 \\
 \text{Angle of major axis} &= 2^\circ.5 \\
 \text{Angle of periastron} &= 176^\circ.6 \\
 \text{Distance of star from centre} &= 0''.194
 \end{aligned}$$

* *Astronomical Journal*, 357.



The accompanying table of computed and observed places shows that these elements are extremely satisfactory. The only large residual is that of 1880, which is probably due to an error of observation incident to the excessive closeness of the components.

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1874.66	15.5	15.2	0.65	0.62	+ 0.3	+0.03	5	Dembowski
1875.65	20.1	20.0	0.54	0.55	+ 0.1	-0.01	4	Dembowski
1876.65	25.8	26.2	0.48	0.48	- 0.4	± 0.00	4	Dembowski
1877.75	35.3	36.2	0.41	0.38	- 0.9	+0.03	7	Dembowski 5; Burnham 2
1878.70	56.4	50.1	0.24	0.29	+ 6.3	-0.05	5-4	Burnham 4; Dembowski 1-0
1879.56	90 $\pm$	71.3	^{elong.} doubtful 0.23	0.23	—	—	2	Burnham
1880.68	133.6	114.7	0.26	0.20	+18.9	+0.06	3	Burnham
1881.54	149.2	145.6	0.26	0.24	+ 3.6	+0.02	5	Burnham
1882.60	167.5	169.1	0.26	0.31	- 1.6	-0.05	3	Burnham
1883.40	183.2	181.7	0.21	0.33	+ 1.5	-0.12	10	Englemann 7; Burnham 3
1884.72	197.6	201.2	0.31	0.33	- 3.6	-0.02	12	Hall 3; Englemann 4; Burnham 5
1885.95	219.8	220.7	0.39	0.28	- 0.9	+0.11	9	Englemann 8; H. Struve 1
1886.86	248.0	247.4	0.30	0.24	+ 0.6	+0.06	8-11	Sch. 7; Englemann 0-4; H. Struve 1-0
1887.70	275.2	271.8	0.31	0.24	+ 3.4	+0.07	18-19	Tar. 5; Ho. 1; Schiaparelli 8; H. Struve 5
1888.65	302.4	296.3	0.30	0.27	+ 6.1	+0.03	8-5	Burnham 5; H. Struve 3-0
1889.68	317.3	313.8	0.34	0.34	+ 3.5	± 0.00	16	β . 5; Schiaparelli 11; H. Struve 6-0
1890.69	325.3	325.2	0.44	0.41	+ 0.1	+0.03	16	Burnham 4; Schiaparelli 12
1891.68	332.4	333.5	0.42	0.48	- 1.1	-0.06	21	β . 4; Hl. 3; Schiaparelli 9; H. Struve 5
1892.66	339.7	339.9	0.51	0.54	- 0.2	-0.03	9	Schiaparelli 5; Burnham 4 [Big. 1-0]
1893.71	341.7	344.8	0.58	0.61	- 3.1	-0.03	24-23	Lv. 2; H.C.W. 2; Ho. 3; Com. 3; Sch. 13;
1894.81	347.9	349.3	0.48	0.65	- 1.4	-0.17	14-13	H.C. Wilson 1-0; Schiaparelli 13
1895.51	352.0	351.9	0.65	0.68	+ 0.1	-0.03	3	See

The present orbit is somewhat more eccentric than those heretofore published, and in this respect it conforms better to the general rule among binaries. That the orbit has an eccentricity of about this magnitude is evident from the rapid motion of the radius-vector in the periastral region, and its slow motion at the present time. The slow, angular motion of the radius-vector during recent years indicates, of course, that the distance of the companion is much increased; and this leads us to remind observers that the present distance is sensibly larger than some have indicated by their measures. At present the distance is probably over 0".65, and for some years will slightly augment.

It does not seem at all probable that the true elements of this remarkable binary can differ materially from those here obtained. Nevertheless, additional exact measures will be valuable in fixing the orbit with great accuracy, and as the star will be relatively easy for several years, observers should give it regular attention. The following is a short ephemeris:

t	θ_c	ρ_c	t	θ_c	ρ_c
1896.51	355.3	0.71	1899.51	4.9	0.72
1897.51	358.6	0.72	1900.51	8.2	0.69
1898.51	1.7	0.72			

4 AQUARIUM = Σ 2729.

$\alpha = 20^h 46^m.1$; $\delta = -6^\circ 1'$.
6, yellow ; 7, yellow.

Discovered by Sir William Herschel, September 3, 1782.

OBSERVATIONS.									
t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1783.55	351.5	—	1	Herschel	1875.62	157.0	0.4 $\pm$	4	Schiaparelli
1802.65	28.9	—	2	Herschel	1877.15	148.7	0.56	3	Dembowski
1825.60	27.5	0.80	2	Struve	1877.70	158.5	0.5 $\pm$	1	Cincinnati
1830.92	13.4	0.69	1	Struve	1879.44	156.4	0.57	5-1	Cincinnati
1832.73	46.0	0.67	2-1	Herschel	1879.76	155.9	0.40	4	Hall
1832.90	23.0	oblonga	1	Struve	1880.78	165.5	0.51	2	Pritchett
1833.77	31.2	0.67	1	Struve	1881.54	159.6	0.52	3	Burnham
1836.05	46.3	0.41	4	Struve	1883.84	182.1	—	1	Seabroke
1839.68	62.2	—	2	Dawes	1884.77	166.8	—	7	Seabroke
1840.72	65.5	0.6 $\pm$	2	Dawes	1885.64	156.1	—	1	Seabroke
1841.51	24.6	0.6 $\pm$	1	Madler	1885.74	167.9	0.46	3	Hall
1841.80	72.7	—	1	Dawes	1886.69	162.5	—	1	Seabroke
1842.82	27.2	0.45	2-1	Madler	1886.74	168.3	0.54	3-2	Leavenworth
1843.70	31.9	0.5 $\pm$	3	Madler	1886.84	174.8	0.47	2	Hall
1843.76	81.7	—	1	Dawes	1887.28	173.4	0.41	7	Schiaparelli
1844.90	23.1	0.5 $\pm$	1	Madler	1887.79	175.9	0.53	3	Hall
1853.70	95.9	0.5 $\pm$	1	Dawes	1887.82	170.5	0.52	2	Tarrant
1854.75	101.7	0.3 $\pm$	1	Dawes	1888.81	172.4	0.48 $\pm$	5	Schiaparelli
1855.	—	—	1	Secchi	1889.51	155.5	—	1	Seabroke
1856.81	107.8	0.3 $\pm$	1	Secchi	1889.88	176.7	0.49 $\pm$	2	Schiaparelli
1862.68	137.5	oblonga	3	Dembowski	1890.78	178.2	0.49	2	Tarrant
1865.71	125 $\pm$	cuneo	1	Secchi	1891.77	178.1	0.50 $\pm$	1	Schiaparelli
1865.74	143.6	—	1	Talmage	1892.70	184.5	0.55	3	Tarrant
1866.08	139.6	oblonga	3	Dembowski	1892.80	181.7	0.33	2-1	Comstock
1866.65	125.5	—	3	Searle	1892.91	187.0	0.4 $\pm$	1	Schiaparelli
1866.66	110.0	—	5	Winlock	1893.81	182.4	0.35 $\pm$	2-1	Comstock
1867.86	141.1	0.30	1	Newcomb	1894.86	186.5	0.38 $\pm$	3	Schiaparelli
1872.88	147.5	oblonga	5	Dembowski	1895.61	193.9	0.30 $\pm$	1	Comstock
					1895.73	184.2	0.33	3	See

This double star is always an exceedingly close and difficult object. SIR WILLIAM HERSCHEL measured the position-angle in 1783, and on repeating his observation in 1802, concluded that in nineteen years the motion had amounted to $37^{\circ}.4$ (*Phil. Trans*, 1804, p. 371). In 1825 the star was measured by STRUVE on two nights; his observations gave $\theta = 25^{\circ}.0$, $\rho = 0''.81$, $\theta = 30^{\circ}.0$, $\rho = 0''.80$. These results do not accord well with those of 1802, but we may infer with DAWES (*Mem. R.A.S.*, vol. xxxv. p. 427) that HERSCHEL'S second observation is erroneous. For it is clear that the angle could not have been the same in 1802 as in 1825, and the subsequent motion of the star shows that STRUVE'S first position is essentially correct. All the early and some of the more recent measures of 4 *Aquarii* are extremely discordant, and great difficulty is experienced in determining what measures ought to be relied upon. Careful sifting of the observations and judicious combinations of individual results will alone insure suitable mean places for the derivation of a satisfactory set of elements. We have relied principally upon the work of SIR WILLIAM HERSCHEL, STRUVE, SIR JOHN HERSCHEL, DAWES, MÄDLER, SECCHI, DEMBOWSKI, HALL, BURNHAM, SCHIAPARELLI and COMSTOCK.

The following elements of 4 *Aquarii* have been published by previous computers:

P	T	e	a	Ω	i	λ	Authority	Source
^{yrs.} 129.8	1752.0	0.46	["] 0.72	[°] 340.2	[°] 56.6	[°] 235.0	Doberck, 1877	A.N., 2287.
126.65	1899.88	0.543	0.7036	177.4	68.51	74.25	See, 1895	A.J., 341.

A revision of my former orbit of this star gives the following elements:

$$\begin{array}{ll}
 P = 129.0 \text{ years} & \Omega = 177^{\circ}.7 \\
 T = 1899.40 & i = 72^{\circ}.53 \\
 e = 0.514 & \lambda = 68^{\circ}.63 \\
 a = 0''.732 & n = +2^{\circ}.7907
 \end{array}$$

Apparent orbit:

$$\begin{array}{ll}
 \text{Length of major axis} & = 1''.288 \\
 \text{Length of minor axis} & = 0''.43 \\
 \text{Angle of major axis} & = 0^{\circ}.3 \\
 \text{Angle of periastron} & = 215^{\circ}.2 \\
 \text{Distance of star from centre} & = 0''.173
 \end{array}$$

The accompanying table of computed and observed places shows a very satisfactory agreement. The present orbit is narrower than the one recently published in the *Astronomical Journal*, 341, but the great discordance of results of individual observers shows that the object has always been extremely close;

and hence we think the chances favor the present orbit, which differs from the previous one chiefly in the higher inclination. It is noticeable that the representation of the more recent observations is sensibly improved.

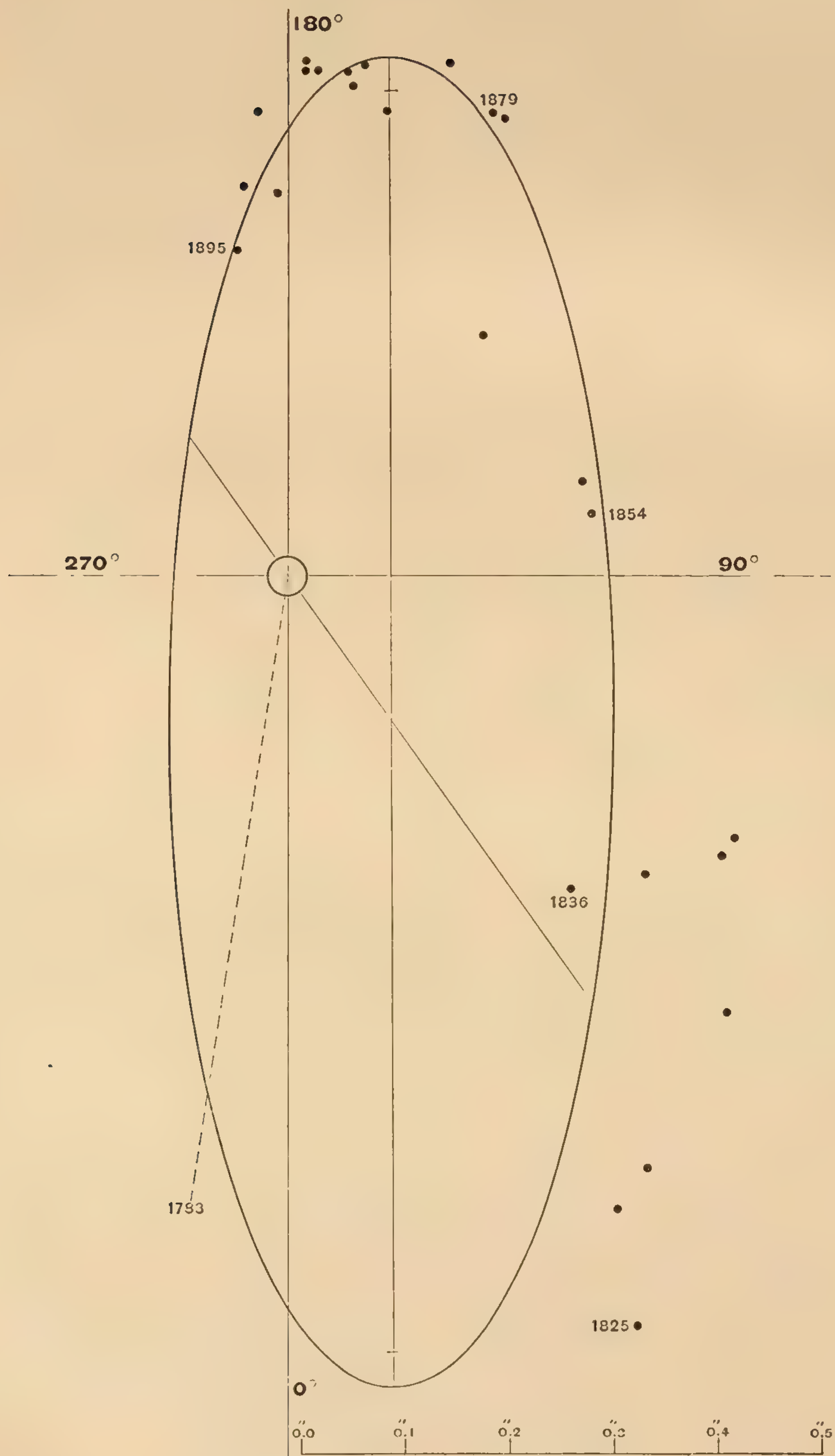
COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1783.55	351.5	352.2	—	0.53	— 0.7	—	1	Herschel
1825.60	25.0	24.0	0.80	0.64	+ 1.0	+0.16	1-2	Struve
1832.18	27.5	31.4	0.69	0.55	— 3.9	+0.14	4-1	Struve 1; Herschel 2-1; Struve 1
1833.77	31.2	33.5	0.67	0.53	— 2.3	+0.14	1	Struve
1836.05	40.9	37.0	0.41	0.50	+ 3.9	—0.09	4	Struve
1841.12	45.0	46.4	0.6 $\pm$	0.43	— 1.4	+0.17	3	Dawes 2; Mädler 1
1842.31	49.9	50.3	0.45	0.40	— 0.4	+0.05	3-1	Dawes 1; Mädler 2-1
1843.73	56.8	53.0	0.5 $\pm$	0.39	+ 3.8	+0.11	4-3	Mädler 3; Dawes 1
1849.30	59.5	68.8	0.5 $\pm$	0.34	— 9.3	+0.16	2	Mädler 1; Dawes 1
1854.75	101.7	85.9	0.3 $\pm$	0.31	+15.8	—0.01	1	Dawes
1856.81	107.8	97.8	0.3 $\pm$	0.31	+10.0	—0.01	1	Secchi
1864.20	131.2	125.4	cuneo	0.34	+ 5.8	—	4	Dembowski 3; Secchi 1
1866.08	139.6	131.2	oblonga	0.36	+ 8.4	—	3	Dembowski 3
1867.86	141.1	136.2	0.30	0.38	+ 4.9	—0.08	1	Newcomb
1872.88	147.5	142.6	oblonga	0.41	+ 4.9	—	5	Dembowski
1876.82	154.7	155.2	0.49	0.47	— 0.5	+0.02	8	Schiaparelli 4; Dembowski 3; Cinn. 1
1879.60	156.2	159.7	0.49	0.49	— 3.5	$\pm$ 0.00	9-5	Cincinnati 5-1; Hall 4
1881.16	162.5	162.1	0.52	0.50	+ 0.4	+0.02	5	Pritchett 2; Burnham 3
1885.74	167.9	168.9	0.46	0.51	— 1.0	—0.05	3	Hall
1886.79	171.5	170.5	0.50	0.51	+ 1.0	—0.01	5-4	Leavenworth 3-2; Hall 2
1887.63	173.3	172.2	0.49	0.50	+ 1.1	—0.01	12	Schiaparelli 7; Hall 3; Tarrant 2
1888.81	172.4	173.5	0.48	0.49	— 1.1	—0.01	5	Schiaparelli
1889.88	176.7	175.3	0.49 $\pm$	0.48	+ 1.4	+0.01	2	Schiaparelli
1890.78	178.2	176.9	0.49	0.47	+ 1.3	+0.02	2	Tarrant
1891.77	178.1	178.5	0.50 $\pm$	0.45	— 0.4	+0.05	1	Schiaparelli
1892.85	181.7	181.0	0.37	0.42	+ 0.7	—0.05	2	Comstock 2-1; Schiaparelli 0-1
1893.25	183.4	181.8	0.45	0.41	+ 1.6	+0.04	5-4	Tarrant 3; Comstock 2-1
1894.86	186.5	185.3	0.38 $\pm$	0.37	+ 1.2	+0.01	3	Schiaparelli
1895.67	189.0	188.8	0.32	0.33	+ 0.2	—0.01	4	Comstock 1; See 3

The period here indicated is not likely to be in error by more than five years, while a variation of ± 0.03 in the eccentricity does not seem probable. It is therefore unlikely that future observations will greatly alter the present elements, but as some improvement is still desirable, astronomers should continue to give this star careful attention. During the next few years the motion will be very rapid, and the object excessively difficult; but for this very reason observations will be the more valuable.

The following is an ephemeris for five years:

t	θ_c	ρ_c	t	θ_c	ρ_c
1896.80	193.5	0.28	1899.80	224.0	0.14
1897.80	199.4	0.24	1900.80	244.1	0.12
1898.80	208.1	0.19			



4 Aquarii = Σ 2729.

δ EQUULEI = σ^5 535.

$\alpha = 21^h 9^m.6$; $\delta = +9^\circ 37'$.
4.5, yellow ; 5.0, yellow.

Discovered by Otto Struve, August 19, 1852.

OBSERVATIONS.

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1852.64	22.5	0.45	1	O. Struve	1881.46	22.1	0.38	4	Burnham
1852.67	18.8	0.43	1	O. Struve	1882.63	9.8	0.29	3	Burnham
1853.91	11.9	0.27	1	O. Struve	1883.55	307.6	0.21	3	Burnham
1854.69	simple		1	O. Struve	1886.84	203.5	0.47	2	Hall
1856.57	simple		1	O. Struve	1886.87	204.6	0.35	6-2	Schiaparelli
1857.67	207.6	0.21	1	O. Struve	1886.91	203.2	0.47	4	Englemann
1857.67	211.8	0.23	1	O. Struve	1887.78	195.2	0.49	2-1	Hough
1858.59	16.8	0.40	1	O. Struve	1887.79	195.8	0.44	5	Tarrant
1859.65	13.5	0.39	1	O. Struve	1887.80	198.7	0.41	4	Hall
1861.57	236 ?	oblong	1	O. Struve	1887.86	195.0	0.33	11-8	Schiaparelli
1865.91	203.3	<0.5	1	O. Struve	1888.69	189.9	0.25	4	Burnham
1869.74	15.6	—	6-0	Harvard	1888.90	187.0	0.15	14-10	Schiaparelli
1870.73	8.0	—	1-0	Dunér	1889.51	163.2	0.10 $\pm$	1	Burnham
1874.67	24.0	oblong	1-0	O. Struve	1889.82	193.1	0.2 $\pm$	1	Hough
1874.73	1.8	cuneiforme	1-0	O. Struve	1889.84	175.0	0.15	3	Schiaparelli
1874.75	221.2	0.33	1	O. Struve	1890.88	single	—	3	Schiaparelli
1877.76	156.4	0.2 $\pm$	1	Burnham	1891.63	31.6	0.20	5	Burnham
1878.65	elong. doubtful		2	Burnham	1891.85	23.4	0.21	5	Schiaparelli
1879.76	150.0	doubtful	2	Hall	1892.39	26.6	0.35	4	Burnham
1880.60	29.1	0.35	5	Burnham	1892.91	22.8	0.30	2	Schiaparelli
					1893.93	16.8	0.25	6	Schiaparelli
					1893.97	200.2	—	1	Bigourdan
					1894.85	simple	—	4	Schiaparelli

The pair was first measured in 1852, and when the observations were repeated the following year it was found that there was a slight diminution in the angle of position as well as in the distance. In 1854 and in 1856 the star was noted as single, but in 1857 the companion appeared in the opposite quadrant, and hence it became evident that the star is a binary in rapid retrograde motion. Continued observation disclosed the fact that the orbit is highly

inclined upon the visual ray, and STRUVE's measures seemed to indicate a period of 6.5 or 13 years. Since 1877 the star has been carefully followed by BURNHAM, and by means of his fine series of observations we are enabled to derive a very satisfactory orbit.

The two orbits heretofore published for this star are as follows:

P	T	e	a	Ω	i	λ	Authority	Source
^{yrs.} 11.48	1892.0	0.20	0.41	24.0	81.8	26.6	Wrublewsky, 1887	A.N., 2771
11.45	1892.80	0.14	0.452	22.2	79.05	0.00	See, 1895	A.N., 3290

An investigation of all the observations leads to the following elements of δ Equulei:

$$\begin{aligned}
 P &= 11.45 \text{ years} & \Omega &= 22^\circ.2 \\
 T &= 1892.80 & i &= 79^\circ.0 \\
 e &= 0.165 & \lambda &= 0^\circ.0 \\
 a &= 0''.452 & n &= -31^\circ.441
 \end{aligned}$$

Apparent orbit:

$$\begin{aligned}
 \text{Length of major axis} &= 0''.904 \\
 \text{Length of minor axis} &= 0''.171 \\
 \text{Angle of major axis} &= 22^\circ.2 \\
 \text{Angle of periastron} &= 22^\circ.2 \\
 \text{Distance of star from centre} &= 0''.075
 \end{aligned}$$

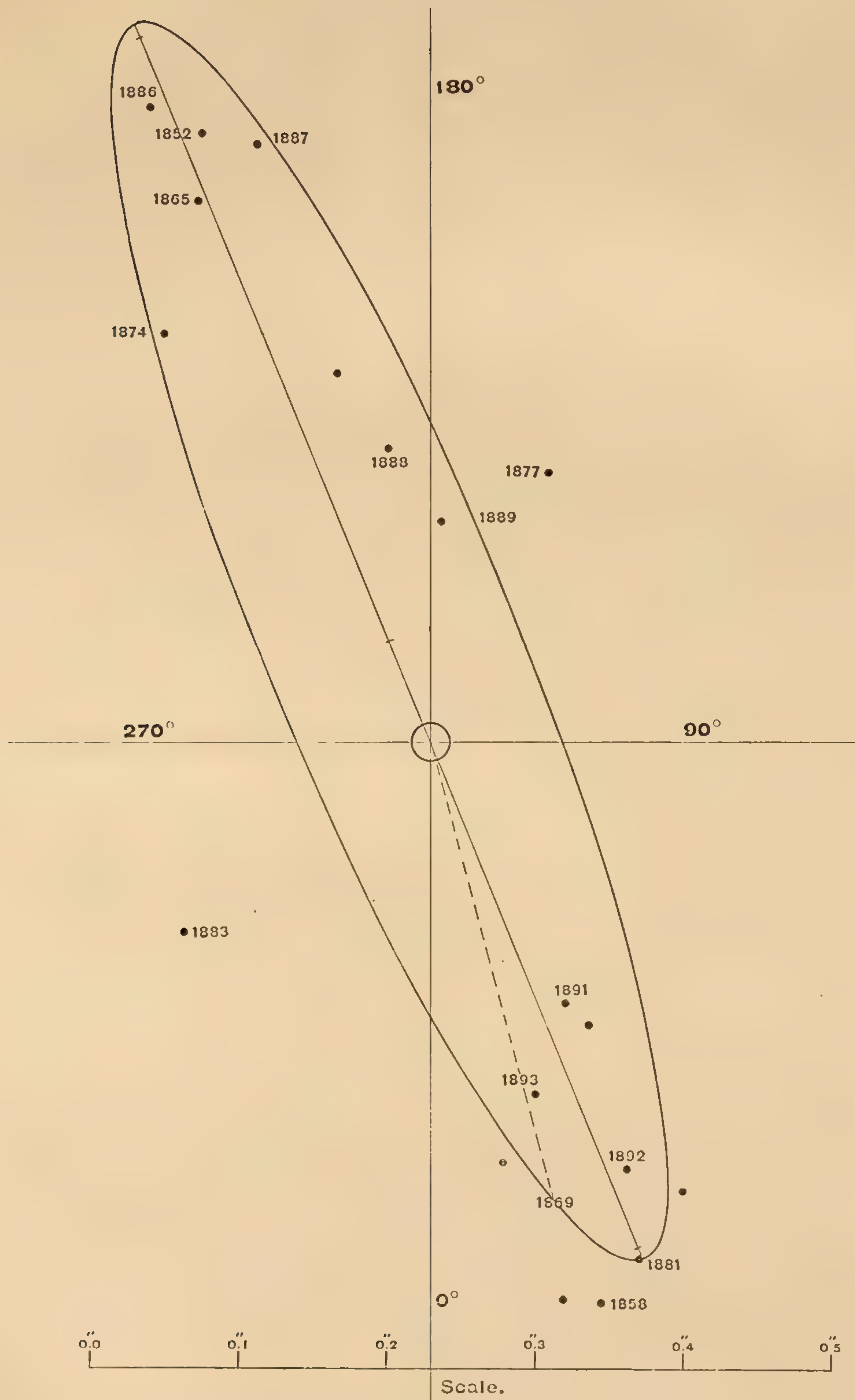
The following table gives a comparison of the computed with the observed places, and shows that the present elements will never require any considerable correction. Only a few large deviations occur, and these are probably to be explained by the extreme difficulty of the object.*

BURNHAM's measure of 1877 is marked "doubtful," and is practically only an estimate, as the object was very difficult to separate.

It will be seen that the eccentricity of this orbit is considerably smaller than that generally found among double stars. It is also remarkable that the real major axis coincides with the line of nodes, so that λ is zero.

δ Equulei and κ Pegasi are the most rapid binaries in the heavens, and on this account are worthy of special attention from observers who have large telescopes. The elements given here need to be tested by further observation. It is especially important to determine the maximum distances of the companion when the angles are about 22° and 202° respectively, as this would furnish a more exact determination of the eccentricity and the major axis.

* *Astronomische Nachrichten*, 3290.



δ Equulei = $0^\circ \Sigma 535 = \Sigma 2777 AB$

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1852.65	200.6	202.5	0.44	0.52	— 1.9	— 0.08	2	O. Struve
1853.91	191.9	196.6	0.27	0.46	— 4.7	— 0.19	1	O. Struve
1857.67	29.7	29.6	0.22	0.33	+ 0.1	— 0.11	2	O. Struve
1858.59	16.8	21.0	0.40	0.38	— 4.2	+ 0.02	1	O. Struve
1859.65	13.5	8.7	0.39	0.25	+ 4.8	+ 0.14	1	O. Struve
1865.91	203.3	192.8	0.4 $\pm$	0.38	+ 10.5	+ 0.02 $\pm$	1	O. Struve
1869.74	15.6	23.5	—	0.38	— 7.9	—	6	Harvard
1870.73	8.0	14.2	—	0.31	— 6.2	—	1	Dunér
1874.74	212.6	206.2	0.33	0.48	+ 6.4	— 0.15	2-1	O. Struve
1877.76	156.4	187.9	0.2 $\pm$	0.30	— 31.5	— 0.10 $\pm$	1	Burnham
1880.60	29.1	29.3	0.35	0.33	— 0.2	+ 0.02	5	Burnham
1881.46	22.1	21.2	0.38	0.37	+ 0.9	+ 0.01	4	Burnham
1882.63	9.8	7.1	0.29	0.24	+ 2.7	+ 0.05	3	Burnham
1883.55	307.6	302.2	0.21	0.09	+ 5.4	+ 0.12	3	Burnham
1886.87	203.8	203.1	0.47	0.52	+ 0.7	— 0.05	12-6	Hall 2; Schiaparelli 6-2; Englemann 4
1887.81	196.2	198.9	0.42	0.50	— 2.7	— 0.08	22-18	Ho. 2-1; Tar. 5; Hall 4; Schiaparelli 11-8
1888.80	188.5	192.9	0.20	0.49	— 4.4	— 0.29	18-14	Burnham 4; Schiaparelli 14-10
1889.72	177.1	180.0	0.15	0.22	— 2.9	— 0.07	5	Burnham 1; Hough 1; Schiaparelli 3
1890.88	single	65.4	—	0.12	—	—	3	Schiaparelli
1891.74	27.5	35.0	0.20	0.26	— 7.5	— 0.06	10	Burnham 5; Schiaparelli 5
1892.65	24.7	23.5	0.32	0.38	+ 1.2	— 0.06	6	Burnham 4; Schiaparelli 2
1893.93	16.8	10.0	0.25	0.26	+ 6.8	— 0.01	6	Schiaparelli
1894.85	simple	324.8	—	0.10	—	—	—	Schiaparelli

The following is a short ephemeris:

t	θ_o	ρ_o	t	θ_o	ρ_o
1896.85	211.1	0.39	1899.85	195.8	0.44
1897.85	205.2	0.50	1900.85	186.4	0.28
1898.85	200.8	0.52			

κ PEGASI = β 989.

$\alpha = 21^h 40^m.1$; $\delta = +25^\circ 11'$.
4.3, yellowish ; 5.0, yellowish.

Discovered by Burnham, August 12, 1880.

OBSERVATIONS.

t	θ_o	ρ_o	n	Observers	t	θ_o	ρ_o	n	Observers
1880.68	137.9	0.27	4	Burnham	1891.61	150.0	0.10	3	Burnham
1883.02	116.0	0.16	1	Englemann	1891.81	144.6	0.13	4	Burnham
1888.78	274.7	0.23	3	Burnham	1891.92	159.0	0.18	3	Schiaparelli
1889.51	262.3	0.14	4	Burnham	1892.39	132.8	0.18	4	Burnham
1890.57	187.1	0.10	4	Burnham	1892.88	131.0	0.20	1	Barnard
					1892.96	135.1	0.20	4	Schiaparelli

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1893.51	121.0	0.29	3	Leavenworth	1894.51	117.6	0.19	7-6	Barnard
1893.77	127.5	0.20	2	Barnard	1894.83	114.8	0.14	4	Lewis
1893.82	130.5	0.25	2-1	Comstock	1894.87	114.7	0.24	6	Schiaparelli
1893.92	123.6	0.27	8	Schiaparelli	1895.62	107.9	0.17	6	Barnard

This remarkable double star was discovered with the 18-inch refractor of the Dearborn Observatory. Its extreme closeness led to the belief that it would prove to be binary,* and accordingly it has been found to be in rapid revolution. DR. ENGLEMAN of Leipzig succeeded in making one measure of the pair in 1883, which indicated a retrograde motion. BURNHAM's measures were continued at the Lick Observatory from 1888 to 1892, and the new data thus obtained enabled him for the first time to get the approximate period of revolution (*Monthly Notices*, March, 1891).

At the request of BURNHAM and the writer, BARNARD has since followed the star, and obtained additional measures which appear to be sufficient to give us a reasonably good approximation to the elements of the orbit. In his first examination of the motion of this pair, BURNHAM made the orbit nearly circular, but the recent observations show that the orbit has about the usual eccentricity prevailing among binaries, and that the inclination of the orbit is very high. In the *Monthly Notices* for November, 1894, MR. LEWIS has given a set of measures recently obtained with the Greenwich 28-inch refractor, and sketched an apparent orbit which would better satisfy the latest observations.

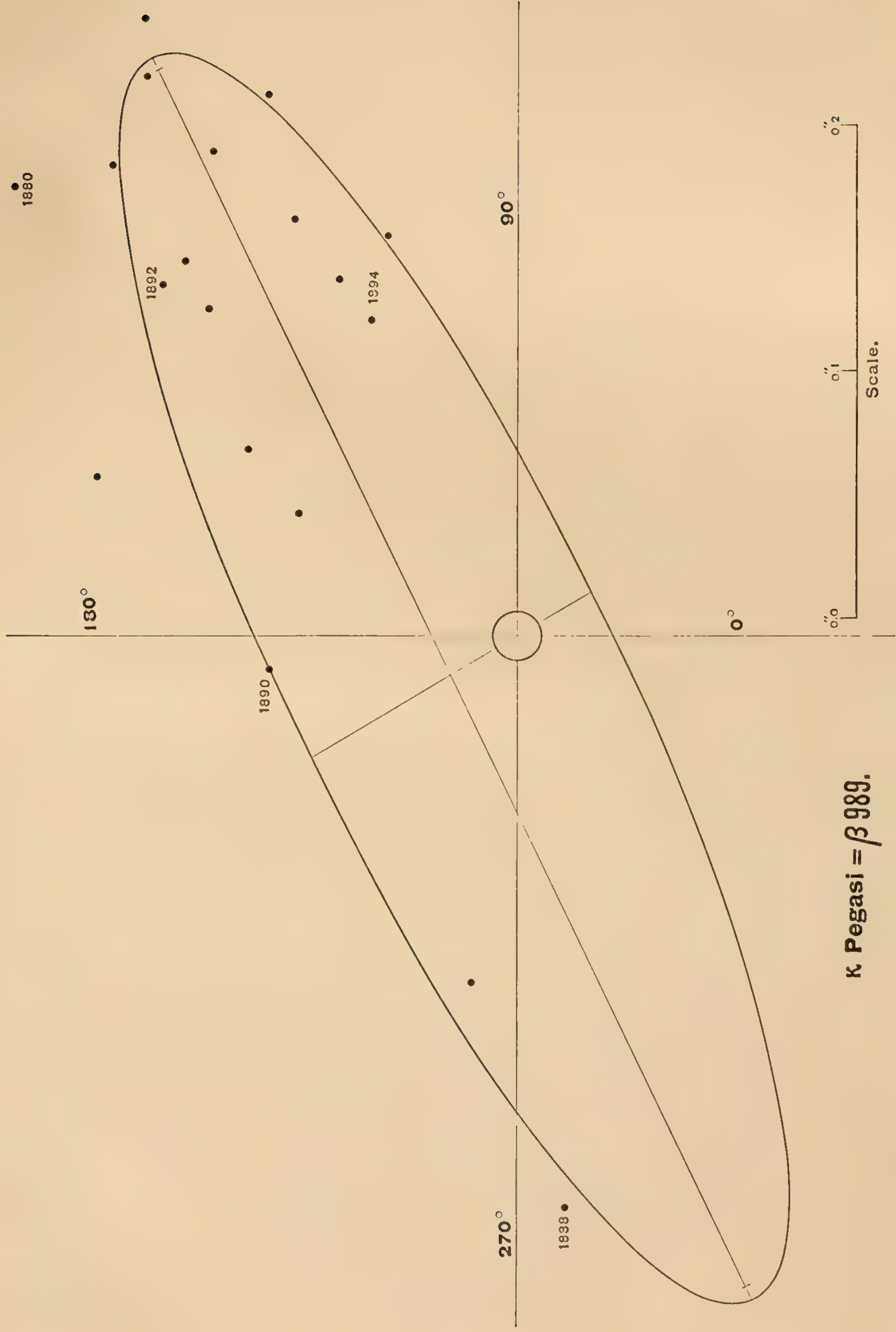
Having collected all the observations of this difficult star, including some unpublished measures kindly furnished by BARNARD last Autumn, we have investigated the orbit by the method of KLINKERFUES, and find the following elements:

$$\begin{array}{ll}
 P = 11.42 \text{ years} & i = 81^\circ.2 \\
 T = 1896.03 & \Omega = 116^\circ.25 \\
 e = 0.49 & \lambda = 89^\circ.2 \\
 a = 0''.4216 & n = -31^\circ.5236
 \end{array}$$

Apparent orbit:

$$\begin{array}{ll}
 \text{Length of major axis} & = 0''.555 \\
 \text{Length of minor axis} & = 0''.130 \\
 \text{Angle of major axis} & = 115^\circ.7 \\
 \text{Angle of periastron} & = 30^\circ.2 \\
 \text{Distance of star from centre} & = 0''.032
 \end{array}$$

* *Astronomische Nachrichten*, 3285.



K Pegasi = β 989.

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1880.68	137.9	136.7	0.27	0.22	+ 1.2	+0.05	4	Burnham
1883.02	116.0	119.5	0.16	0.27	- 3.5	-0.11	1	Englemann
1888.78	274.7	274.1	0.23	0.21	+ 0.6	+0.02	3	Burnham
1889.51	262.3	257.9	0.14	0.15	+ 4.4	-0.01	4	Burnham
1890.57	187.1	191.5	0.10	0.10	- 4.4	$\pm$ 0.00	4	Burnham
1891.61	150.0	145.0	0.10	0.18	+ 5.0	-0.08	3	Burnham
1891.81	144.6	140.2	0.13	0.20	+ 4.4	-0.07	4	Burnham
1891.92	159.0	139.2	0.18	0.20	+19.8	-0.02	3	Schiaparelli
1892.39	132.8	133.2	0.18	0.24	- 0.	-0.06	4	Burnham
1892.88	131.0	129.1	0.20	0.26	+ 1.9	-0.06	1	Barnard
1892.96	135.1	128.2	0.20	0.26	+16.9	-0.06	4	Schiaparelli
1893.51	121.0	125.5	0.29	0.27	- 4.5	+0.02	3	Leavenworth
1893.77	127.5	123.2	0.20	0.28	+ 4.3	-0.08	2	Barnard
1893.82	130.5	123.0	0.25	0.28	+ 7.5	-0.03	2-1	Comstock
1893.92	123.6	122.2	0.27	0.28	+ 1.4	-0.01	8	Schiaparelli
1894.51	117.6	118.8	0.19	0.26	- 1.2	-0.07	7-6	Barnard
1894.83	114.8	116.7	0.14	0.25	- 1.9	-0.11	4	Lewis
1894.87	114.7	116.6	0.24	0.25	- 1.9	-0.01	6	Schiaparelli
1895.62	107.9	106.7	0.17	0.16	+ 1.2	+0.01	6	Barnard

EPHEMERIS.

t	θ_c	ρ_c	t	θ_c	ρ_c
1896.80	299.4	0.21	1899.80	279.0	0.24
1897.80	292.6	0.27	1900.80	260.4	0.16
1898.80	287.0	0.28			

The agreement must be considered very satisfactory when account is taken of the extreme closeness of the components, and the high inclination of the orbit, which permits a small error in angle to have a marked effect on the distance. From an examination of all the measures it seems probable that most observers have underestimated the distances, and this certainly must have been the case with DR. ENGLEMAN, who used only a 7.5-inch refractor, and therefore could not have divided the components at a distance of 0".16. The computed distance is therefore much more probable, and especially since the elements are based principally upon the excellent measures of BURNHAM and BARNARD, made with the 36-inch refractor of the Lick Observatory.

BURNHAM has repeatedly called the attention of astronomers to the high importance of systematically following such extremely rapid binaries with large telescopes, so that we could in a few years derive orbits, which, in the case of most stars, would require the observations of centuries.

We would beg to add that it is not only important to observe κ Pegasi annually, but especially at certain critical parts of its orbit, where measures would enable us to fix the eccentricity and the inclination more accurately. Thus, according to the above elements, the minimum distance will occur just

after periastron passage in 1896.03, and measures made on either side of the periastron will be very valuable. At the minimum distance ($0''.034$) the star will be single in the largest telescope in the world, but it would be important to ascertain just when this disappearance takes place, and how long it lasts. According to the above orbit, the companion ought to be visible in a 30-inch refractor until August, 1895, and hence we suggest that observers should watch for it during the Summer of 1895 and the Autumn of 1896. Good observations at these epochs will be of the greatest value in improving the elements of the orbit.

85 PEGASI = β 733.

$\alpha = 23^{\text{h}} 56^{\text{m}}.9$; $\delta = +26^{\circ} 34'$.
6, yellowish ; 10, bluish.

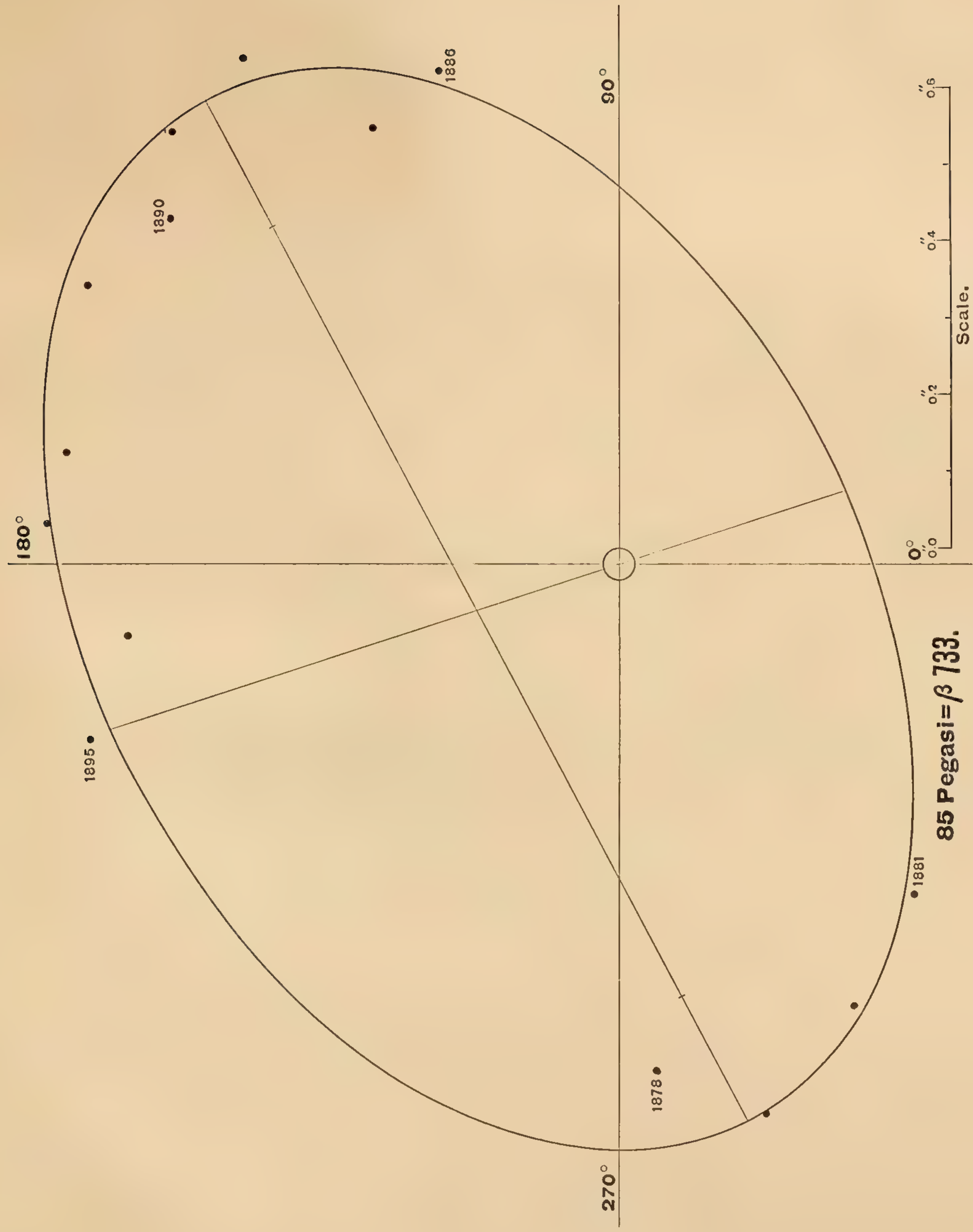
Discovered by Burnham in 1878.

OBSERVATIONS.

t	θ_0	ρ_0	n	Observers	t	θ_0	ρ_0	n	Observers
1878.73	274.0	0.67	3	Burnham	1889.59	134.7	0.94	5	Burnham
1879.46	284.6	0.75	5	Burnham	1889.90	137.0	0.70	5	Schiaparelli
1880.59	298.3	0.65	5	Burnham	1890.55	139.0	0.78	4	Burnham
1880.79	297.2	0.66	3-2	Hall	1890.96	146.4	0.71	6	Schiaparelli
1881.54	311.5	0.58	1	Burnham	1891.56	151.8	0.79	3	Burnham
1882.62	89.4	0.64	1	O. Struve	1891.94	152.7	0.78	3	Schiaparelli
1883.75	333 $\pm$	—	1	Burnham	1892.75	169.7	0.57	1	Burnham
1886.91	109.1	0.79	3	Hall	1892.94	167.3	0.74	4	Schiaparelli
1886.98	111.0	0.58	1	Schiaparelli	1893.96	176.1	0.75	6-3	Schiaparelli
1887.91	119.3	0.66	1	Schiaparelli	1894.54	178.6	0.84	5	Barnard
1888.69	126.7	0.95	5	Burnham	1894.88	251.8	0.85	1	Lewis
1888.96	124.1	0.83	3	Hall	1894.93	188.6	0.65	2-1	Schiaparelli
1888.96	128.3	0.70	7	Schiaparelli	1895.65	190.2	0.80	10-9	Barnard
					1895.73	198.4	0.73	3	See
					1895.74	204.8	0.75	2	Moulton

Since BURNHAM's discovery of this rapid binary, the companion has described an arc of 285° .* The components are of the 6th and 11th magnitudes, and so great an inequality in brightness combined with the closeness of the pair, renders exact measurement very difficult. Therefore it is not strange that

* *Astronomische Nachrichten*, 3339.



85 Pegasi = β 733.

the position-angles as well as the distances obtained by the same or by different observers should occasionally exhibit sensible discrepancies. Yet when the measures are properly combined into suitable yearly means we obtain a series of places which will give an orbit that is substantially correct.

The first orbit of this pair was computed by PROFESSOR SCHAEBERLE in 1889; his elements are:

$$\begin{array}{ll} P = 22.3 \text{ years} & \Omega = 306^\circ.1 \\ T = 1884.00 & i = 68^\circ.6 \\ e = 0.35 & \lambda = 70^\circ.3 \\ a = 0''.96 & n = +16^\circ.144 \end{array}$$

This orbit represents the measures prior to 1891 with the desired accuracy, but the error in angle rapidly accumulated and in 1892 surpassed 20° . Accordingly, PROFESSOR GLASENAPP attempted an improvement of the orbit (*A.N.* 3145), and obtained a set of elements which rendered the residuals in angle exceedingly small:

$$\begin{array}{ll} P = 17.487 \text{ years} & \Omega = 307^\circ.32 \\ T = 1884.21 & i = 66^\circ.74 \\ e = 0.164 & \lambda = 69^\circ.73 \\ a = 0''.80 & n = +20^\circ.586 \end{array}$$

Nevertheless the ephemeris computed by PROFESSOR GLASENAPP has signally failed of its purpose, as the error now amounts to about 80° . As the investigation was based wholly on angles of position we may infer that these coördinates were affected by sensible systematic errors, which might the more easily result from the inequality of the stars.

The careful measures which I recently secured at the Washburn Observatory (*A.J.* 359) have enabled me to make a new determination of the orbit based on all the material of a trustworthy character. We find the following elements of 85 *Pegasi*:

$$\begin{array}{ll} P = 24.0 \text{ years} & \Omega = 116^\circ.3 \\ T = 1883.80 & i = 55^\circ.6 \\ e = 0.388 & \lambda = 265^\circ.4 \\ a = 0''.8904 & n = +15^\circ.0 \end{array}$$

Apparent orbit:

$$\begin{array}{ll} \text{Length of major axis} & = 1''.52 \\ \text{Length of minor axis} & = 1''.00 \\ \text{Angle of major axis} & = 118^\circ.0 \\ \text{Angle of periastron} & = 18^\circ.2 \\ \text{Distance of star from centre} & = 0''.197 \end{array}$$

The accompanying table gives a comparison of the computed with the observed places.

COMPARISON OF COMPUTED WITH OBSERVED PLACES.

t	θ_o	θ_c	ρ_o	ρ_c	$\theta_o - \theta_c$	$\rho_o - \rho_c$	n	Observers
1878.73	274.0	275.5	0.67	0.77	-1.5	-0.10	3	Burnham
1879.46	284.6	282.2	0.75	0.76	+2.4	-0.01	5	Burnham
1880.69	297.7	294.4	0.66	0.69	+3.3	-0.03	8-7	Burnham 5; Hall 3-2
1881.54	311.5	309.0	0.58	0.58	+2.5	± 0.00	1	Burnham
1886.94	110.1	113.4	0.69	0.69	-3.3	± 0.00	4	Hall 3; Schiaparelli
1887.91	119.3	122.8	0.66	0.77	-3.5	-0.11	1	Schiaparelli
1888.87	126.4	130.8	0.83	0.81	-4.4	+0.02	15	β 5; Hall 3; Schiaparelli 7
1889.74	135.8	137.8	0.82	0.83	-2.0	-0.01	10	Burnham 5; Schiaparelli 5
1890.76	142.7	146.0	0.75	0.83	-3.3	-0.08	10	Burnham 4; Schiaparelli 6
1891.75	152.2	154.7	0.79	0.81	-2.5	-0.02	6	Burnham 3; Schiaparelli 3
1892.85	168.5	165.0	0.74	0.78	+3.5	-0.04	5-4	Burnham 1-0; Schiaparelli 4
1893.96	176.1	176.4	0.75	0.75	-0.3	± 0.00	6-3	Schiaparelli
1894.93	188.6	187.5	0.65	0.72	+1.1	-0.07	2-1	Schiaparelli
1895.73	198.4	197.4	0.73	0.70	+1.0	+0.03	3	See

We are justified in predicting that the true period of 85 *Pegasi* will not differ from the value given above by more than one year, and that the error of the eccentricity will not surpass ± 0.02 . The good representation of the angles and distances shows that the other elements are equally satisfactory. The foregoing elements will therefore never be greatly changed; but some improvement is desirable, and observers with great telescopes should continue to give this important system regular attention. The following is an ephemeris for the next five years:

t	θ_c	ρ_c	t	θ_c	ρ_c
1896.70	209.6	0.70	1899.70	245.8	0.74
1897.70	222.4	0.69	1900.70	256.1	0.76
1898.70	234.5	0.71			

CHAPTER III.

RESULTS OF RESEARCHES ON THE ORBITS OF FORTY BINARY STARS, WITH GENERAL CONSIDERATIONS RESPECTING THE STELLAR SYSTEMS.

§ 1. *Elements of the Orbits of Forty Binary Stars.*

IN THE preceding chapter we have presented detailed researches on the orbits of forty stars. To enable the reader to grasp readily the existing state of our knowledge, we have also included diagrams of the apparent ellipses, and of the mean observations from which the elements were derived. In many cases we have seen that the observations are relatively rough, and that while the errors are small absolutely, they are yet very large in comparison with the minute quantities measured. Under these circumstances it seemed useless to attempt a Least-Square adjustment of the residuals, and hence we have throughout employed graphical methods, and arrived at the adopted elements by successive approximations of an empirical character. Accordingly, the orbits are not definitive, but for reasons indicated in the several cases the changes which future observations may necessitate will be confined within narrow limits.

In the following Table we give a summary of the elements, with the probable uncertainty still attaching to the period and the eccentricity. From the variations of these elements it is easy to see about the extent of the alterations which may be required in the adopted values of the other elements. The final changes which future observations may produce in any given orbit can not yet be determined with certainty, and hence our variations may occasionally turn out somewhat too small; but as care has been exercised to avoid over-estimation of the accuracy of results, the values here indicated ought not to prove very deceptive.

In glancing over the apparent orbits of the preceding chapter the reader should remember that the adopted elements depend not only on the agreement of the observed distances with the apparent ellipses, but also on the accuracy with which the law of areas is satisfied. These two criteria seem to justify the comparatively small variations indicated in the Table of elements; but as

the orbits here presented depend essentially on the observations employed, and as our choice is to some extent a matter of judgement, it is not certain that we have always arrived at the best results.

RESULTS OF RESEARCHES ON THE

Star	α	δ	P	T	e	a''	Ω	i	λ
$\Sigma 3062$	0 1 ^{h m}	+57° 53'	104.61 ^{yrs.} ± 2.0	1836.26	0.450 ± 0.02	1.3712	47.15	43.85	90.9
η Cassiopeae = $\Sigma 60$	0 42.9	+57 18	195.76 ± 10.0	1907.84	0.514 ± 0.03	8.2128	46.1	45.95	217.87
γ Androm. BC = $O\Sigma 38$	1 57.8	+41 51	54.0 ± 1.0	1892.1	0.857 ± 0.02	0.3705	113.4	77.85	200.1
α Can. Maj. = Sirius	6 40.4	-16 34	52.20 ± 2.0	1893.50	0.620 ± 0.02	8.0316	34.3	46.77	131.03
F. 9 Argūs = $\beta 101$	7 47.1	-13 38	22.0 ± 1.0	1892.30	0.700 ± 0.02	0.6549	95.5	77.72	75.28
ζ Cancri AB = $\Sigma 1196$	8 6.2	+17 58	60.0 ± 0.5	1870.40	0.340 ± 0.03	0.8579	88.7	7.4	264.0
$\Sigma 3121$	9 12.1	+29 0	34.0 ± 1.0	1878.30	0.330 ± 0.03	0.6692	28.25	75.0	127.52
ω Leonis = $\Sigma 1356$	9 23.1	+ 9 30	116.20 ± 1.0	1842.10	0.537 ± 0.01	0.8824	146.70	63.47	124.22
φ Urs. Maj. = $O\Sigma 208$	9 45.3	+54 33	97.0 ± 5.0	1884.0	0.440 ± 0.03	0.3440	160.3	30.5	15.9
ξ Urs. Maj. = $\Sigma 1523$	11 12.9	+32 6	60.0 ± 0.1	1875.22	0.397 ± 0.005	2.5080	100.8	55.92	126.33
$O\Sigma 234$	11 25.4	+41 50	77.0 ± 5.0	1880.10	0.302 ± 0.04	0.3467	157.5	50.8	206.8
$O\Sigma 235$	11 26.7	+61 38	80.0 ± 5.0	1834.30	0.324 ± 0.05	0.8690	81.7	49.32	137.78
γ Centauri = H ₂ 5370	12 36	-48 25	88.0 ± 3.0	1848.0	0.800 ± 0.03	1.0232	4.6	62.15	194.3
γ Virginis = $\Sigma 1670$	12 36.6	- 0 54	194.0 ± 4.0	1836.53	0.897 ± 0.005	3.9890	50.4	31.0	270.0
F. 42 Com. Ber. = $\Sigma 1728$	13 5.1	+18 4	25.56 ± 0.1	1885.69	0.461 ± 0.01	0.6416	11.9	90 $\pm$	280.5
$O\Sigma 269$	13 28.3	+35 46	48.8 ± 1.0	1882.80	0.361 ± 0.05	0.3248	46.2	71.3	32.63
25 Can. Ven. = $\Sigma 1768$	13 33	+36 48	184.0 ± 25.0	1866.0	0.752 ± 0.05	1.1307	123.0	33.5	201.0
α Centauri	14 32.6	-60 25	81.10 ± 0.3	1875.70	0.528 ± 0.005	17.700	25.15	79.30	52.0
$O\Sigma 285$	14 41.7	+42 48	76.67 ± 5.0	1882.53	0.470 ± 0.05	0.3975	62.2	41.95	162.23
ξ Boötis = $\Sigma 1888$	14 46.8	+19 31	128.0 ± 1.0	1903.90	0.721 ± 0.02	5.5578	10.5	52.28	239.25
η Cor. Bor. = $\Sigma 1937$	15 19.1	+30 39	41.60 ± 0.1	1892.50	0.267 ± 0.01	0.9165	27.1	58.5	217.57
μ^2 Boötis = $\Sigma 1938$	15 20.7	+37 43	219.42 ± 10.0	1865.30	0.537 ± 0.03	1.2679	163.8	43.9	329.75
$O\Sigma 298$	15 32.4	+40 9	52.0 ± 1.0	1883.0	0.581 ± 0.02	0.7989	1.9	60.9	26.1
γ Cor. Bor. = $\Sigma 1967$	15 38.5	+26 36	73.0 ± 2.0	1841.0	0.482 ± 0.05	0.7357	110.7	82.63	97.95
ξ Scorp. AB = $\Sigma 1998$	15 58.9	-11 5	104.0 ± 4.0	1864.60	0.131 ± 0.05	1.3612	9.5	70.3	111.6
σ Cor. Bor. = $\Sigma 2032$	16 11	+34 7	370.0 ± 25.0	1821.80	0.540 ± 0.04	3.8187	30.5	47.48	47.7
ζ Herculis = $\Sigma 2084$	16 37.6	+31 47	35.0 ± 0.3	1864.80	0.497 ± 0.03	1.4321	37.5	51.77	101.7
β 416 = Lac. 7215	17 12.1	-34 52	33.0 ± 1.0	1891.85	0.512 ± 0.03	1.2212	144.6	37.35	86.1
$\Sigma 2173$	17 25.3	- 0 59	46.0 ± 0.4	1869.50	0.200 ± 0.03	1.1428	153.7	80.75	322.2
μ^1 Herculis BC = A.C. 7	17 42.6	+27 47	45.0 ± 1.0	1879.80	0.219 ± 0.02	1.3900	61.4	64.28	180.0
τ Ophiuchi = $\Sigma 2262$	17 57.6	- 8 11	230.0 ± 15.0	1815.0	0.592 ± 0.05	1.2495	76.4	57.6	18.05
F. 70 Ophiuchi = $\Sigma 2272$	18 0.4	+ 2 33	88.3954 ± 1.0	1896.4661	0.500 ± 0.02	4.548	125.7	58.42	198.25
F. 99 Herculis = A.C. 15	18 3.2	+30 33	54.5 ± 3.0	1887.70	0.781 ± 0.02	1.014	indeter.	0.0	(*)
ζ Sagittarii	18 56.3	-30 1	18.85 ± 1.0	1878.80	0.279 ± 0.02	0.6860	69.3	67.32	328.1
γ Coronae Australis	18 59.6	-37 12	152.7 ± 5.0	1876.80	0.420 ± 0.02	2.453	72.3	34.0	180.2
β Delphini = $\beta 151$	20 32.9	+14 15	27.66 ± 1.0	1883.05	0.373 ± 0.03	0.6724	3.9	61.35	164.93
F. 4 Aquarii = $\Sigma 2729$	20 46.1	- 6 1	129.0 ± 5.0	1899.40	0.514 ± 0.03	0.7320	177.7	72.53	68.63
δ Equulei AB = $O\Sigma 535$	21 9.6	+ 9 37	11.45 ± 0.2	1892.80	0.165 ± 0.02	0.452	22.2	79.0	0.00
κ Pegasi = $\beta 989$	21 40.1	+25 11	11.42 ± 0.4	1896.03	0.490 ± 0.1	0.4216	116.25	81.2	89.2
F. 85 Pegasi = $\beta 733$	23 56.9	+26 34	24.0 ± 1.0	1883.80	0.388 ± 0.02	0.8904	116.3	55.6	256.4

(*) Angle Per. = $169^\circ.5$.

In the course of the next twenty years a sensible improvement can be effected in the orbits of rapidly moving stars, such as κ Pegasi; but mean-

while the elements here adopted will give ephemerides sufficiently exact for the use of observers.

Vigorous prosecution of the measurement of double stars will furnish the

ORBITS OF FORTY BINARY STARS.

n	Maj. Axis App. Orbit	Min. Axis App. Orbit	Angle of Maj. Axis	Angle of Periastr.	Star from Center	$\frac{\rho}{\alpha}$	$\pm \frac{\kappa}{\rho}$	Magnitude	Light-ratio	Colors	Proper Motion	Γ	Γ'
$^{\circ}$	$''$	$''$	$^{\circ}$	$^{\circ}$	$''$						$''$	$^{\circ}$	$^{\circ}$
+ 3.4414	2.526	1.984	45.7	138.4	0.446	0.572	0.261	6.9 ; 7.5	1 : 1.75	yellowish bluish white	0.267	67.4	59.3
+ 1.8390	15.81	10.24	55.8	254.5	3.80	1.320	0.716	4 ; 7	1 : 15.85	yellow purple	1.199	70.2	56.0
- 6.6667	0.706	0.084	109.9	289.0	0.298	0.611	0.232	5.5 ; 7	1 : 3.99	bluish bluish	. .	13.1	32.2
- 6.8966	14.63	9.50	50.7	252.4	4.16	1.161	0.715	1 ; 10	1 : 3981	white yellow	1.306	83.0	70.7
+ 16.3636	0.941	0.267	99.2	134.5	0.152	0.595	0.977	5.7 ; 6.3	1 : 1.74	yellow yellow	. .	54.0	68.3
- 6.0000	1.704	1.632	8.8	184.9	0.290	0.659	0.053	5.5 ; 6.2	1 : 1.91	yellow yellow	0.115	62.4	67.6
+ 10.5883	1.318	0.349	27.4	189.6	0.142	0.670	0.716	7.2 ; 7.5	1 : 1.32	white yellowish	0.523	81.1	56.2
+ 3.0981	1.576	0.738	141.1	293.4	0.317	0.460	0.387	6 ; 7	1 : 2.51	yellow yellow	0.040	13.4	65.7
+ 3.7114	0.690	0.530	167.6	174.1	0.149	1.015	0.032	5.5 ; 5.5	1 : 1	yellowish yellowish	0.028	26.4	64.2
- 6.0000	4.760	2.700	104.6	318.0	0.750	0.665	0.062	4 ; 5	1 : 2.51	yellow yellowish	0.736	59.4	57.5
+ 4.6754	0.695	0.437	158.0	355.2	0.098	0.924	0.399	7 ; 7.8	1 : 2.09	yellowish yellowish	0.115	42.1	64.6
+ 4.5000	1.682	1.020	72.8	231.1	0.242	0.912	0.577	6 ; 7.8	1 : 5.25	yellowish yellowish	0.129	89.4	19.3
- 4.0911	2.100	0.580	0.1	177.8	0.794	0.198	0.438	4 ; 4	1 : 1	yellowish yellowish	. .	83.3	74.0
- 1.8557	6.824	3.530	140.4	140.4	3.062	0.172	0.403	3 ; 3.2	1 : 1.20	yellow yellow	0.578	54.2	50.8
$\pm$ 14.0867	1.147	0.00	11.9	11.9	0.054	0.575	0.187	6 ; 6	1 : 1	orange orange	0.488	84.9	84.9
+ 7.3771	0.64	0.20	47.7	57.8	0.102	0.779	0.849	7.3 ; 7.7	1 : 1.45	yellowish yellowish	. .	69.3	60.5
- 1.9565	1.910	1.08	108.9	285.4	0.714	0.525	0.298	5 ; 8.5	1 : 25.12	white blue	0.114	19.8	47.3
+ 4.4390	32.18	6.16	27.25	38.65	5.90	0.670	0.982	1 ; 2	1 : 2.51	or. yellow or. yellow	3.685	47.8	88.1
- 4.6953	0.788	0.522	67.1	255.3	0.182	0.851	0.146	7.5 ; 7.6	1 : 1.10	yellowish whitish	. .	48.0	47.7
- 2.8125	9.07	5.76	167.7	144.7	2.94	1.100	0.790	4.5 ; 6.5	1 : 6.31	yellow purple	0.161	80.4	24.7
+ 8.6538	1.804	0.934	28.7	229.0	0.209	1.152	0.661	5.5 ; 6	1 : 1.59	yellowish yellowish	0.217	30.0	89.5
- 1.6407	2.656	1.480	173.5	186.7	0.638	0.912	0.419	6.5 ; 8	1 : 3.98	white white	0.194	11.5	77.5
+ 6.9231	1.546	0.656	186.9	15.3	0.427	0.582	0.695	7 ; 7.4	1 : 1.45	yellowish yellowish	0.500	25.5	83.1
- 4.9315	1.300	0.175	111.3	329.6	0.068	0.725	0.973	4 ; 7	1 : 15.85	yellow blue	0.115	78.0	89.6
+ 3.4616	2.696	0.884	9.6	150.2	0.085	0.935	0.639	5 ; 5.2	1 : 1.20	yellow yellow	0.098	29.5	71.0
+ 0.9730	7.08	4.71	42.4	66.9	1.735	0.750	0.631	6 ; 7	1 : 2.51	yellow bluish	0.342	18.5	88.7
- 10.2843	2.498	1.752	43.1	289.0	0.455	1.180	0.559	3 ; 6	1 : 15.85	yellow bluish	0.613	86.4	21.2
- 9.0908	2.76	2.38	142.5	59.5	0.61	1.042	0.449	6.4 ; 7.8	1 : 3.63	yellowish yellowish	. .	71.9	86.0
- 7.8261	2.22	0.35	154.5	160.8	0.18	0.835	0.970	6 ; 6	1 : 1	yellow yellow	0.185	53.6	60.6
+ 8.000	2.78	1.148	61.4	241.4	0.304	0.822	0.698	9.4 ; 10	1 : 1.74	bluish white bluish	0.811	48.3	83.1
+ 1.5652	2.46	1.09	80.0	85.8	0.712	0.475	0.781	5 ; 6	1 : 2.51	yellowish yellowish	0.025	49.9	72.0
- 4.0728	9.00	4.17	122.9	295.8	2.198	1.475	0.848	4.5 ; 6.0	1 : 3.98	yellow purplish	1.123	76.7	88.9
+ 6.6055	2.028	1.278	169.5	169.5	0.792	. . .	. . .	6.0 ; 11.7	1 : 190.55	yellow purple	0.136	67.9	67.9
- 19.098	1.300	0.423	74.8	82.8	0.168	1.223	0.430	3.9 ; 4.4	1 : 1.59	yellow yellow	. .	44.6	59.5
- 2.3575	4.906	3.661	72.2	252.1	1.033	1.045	0.180	5.5 ; 5.5	1 : 1	yellowish yellowish	. .	75.7	87.3
+ 13.015	1.060	0.477	2.5	176.6	0.194	0.645	0.828	4 ; 6	1 : 6.31	yellow yellowish	0.089	53.2	30.4
+ 2.7907	1.288	0.43	0.3	215.2	0.173	1.480	0.616	6 ; 7	1 : 2.51	yellow yellow	0.064	29.8	60.5
- 31.441	0.904	0.171	22.2	22.2	0.075	0.930	0.532	4.5 ; 5.0	1 : 1.59	yellow yellow	0.300	20.1	39.4
- 31.5236	0.555	0.130	115.7	30.2	0.032	1.390	0.503	4.3 ; 5.0	1 : 1.91	yellowish yellowish	. .	68.6	87.9
+ 15.0	1.52	1.00	118.0	18.2	0.197	0.610	0.062	6.0 ; 10	1 : 39.81	yellowish bluish	1.288	80.3	65.8

material for one hundred orbits at the end of another half century, and accordingly such effort is urgently demanded by the highest interests of science.

§ 2. *Relative Velocity of the Companion in the Line of Sight
for the Epoch 1896.50.*

When the elements of the orbit are known, the theory developed in §5, Chapter I, first published in the *Astronomische Nachrichten*, No. 3314, enables us to predict the relative motion of the companion of a binary in the line of sight for any given time. The columns marked $\frac{\rho}{a}$ and $\pm \frac{\kappa}{\rho}$ in the foregoing Table contain the desired results for the epoch 1896.50. The numbers in the column $\frac{\rho}{a}$ express the orbital velocities in units of the radius of the hodograph. As the scale of this radius is unknown, except in a very few cases, we are not able to express this velocity in kilometres or in other absolute units; but when the parallaxes are determined this may be readily accomplished. The column as it stands, however, shows the rate of orbital motion, compared to what is approximately the average velocity, and we are thus enabled to select those stars which have a rapid orbital motion. If the motion of any given pair be rapid, and also mainly in the line of sight, as in the case of 70 *Ophiuchi*, the system so circumstanced will be favorable for spectroscopic measurement. The column $\pm \frac{\kappa}{\rho}$ shows what part of the orbital motion is in the line of sight, and this enables us to select for measurement with the Spectrograph those pairs which have a large orbital velocity with the major portion of it towards or from the earth.

The stars at present the most favorably situated for measurement of the relative motion in the line of vision are: η *Cassiopeae*, α *Canis Majoris*, 9 *Argûs*, ξ *Boötis*, γ *Coronae Borealis*, Σ 2173, 70 *Ophiuchi*, β *Delphini*, and α *Centauri*.

Adopting parallaxes of 0".75, 0".162, and 0".154 for α *Centauri*, 70 *Ophiuchi*, and η *Cassiopeae* respectively, we find the line-of-sight components for the several systems to be 6.66, 13.95, 8.89, where the unit is the kilometre. These quantities are well within the limit of spectroscopic measurement, and therefore an experimental determination offers an attractive problem to observers occupied with this branch of Astronomy.

It will be seen that several of the above stars are wide, while others are very close. If the two spectra can be photographed on the same plate, the lines being only slightly displaced by the relative motion of the stars, as in the case of spectroscopic binaries, the close pairs ought to be as easily measured as the wide ones, whose spectra could perhaps be photographed separately.

In any case the prosecution of these researches with the powerful spectroscopic appliances of the great telescopes of our time is an urgent *desideratum*.

of Astronomy. And until the relative motions of visible systems are thus determined there will remain some doubt as to the reality of the so-called spectroscopic binaries; not that any one doubts the theoretical validity of the DÖPPLER-HUGGINS principle, but rather that other explanations of the phenomena interpreted as spectroscopic binaries are considered possible. Moreover, the great interest attaching to investigations which will give the absolute dimensions, parallaxes and masses of binary systems, as well as the possibility of testing the validity of the law of gravitation, ought to induce astronomers to prosecute these studies with a zeal commensurate with their real importance.

Owing to the small size of the earth's orbit, it seems that our principal hope for knowledge of the dimensions of the universe must be based upon this method. The change in wave-length due to motion in the line of sight was originally pointed out by DÖPPLER, but HUGGINS was the first to apply the Spectroscope to the heavenly bodies, and to reduce DÖPPLER's principle to actual practice, and to assign it a place in modern Astronomy. The application of the principle to the determination of the dimensions of binary systems was first proposed by FOX TALBOT. But as his theory was restricted to the case of circular motion, it could not be applied to the eccentric orbits described by the stars, and accordingly it has since been somewhat varied and extended by others. The theory which we have developed is entirely general for ellipses of every possible eccentricity, and from the point of view of rigor and generality leaves nothing to be desired.

§3. *Investigation of a Possible Relation of the Orbit-Planes of Binary Systems to the Plane of the Milky Way.*

Owing to the well known arrangement of the stars and sharply-defined nebulae with respect to the Milky Way, it has been suggested that some relation might exist between the planes of the stellar orbits and this fundamental plane of the universe. An examination of this question is worthy of the attention of astronomers, and accordingly we shall compute the inclinations of the foregoing orbits by the formulae developed in the *Berliner Astronomisches Jahrbuch* for 1832. The method of transformation which ENCKE has employed enables us to refer the plane of a double-star orbit to any absolute plane in space.

Let us pass a plane through the central star parallel to the equator. The pole of this plane will meet the celestial sphere at the same point as the pole

of the heavens. Consider the triangle connecting the pole of the equator with the poles of the real and of the apparent orbit. The pole of the apparent orbit is determined by the right ascension and declination of the star (α, δ). Let the coördinates of the pole of the real orbit referred to the same axes be A and D , and let Ω' be the angle which the great circle passing through the poles of the real and apparent orbits makes with the meridian. The arc joining the poles of the orbits is the inclination, i , and this is one of the elements given in the foregoing Table. From the resulting spherical triangle we have

$$\begin{aligned}\sin D &= \cos i \sin \delta + \sin i \cos \delta \cos \Omega' = m \cos (M - \delta), \\ \cos D \sin (\alpha - A) &= \sin i \sin \Omega', \\ \cos D \cos (\alpha - A) &= \cos i \cos \delta - \sin i \sin \delta \cos \Omega' = m \sin (M - \delta),\end{aligned}$$

where

$$\sin i \cos \Omega' = m \cos M,$$

and

$$\cos i = m \sin M.$$

Then

$$\begin{aligned}\tan M &= \frac{1}{\tan i \cos \Omega'}, \\ \tan (\alpha - A) &= \frac{\sin (M - \delta)}{\cos M \tan \Omega'}, \\ \tan D &= \frac{\cos (\alpha - A)}{\tan (M - \delta)}.\end{aligned}$$

When the right ascension and declination of the pole of the real orbit have been determined, we may pass a plane through the central star parallel to the Milky Way. In the spherical triangle which joins the pole of this plane with the pole of the real orbit and the pole of the heavens, the inclination of the real orbit to the plane of the Milky Way is given by the arc connecting their poles. Thus we have

$$\cos \Gamma = \sin D \sin \delta' + \cos D \cos \delta' \cos (A - \alpha'),$$

where α' and δ' denote the coördinates of the north pole of the Milky Way.

In our computations the coördinates of the north pole of the Milky Way are taken on the authority of SIR JOHN HERSCHEL, who found

$$\alpha' = 12^{\text{h}} 47^{\text{m}} ; \quad \delta' = +27^\circ.$$

There are two solutions for Γ , owing to the two values of A and D incident to the indetermination of the ascending node; and the resulting inclinations to the Galaxy are tabulated as Γ and Γ' . Now, we do not know which of these two possible inclinations to the Milky Way is correct, but since it is impossible to select from either column any one prevailing angle, much less an evanescent inclination, we conclude that the orbits are not directly related to the Milky Way, or to any other fundamental plane of the heavens. Thus it is clear that the orbit-planes lie at all possible angles to the Milky Way, with no

marked relation to the general scheme which distinguishes the arrangement of the stars and well-defined nebulae. The consideration that the size of a stellar orbit is small compared to the dimensions of the Milky Way, and that the number of such systems is very great, might have enabled us to anticipate this result as probable *à priori*, since the condensation of nebulous matter to so many centres would almost of necessity have produced rotations in all possible planes, and even if confined originally to one plane the parallelism would have been disturbed by the action of foreign bodies during the ages required for the development of the visible universe.

§4. *High Eccentricities a Fundamental Law of Nature.*

It thus appears that the inclinations of the orbit-planes bear no definite relation to any given plane of the heavens, and an examination of the periods of revolution shows that this element likewise has no characteristic property. The periods are found to range from 11 to 370 years.

It is evident that such elements as T , a , Ω , i , λ , can have no relation to physical causes, and an inspection of the Table shows no trace of such a connection. When, however, we came to deal with the eccentricity the case is different. The results given in the preceding Table establish a most remarkable law, which is of fundamental importance in our theory of the origin and development of the stellar systems, and is besides of practical value to working astronomers.

On glancing over the eccentricities it is found that while nearly all values exist, few, if any, are very small like those of the planets and satellites, nor are any very large like those of the long-period comets. The smallest eccentricity is that of ξ *Scorpii*, $e = 0.131$, the largest that of γ *Virginis*, $e = 0.897$, the mean value for the forty orbits, $e = 0.482$.

Let us take the x -axis as the axis of eccentricity, and the y -axis as the axis of number of orbits, and divide the interval from $e = 0.0$ to $e = 1.0$ into a convenient number of parts. Then, if we erect ordinates denoting the number of orbits falling in the given intervals, and connect the points thus determined, we shall be able to illustrate the distribution of orbits as regards the region of eccentricity.

We find no orbits between 0.0 and 0.1; two between 0.1 and 0.2; four between 0.2 and 0.3; eight between 0.3 and 0.4; nine between 0.4 and 0.5; nine between 0.5 and 0.6; two between 0.6 and 0.7; four between 0.7 and 0.8;

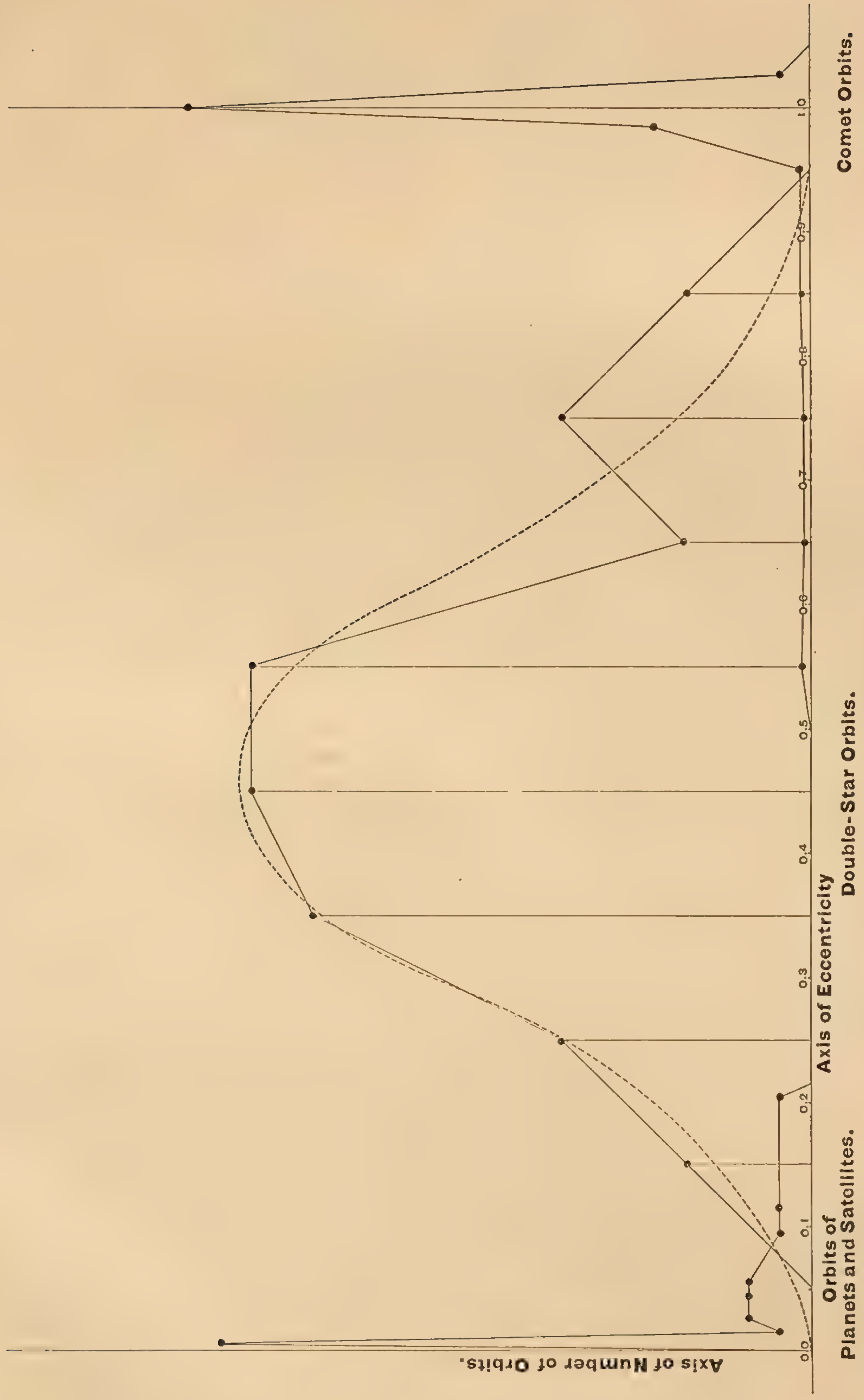
two between 0.8 and 0.9; none between 0.9 and 1.0. The distribution is illustrated by the broken line in the accompanying figure. Since the number of orbits is finite, the figure is an irregular line; if the number were indefinitely increased, the figure ought to become approximately a smooth curve.

It is evident, therefore, that the true curve of distribution of orbits resembles a probability curve with maximum near 0.482; the slope in either direction is gradual, but the curve vanishes before it reaches zero and unity. We have drawn a pointed curve to illustrate what is conceived to be the probability curve for the distribution of orbits, but it is based on forty orbits only, and therefore is necessarily provisional. We may observe, however, that forty is a number sufficiently large to realize the essential conditions underlying the theory of probability, and accordingly we are justified in the inference that the nature of the curve here indicated will never be greatly changed. There is an irregularity in the broken line between 0.6 and 0.7, which may be attributed to the effect of chance; if the number of orbits were greatly increased this gap would be filled up. In general, there will be irregularities in the distribution so long as the number of orbits is finite, but they ought to become less marked as the number is increased.

Thus, it is clear that in whatever intervals the axis of eccentricity be divided, and however the number of orbits be increased, there will remain in the curve of distribution a conspicuous maximum near 0.482, with a gradual slope in both directions. The following table shows the eccentricities of the orbits of the planets and satellites (*Inaugural Dissertation*, Berlin, 1893, p. 58):

Planet	Eccentricity	Mean Eccentricity	Planet	Eccentricity
<i>Venus</i>	0.00684	} 0.06026 {	<i>Jupiter</i>	0.04825
<i>Neptune</i>	0.00896		<i>Saturn</i>	0.05607
<i>Earth</i>	0.01677		<i>Mars</i>	0.09326
<i>Uranus</i>	0.04634		<i>Mercury</i>	0.20560

Satellite	Eccentricity	Mean Eccentricity	Satellite	Eccentricity	Mean Eccentricity
Satellite of <i>Neptune</i>		} These orbits appear to be circular {	<i>V</i> (BARNARD)		} These orbits appear to be circular {
<i>Ariel</i>			<i>Io</i>		
<i>Umbriel</i>			<i>Europa</i>		} 0.0325 {
<i>Titania</i>			<i>Ganymede</i>	0.0013	
<i>Oberon</i>			<i>Deimos</i>	0.0057	
<i>Mimas</i>			<i>Phobos</i>	0.0066	
<i>Enceladus</i>			<i>Calypso</i>	0.0072	} 0.0325 {
<i>Tethys</i>			<i>Iapetus</i>	0.0296	
<i>Dione</i>			<i>Titan</i>	0.0299	
<i>Rhea</i>			<i>Moon</i>	0.05491	
			<i>Hyperion</i>	0.1189	



The orbits of several satellites appear to be circular, or rather the eccentricity is found to be insensible in consequence of the errors of observation. We shall not underestimate these unknown eccentricities if we assign to them the mean value of the known eccentricities of the satellite orbits (0.0325). Making this maximum assumption we find that the average eccentricity for the solar system—the eight great planets and their twenty-one satellites—cannot surpass 0.0389.

In these considerations we have omitted the comets and the asteroids, because the former have been drawn to our system from outer space, while the latter have originated by an anomalous process, and depart so radically from the other bodies of the system that they cannot be considered as a type of planetary evolution, but rather as an abnormal development. It is also to be remarked that the eccentricities of the orbits of the planets and satellites are still involved in some small degree of uncertainty, and moreover they will vary from century to century owing to the cumulative effects of the secular variations and of the long-period inequalities. Notwithstanding these changes it is clear that the values of the eccentricities given above represent the true nature of the solar system.

It follows, therefore, that the average eccentricity among the double stars is more than twelve times that found in the planetary system, and this extraordinary result is manifestly the expression of a fundamental law of nature.

The eccentricities of the orbits of the stars discussed in this work are still subject to slight changes, but there is reason to believe that the average value (0.482) will never be altered except by a very small quantity. The apparent orbits given in the preceding chapter enable the reader to make a direct inspection of the linear eccentricity, and he may thus judge of the magnitude of this element, as well as of the changes it is likely to undergo. In order to minimize the uncertainty in our final data, we have purposely restricted our researches to the forty orbits which were capable of the most exact determination. Since the orbits of the forty stars will undergo no sensible improvement, at least for a good many years, it seemed of interest to present also figures of the real orbits.

In the accompanying illustrations the orbits are arranged in the order of eccentricity, and the reader is thus enabled to examine the different degrees of elongation. Accordingly, it appears that while the orbits are much more eccentric than those of the planets and satellites, they are yet much less eccentric than those of the long-period comets.

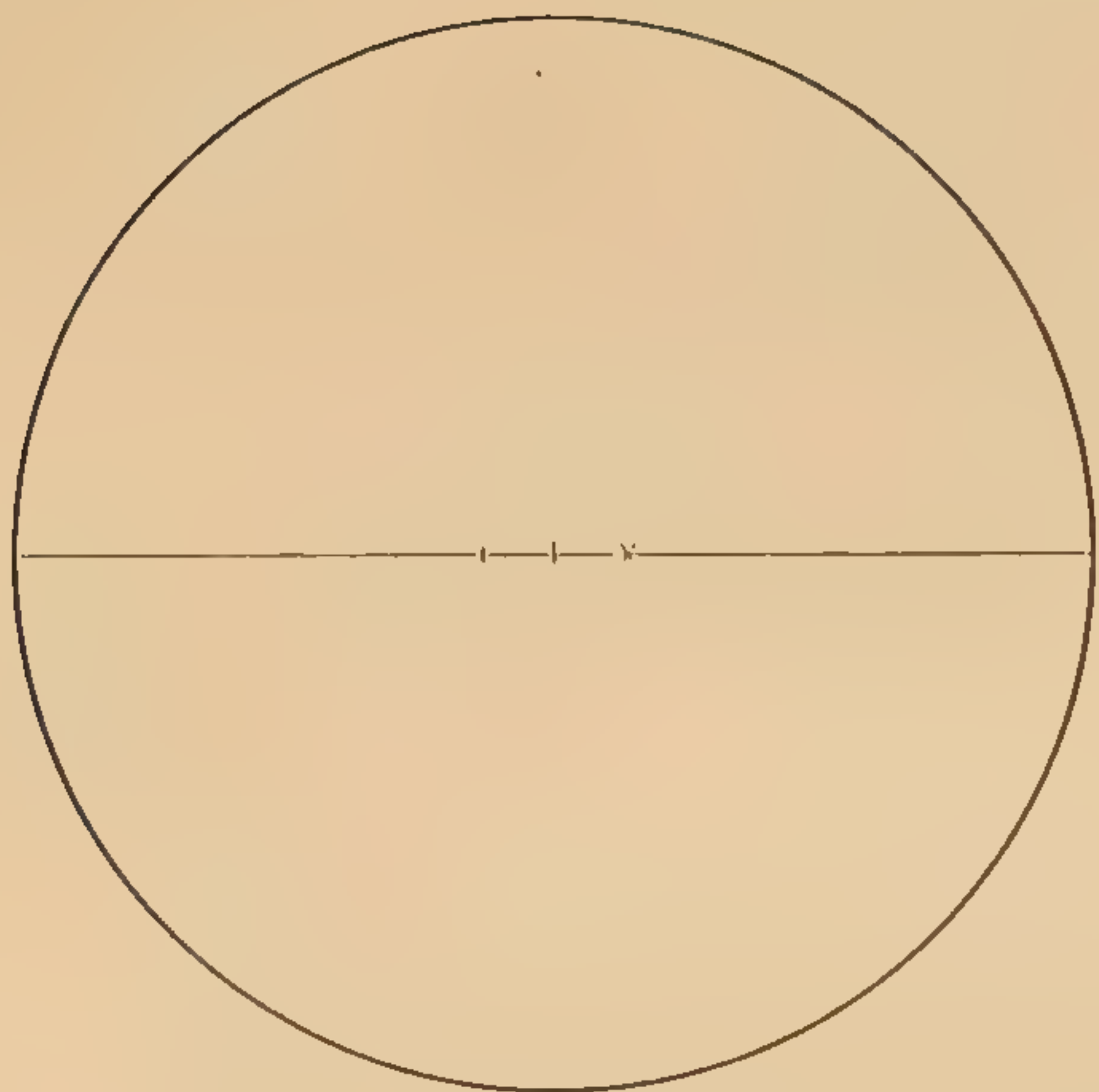
In the preceding diagram we have drawn one broken line to illustrate the

distribution of the orbits of comets, and another for the distribution of the orbits of the planets and satellites. The number of cometary orbits is so large that in this case the scale of ordinates had to be very much reduced. An inspection of these curves shows that the planetary orbits are heaped up about a very small eccentricity, while the cometary orbits cluster around the parabolic eccentricity. This characteristic of the orbits of comets indicates, as LAPLACE first pointed out, that these bodies have been drawn to our system from the regions of the fixed stars; and therefore their eccentricities surpass, equal or approximate unity. Some of the comets have passed near the larger planets, and thus suffered perturbations which have reduced their eccentricities; and hence the curve slopes down gradually on the side towards the origin. The right branch of the curve is but little known, since the great perihelion distance of hyperbolic comets enables them to pass through our system unnoticed, unless they happen to be very bright.

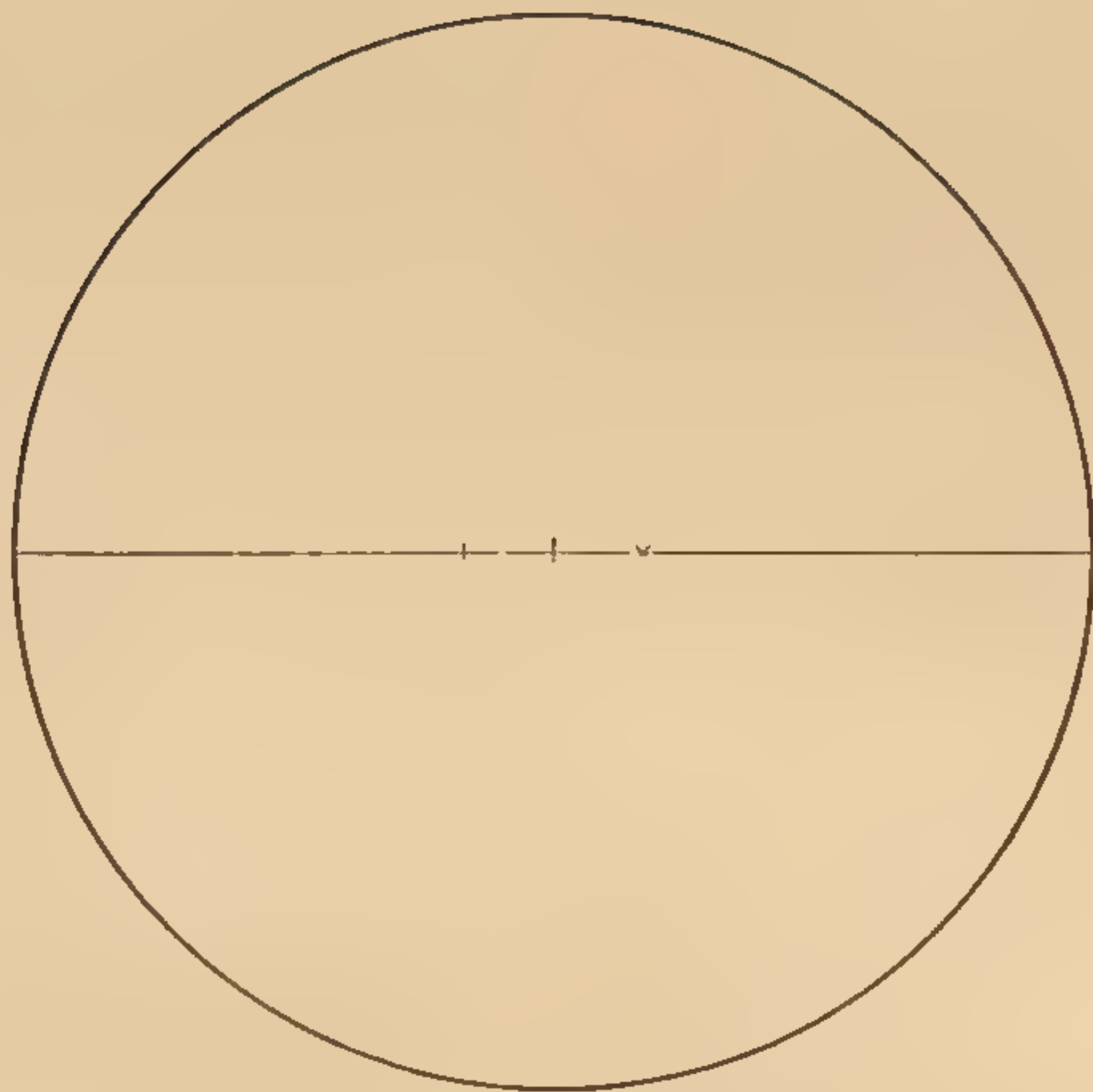
Thus it is evident that the tendency of double-star orbits is to group about a mean eccentricity which is almost equally removed from the two extremes presented in the solar system. Orbits which are so much elongated have no close analogy with those of the planets and satellites; on the other hand their lack of very great eccentricities excludes them from the category of comets, and does not permit us to assign to these systems a *fortuitous* origin. We shall see hereafter that the orbits were originally nearly circular; in the course of immeasurable ages they have been gradually expanded and elongated by the working of tidal friction in the bodies of the stars. The visible elongation of the orbits thus enables us to trace the changes of the stellar systems through millions of years, and to throw light upon the problems connected with their evolution.

In discussing the motion of γ *Virginis*, SIR JOHN HERSCHEL long ago remarked that "the eccentricity is, physically speaking, by far the most important of all the elements," and now we see that this element, which depends wholly on micrometrical measures, and is independent of the parallaxes and relative masses of the stars, gives the sole clue to the evolution of the stellar systems, and will some day enable us to lay a secure foundation for scientific Cosmogony.

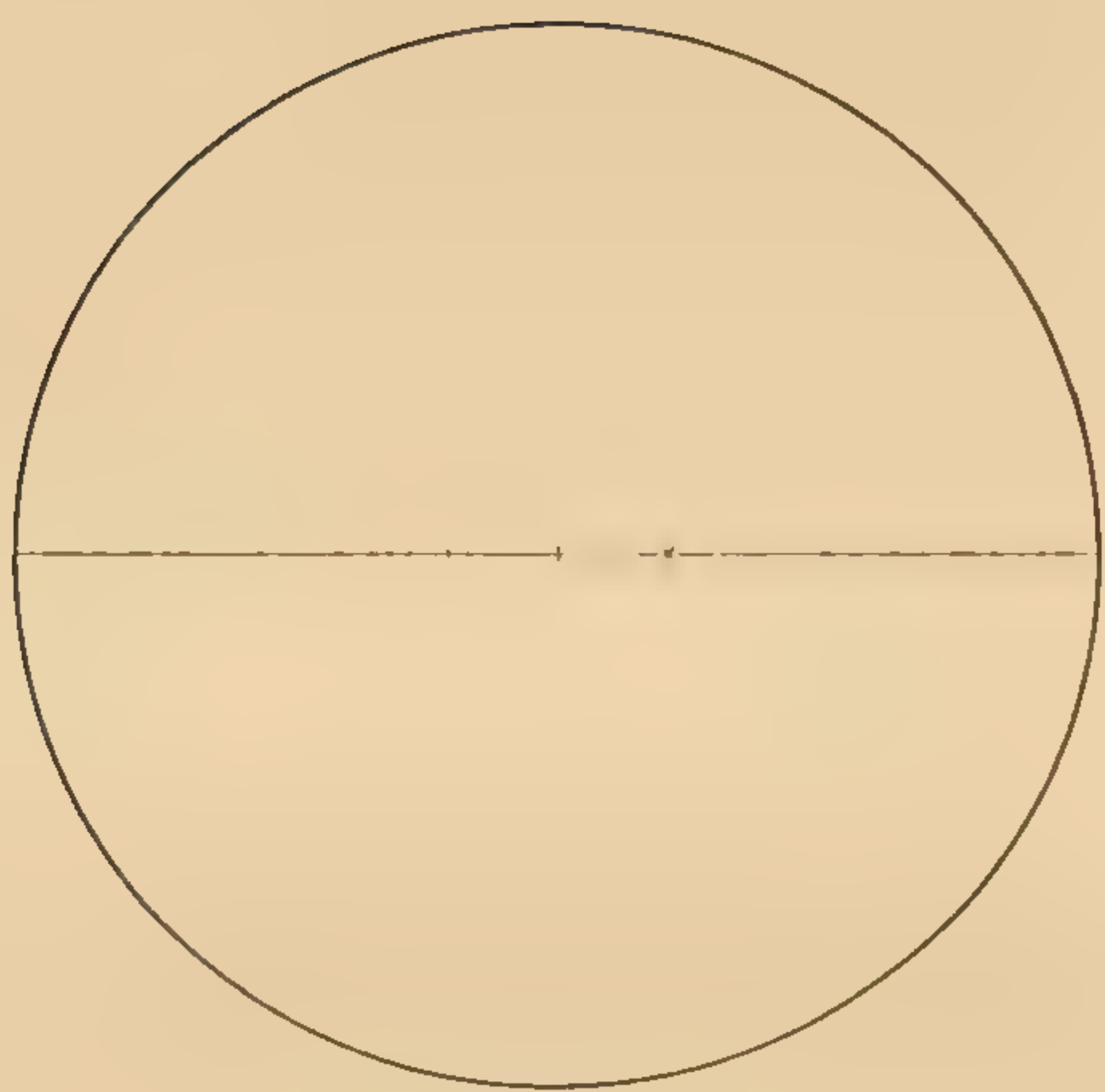
We may observe that besides throwing light upon the past condition of the universe the general law of the eccentricity here established will also be useful to practical astronomers. The eccentricity of any given orbit may depart considerably from the mean here indicated as the most probable value, yet the tendency towards this region will on the whole prove useful to computers.



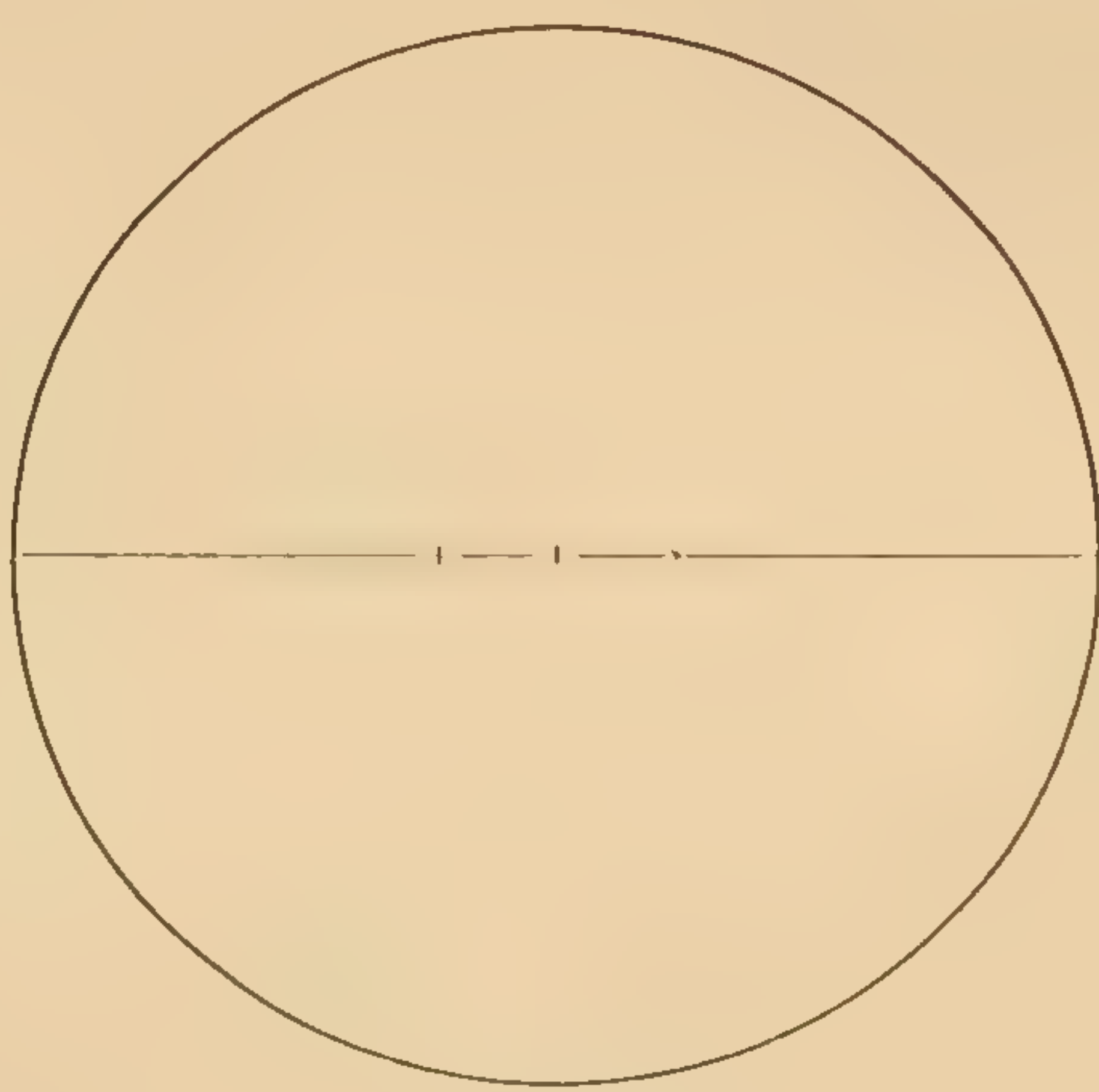
ξ Scorpii.



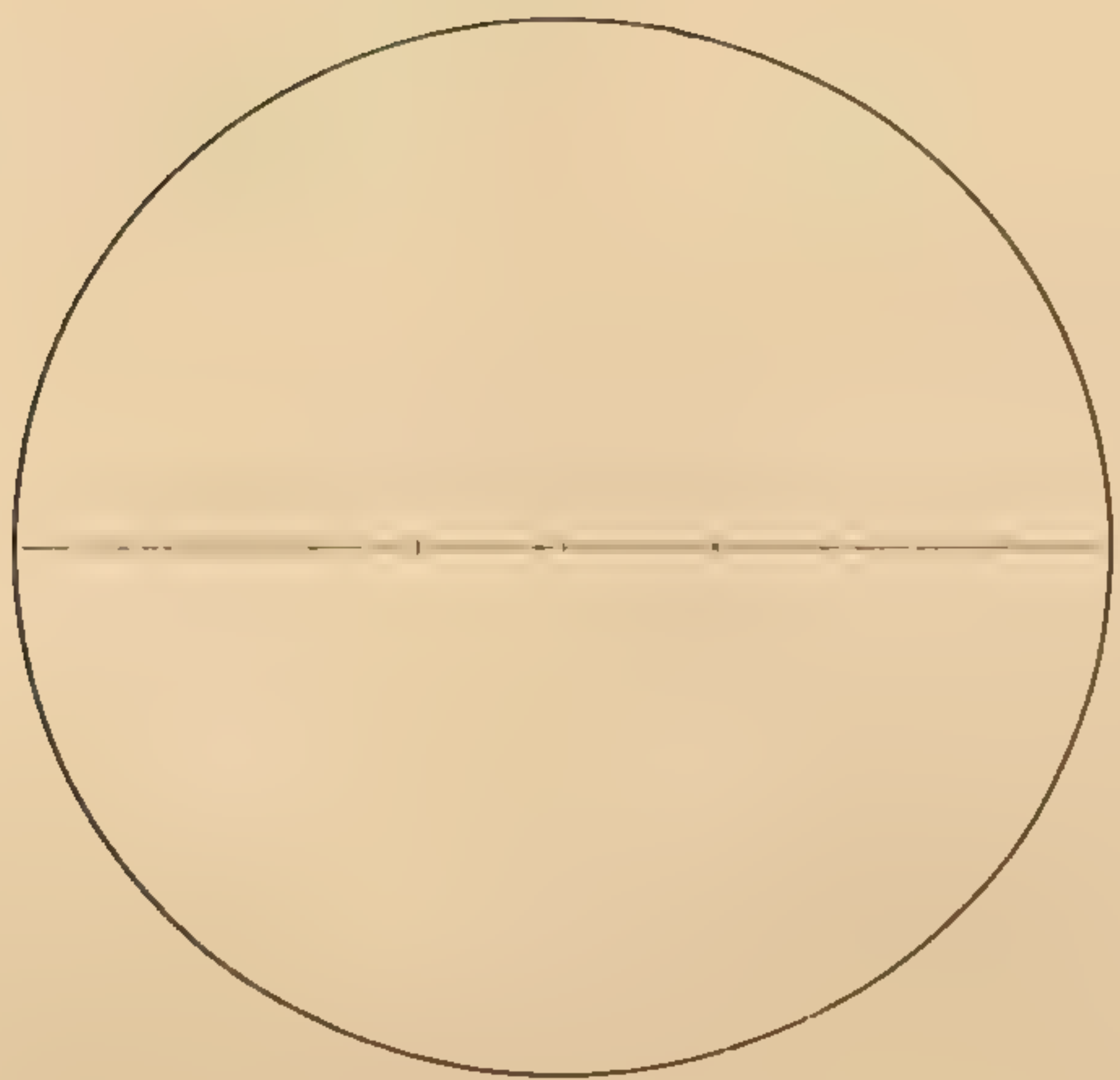
δ Equulei.



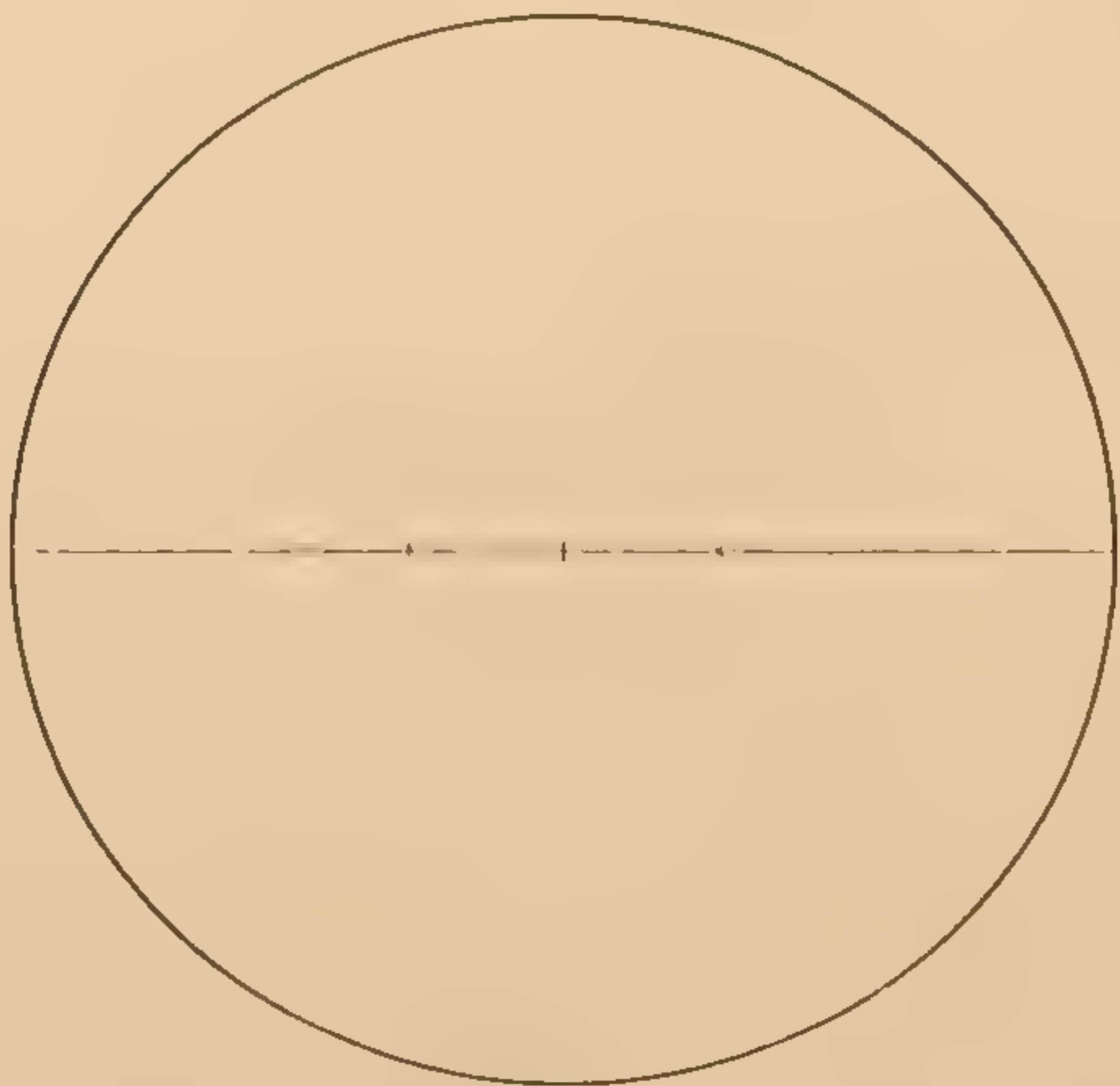
Σ 2173.



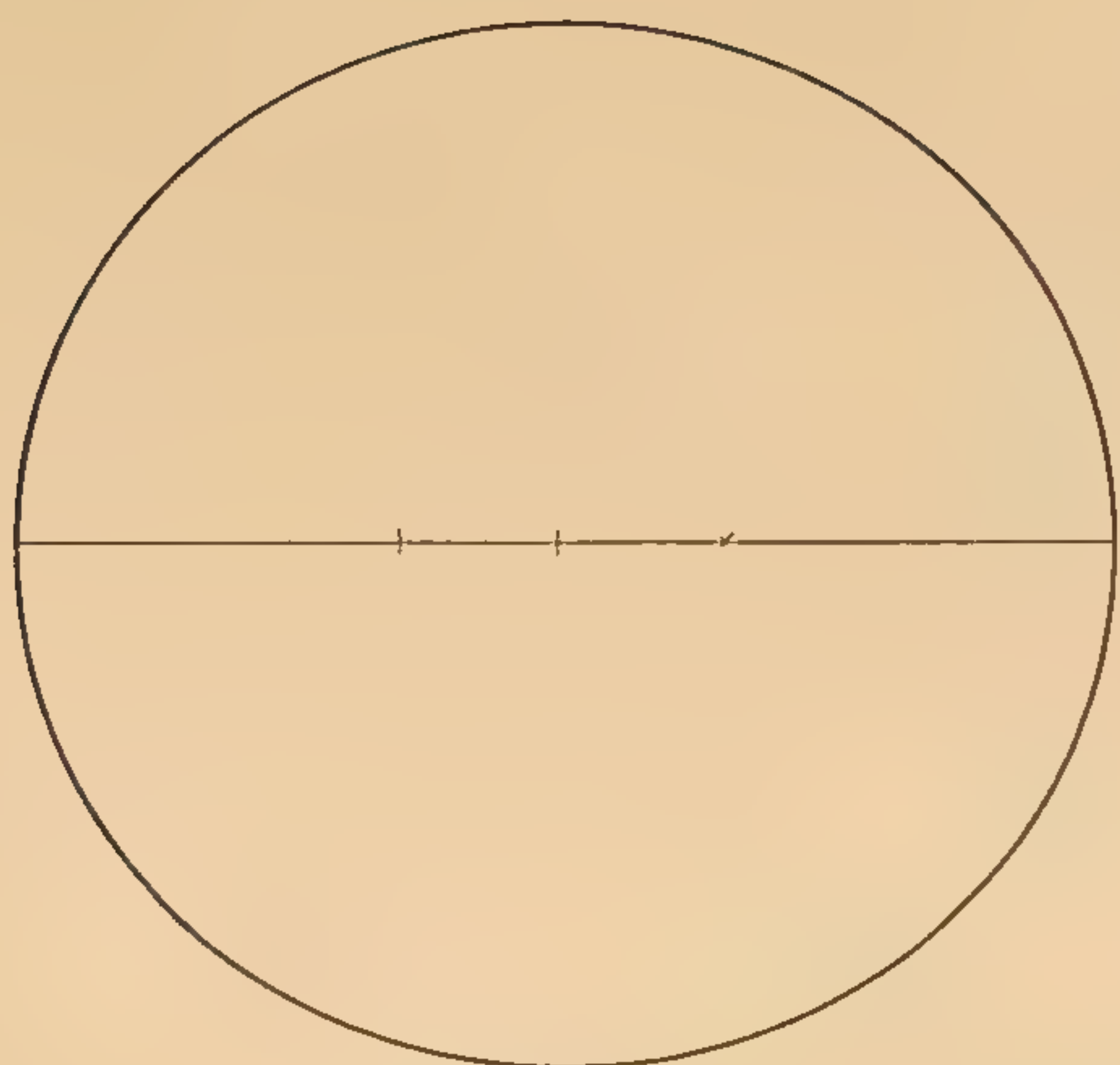
μ' Herculis.



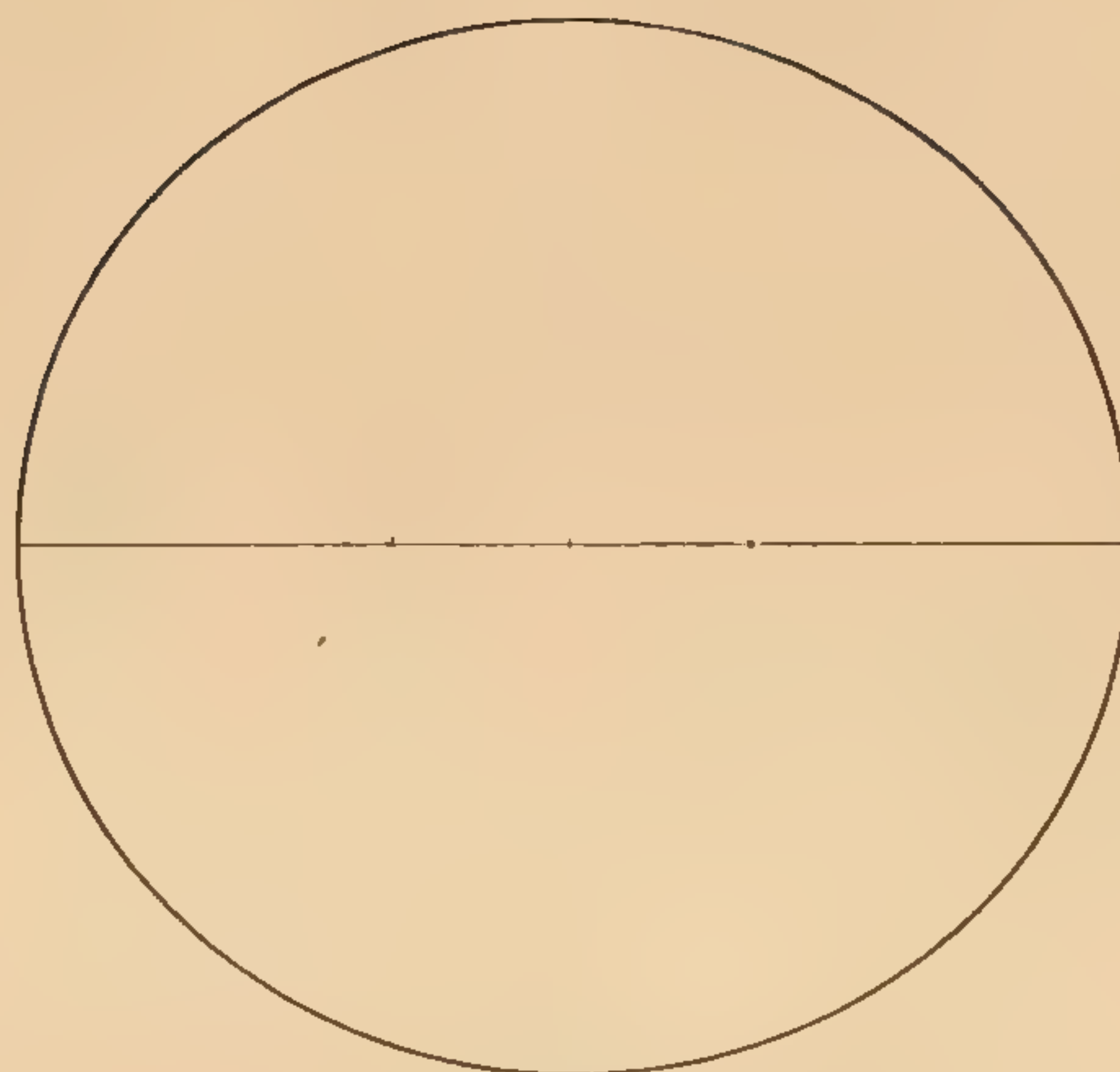
η Coronae Borealis.



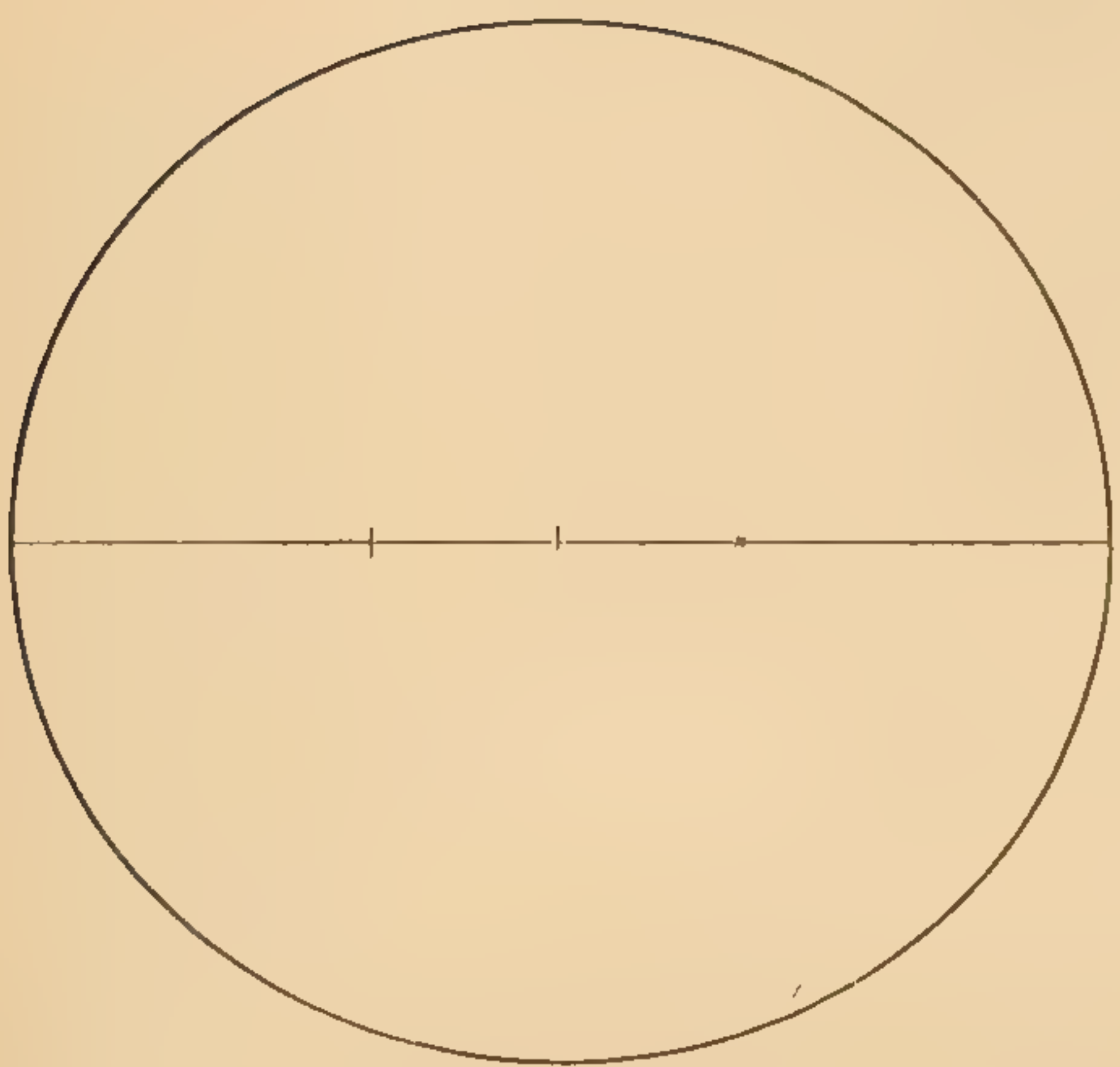
ζ Sagittarii.



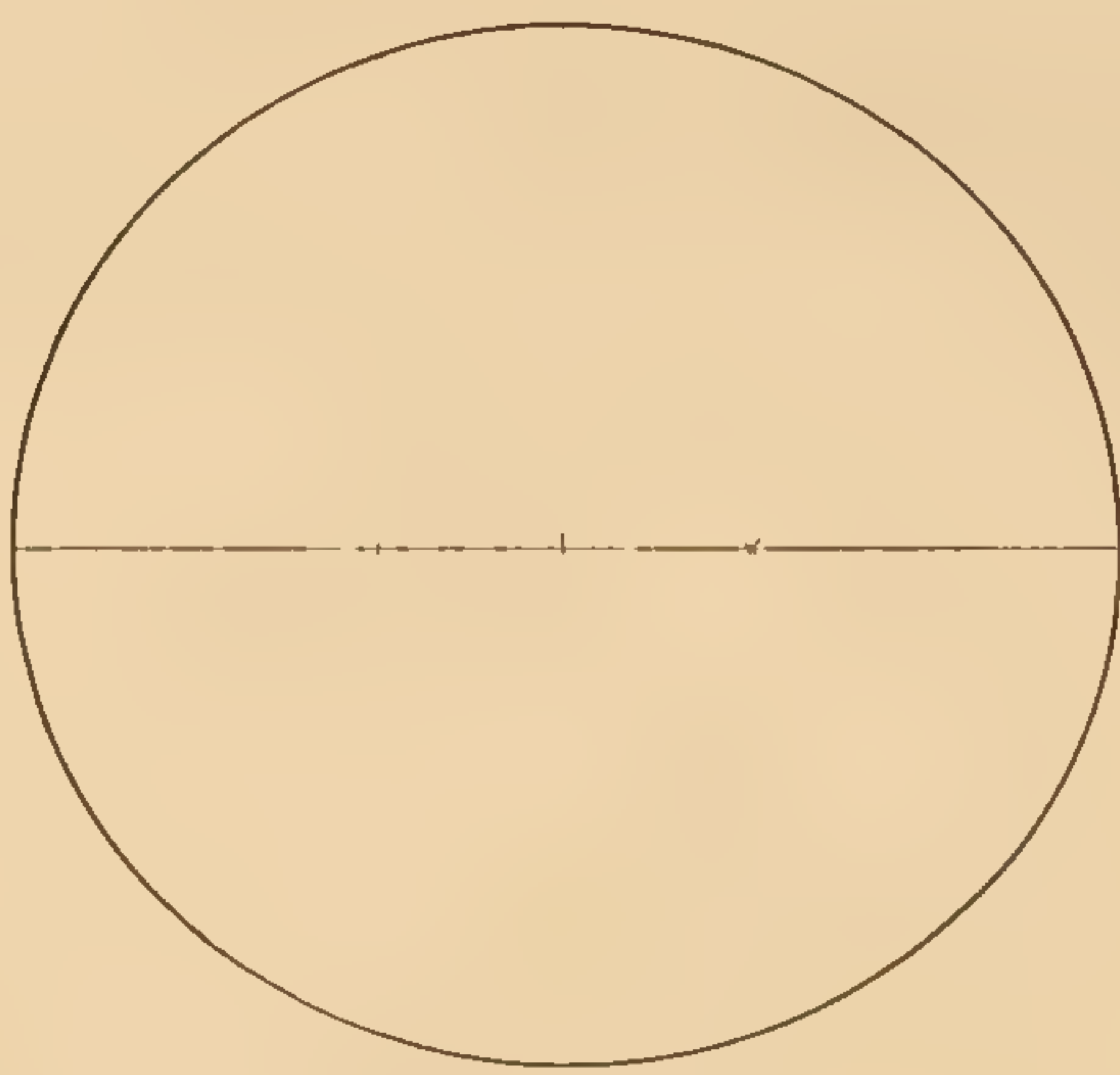
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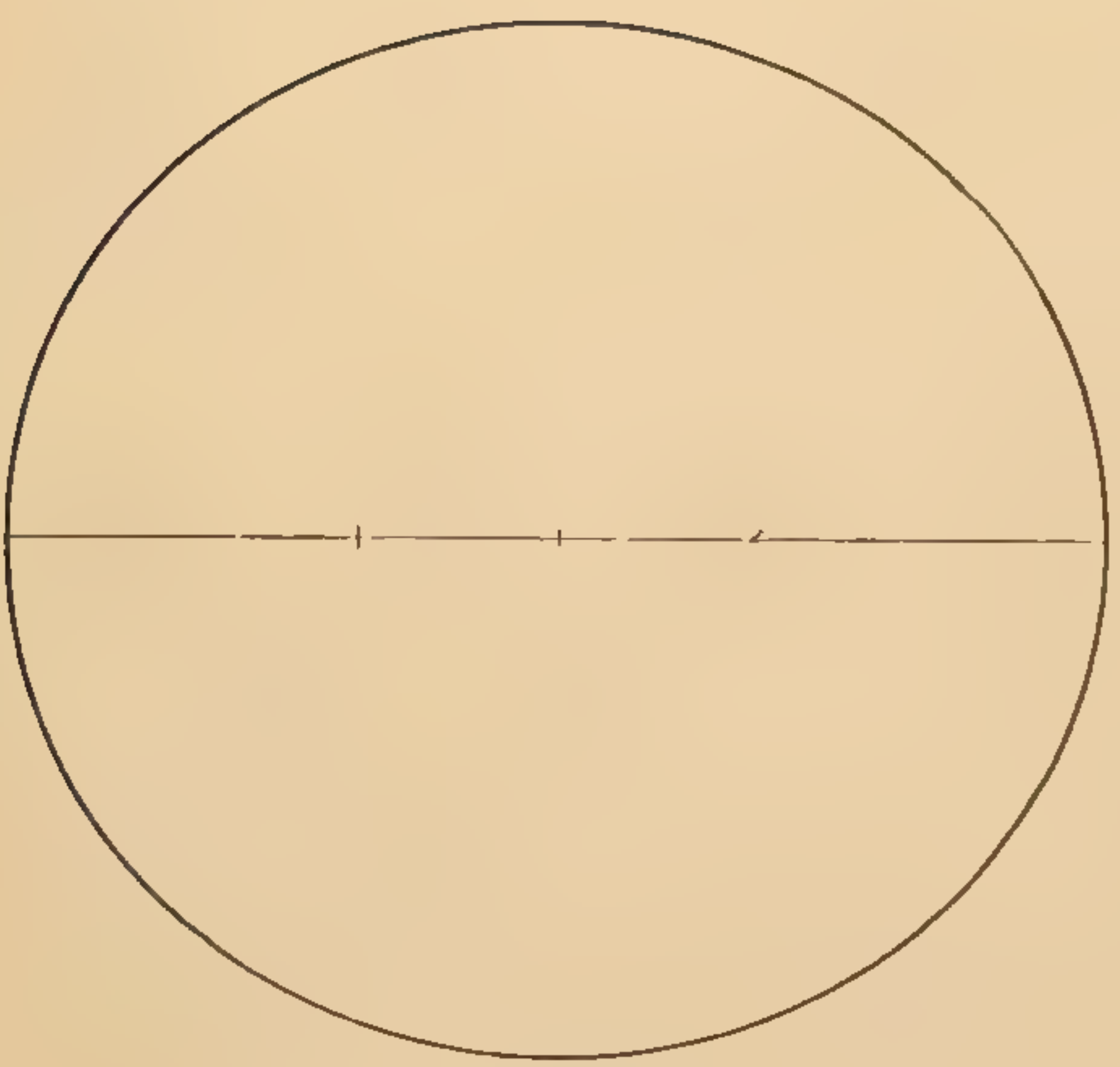
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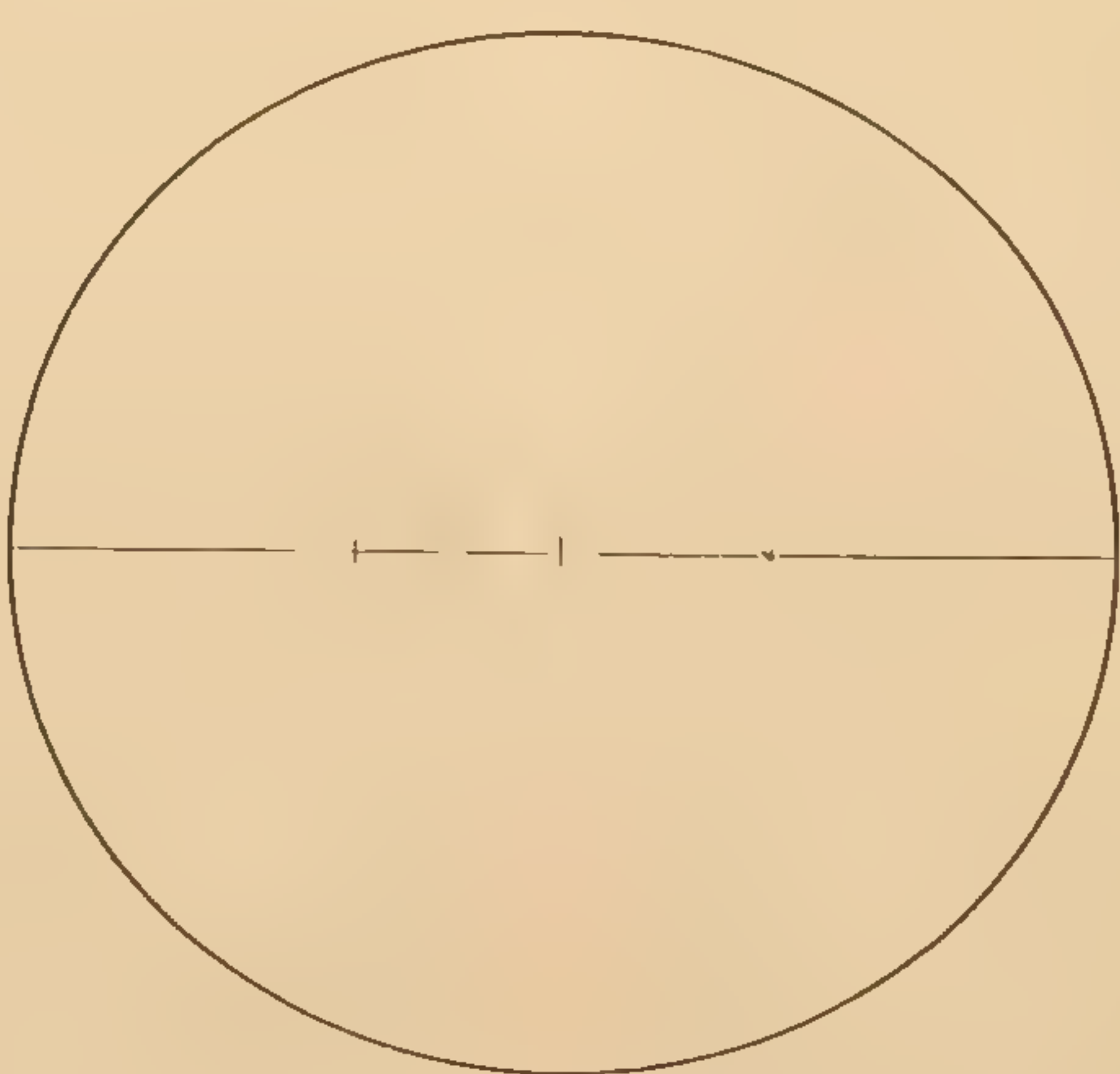
$\Sigma 3121.$



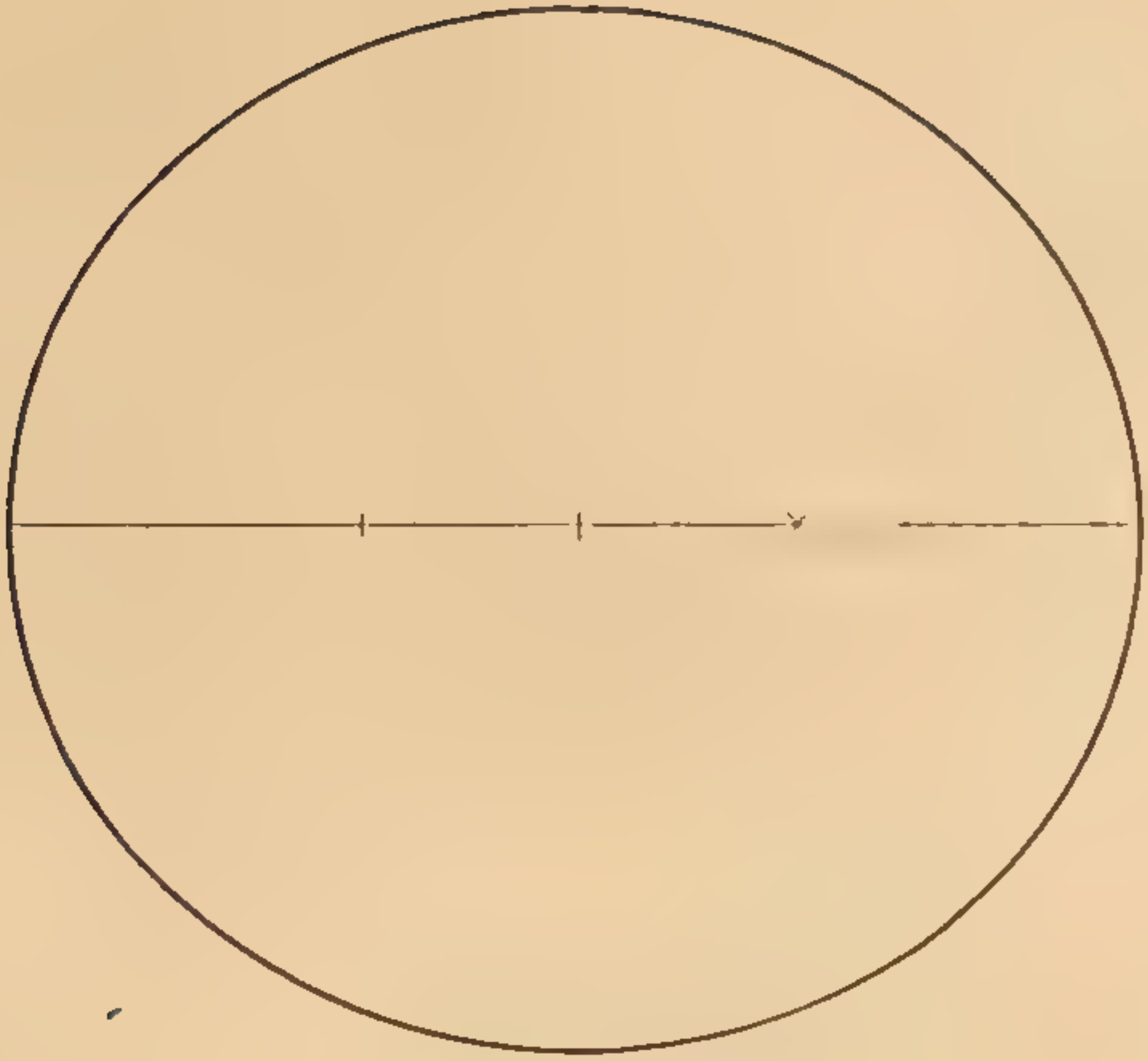
ζ Cancri.



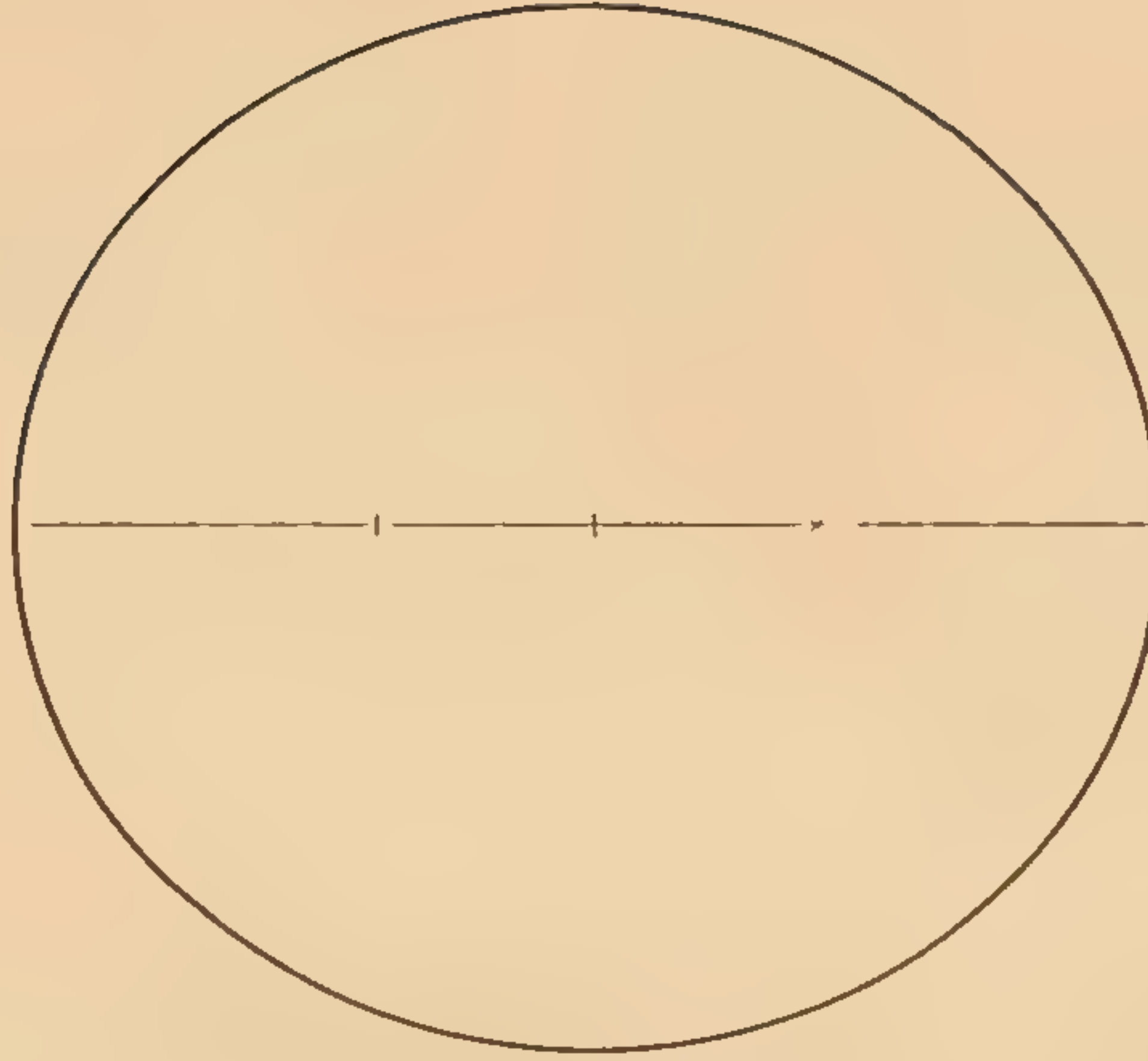
$0 \Sigma 269.$



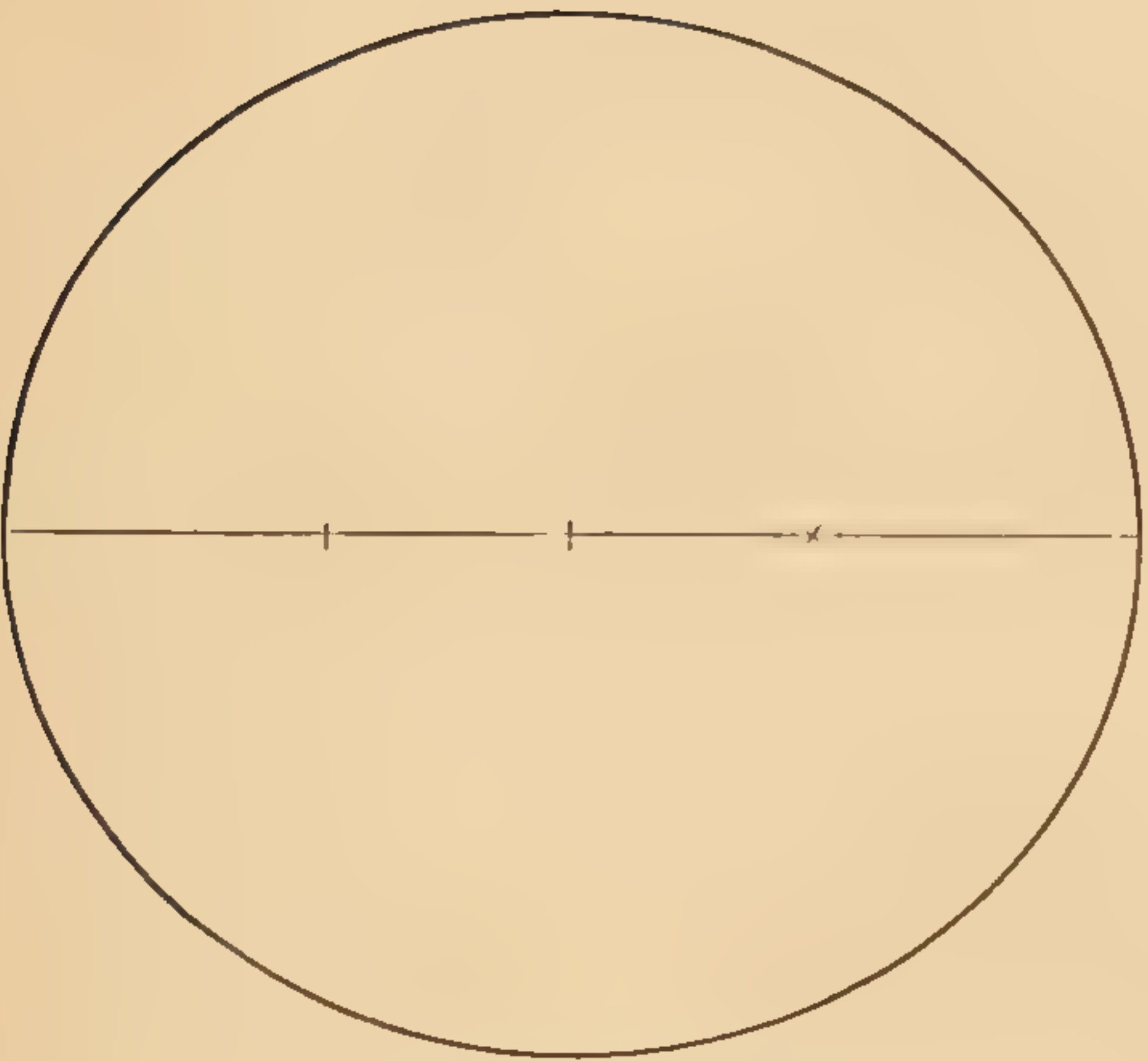
β Delphini.



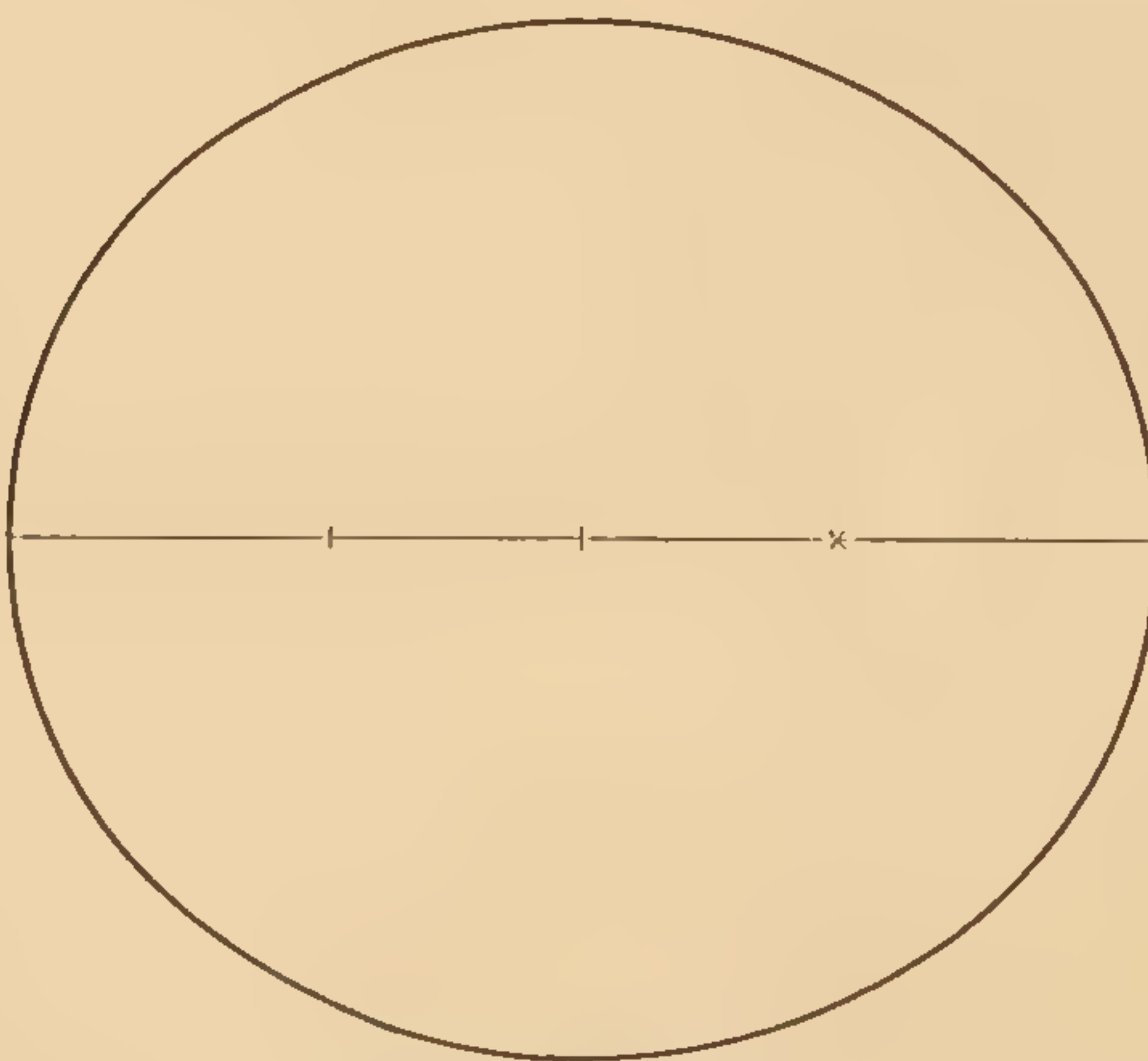
85 Pegasi.



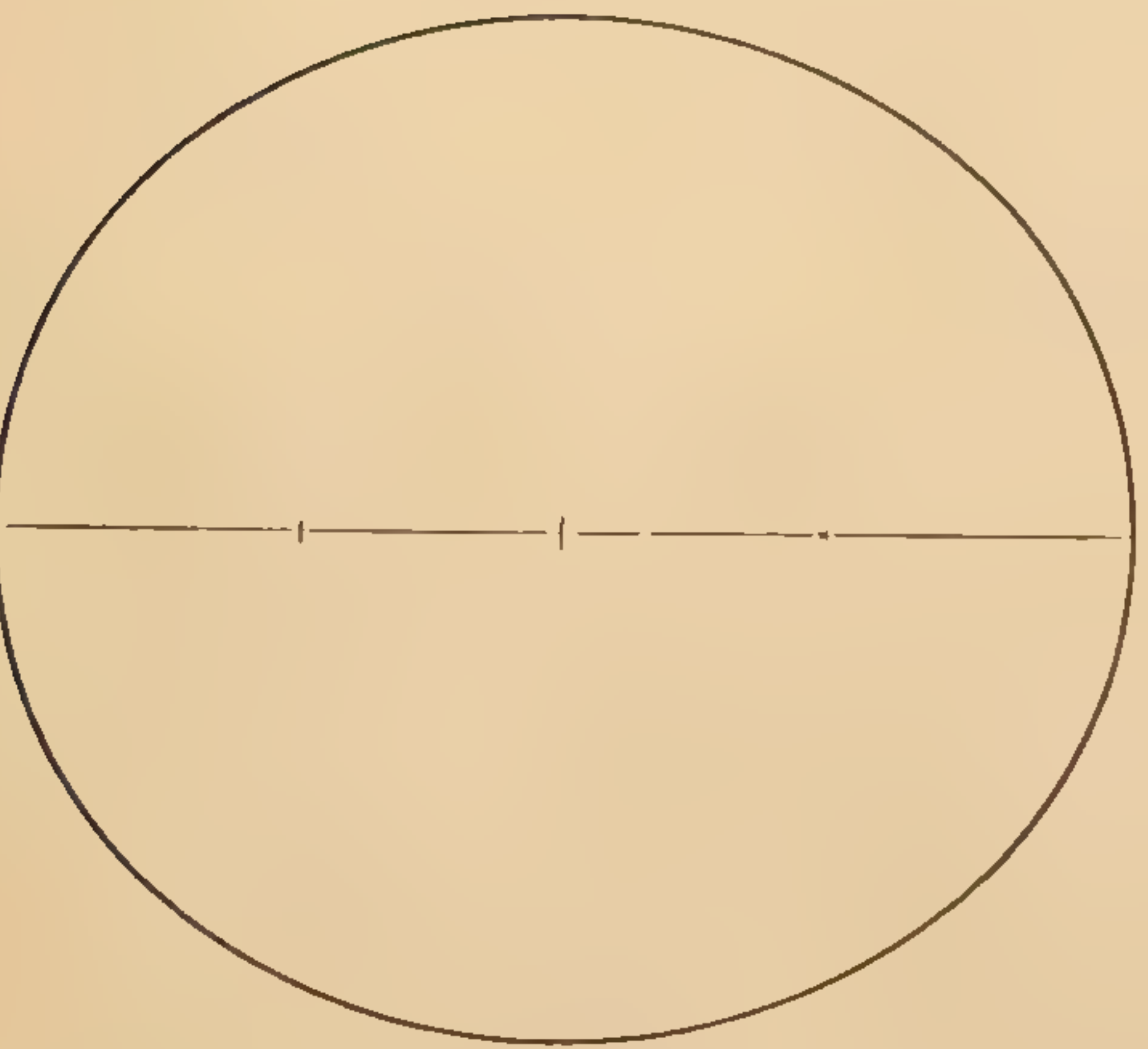
ξ Ursae Majoris.



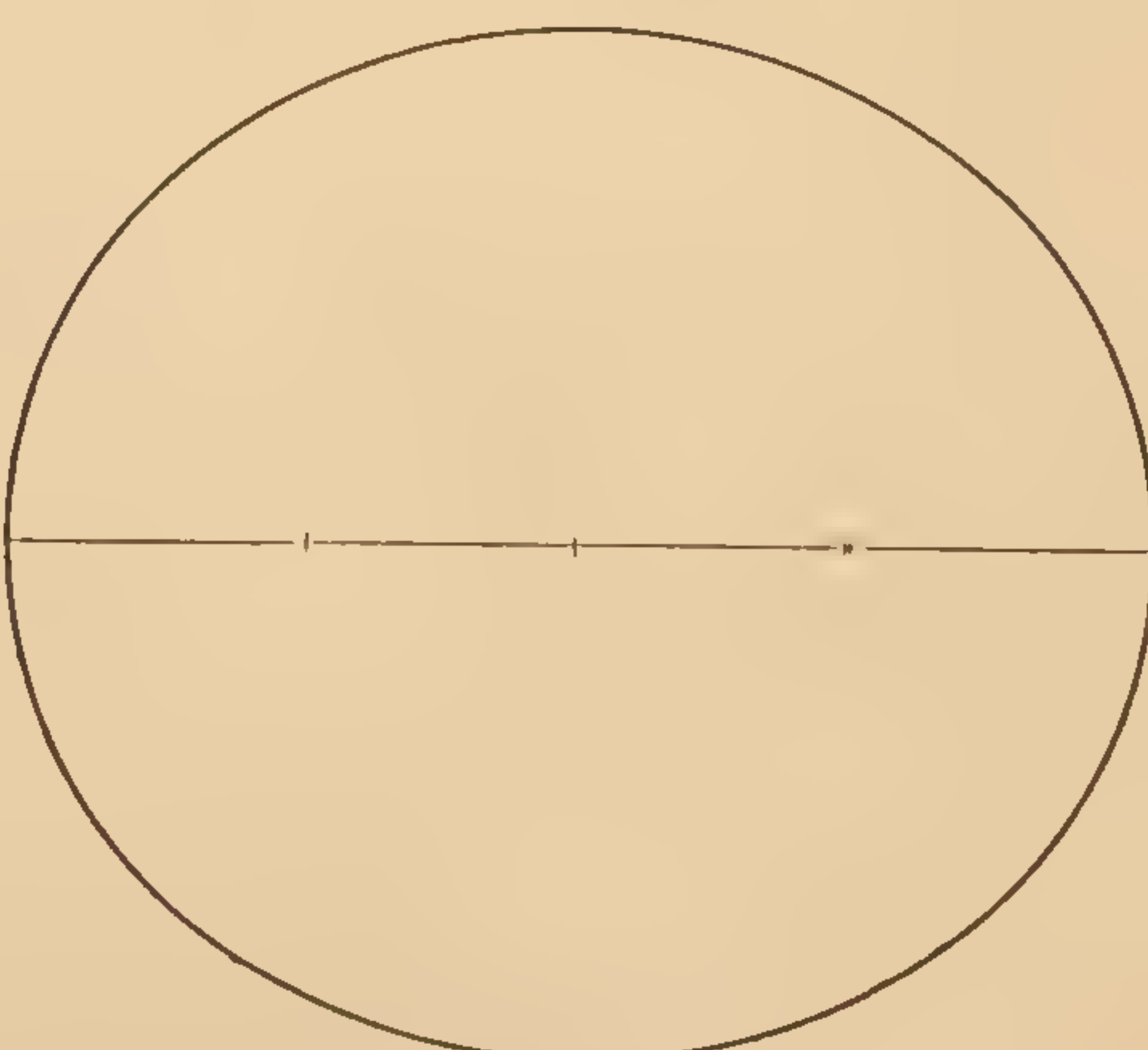
γ Coronae Australis.



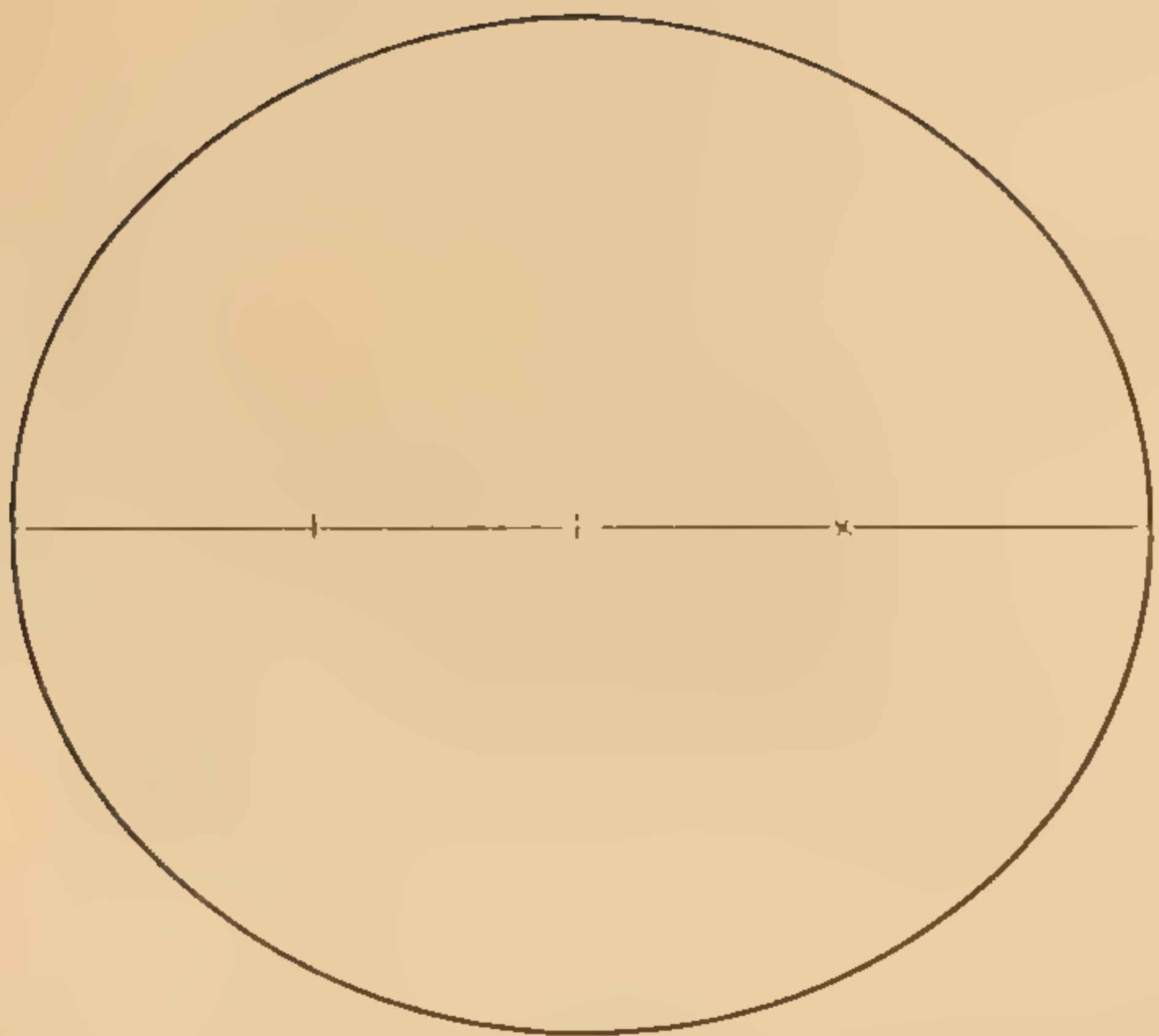
φ Ursae Majoris.



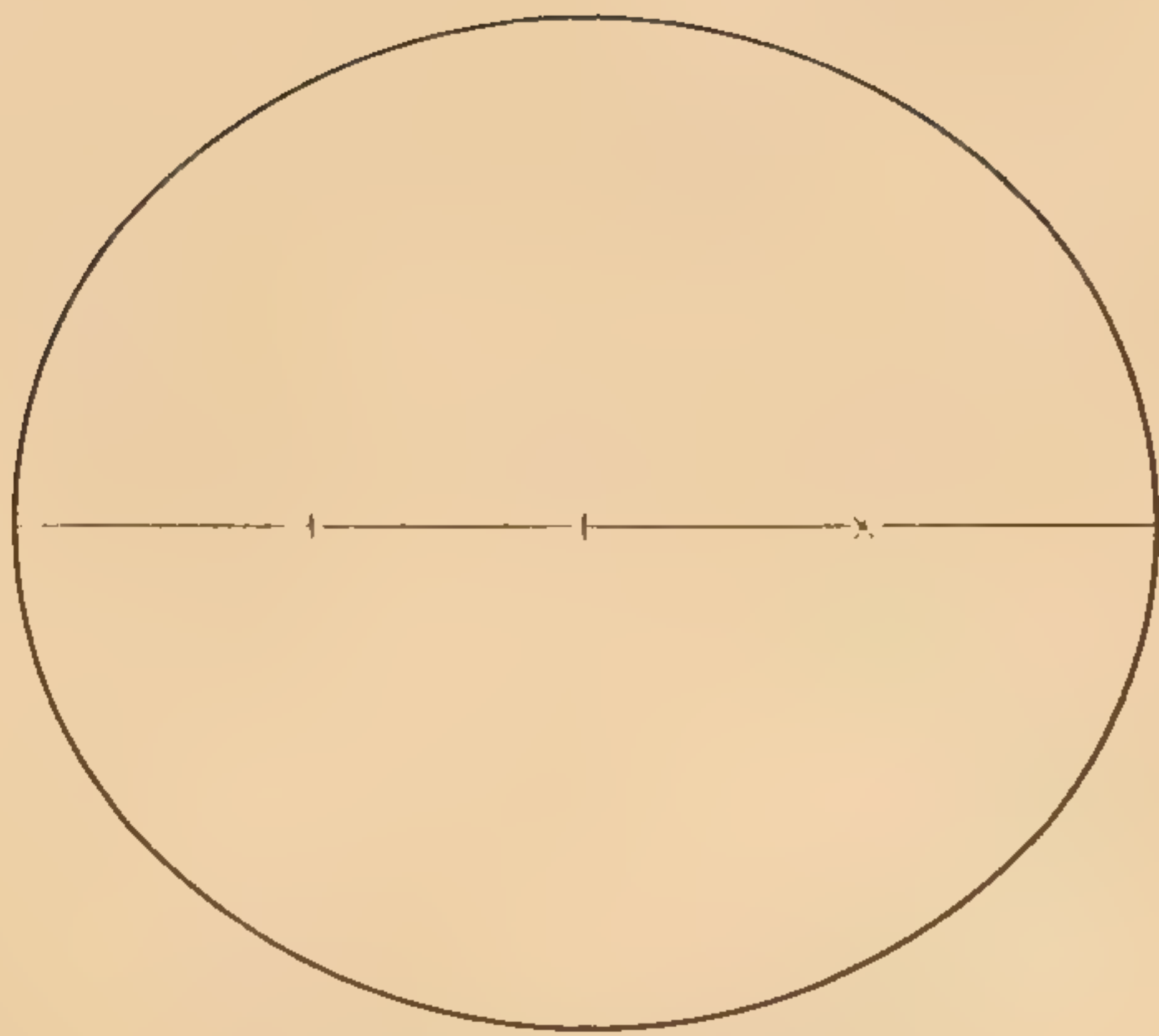
Σ 3062.



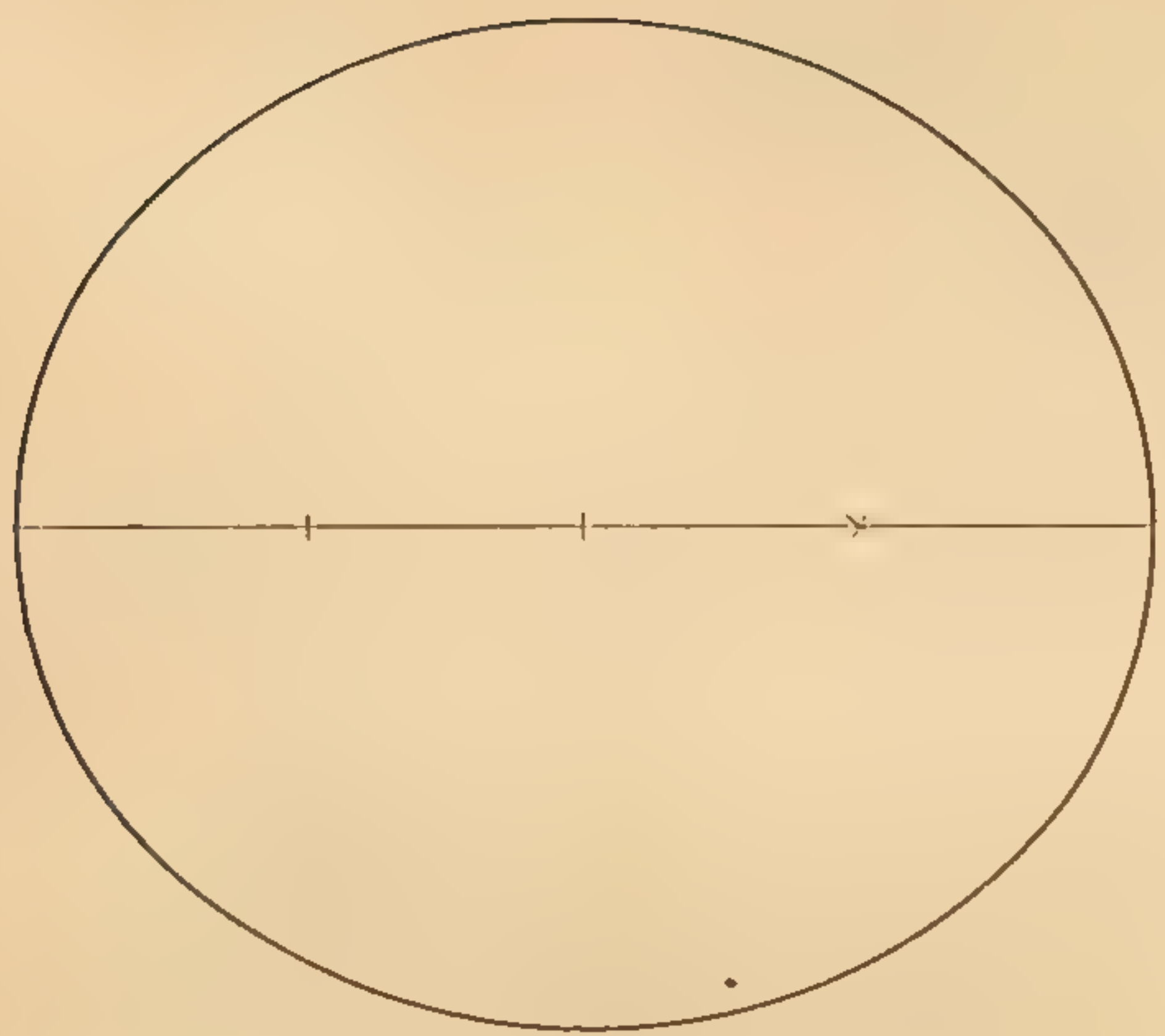
42 Comae Berenices.



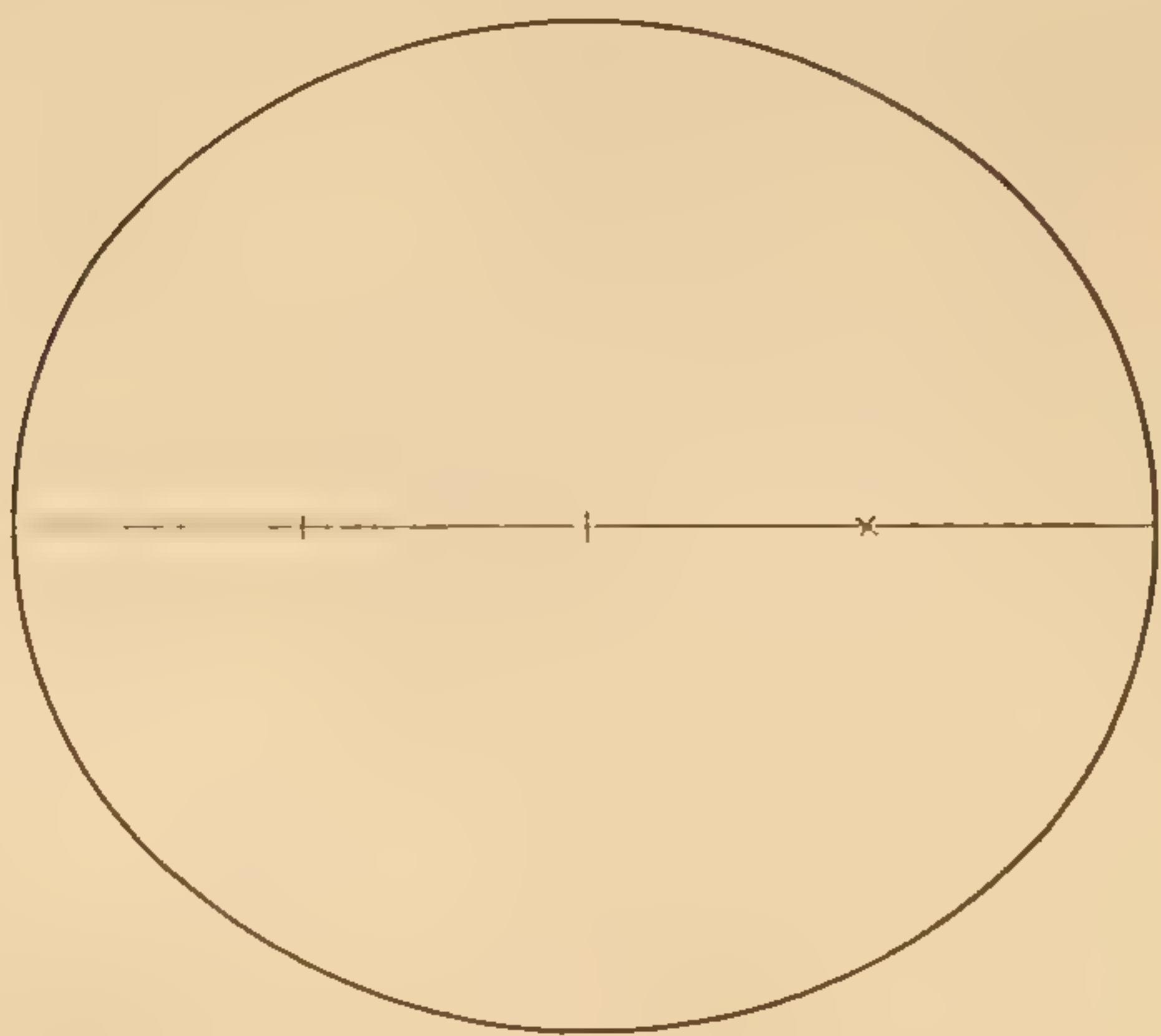
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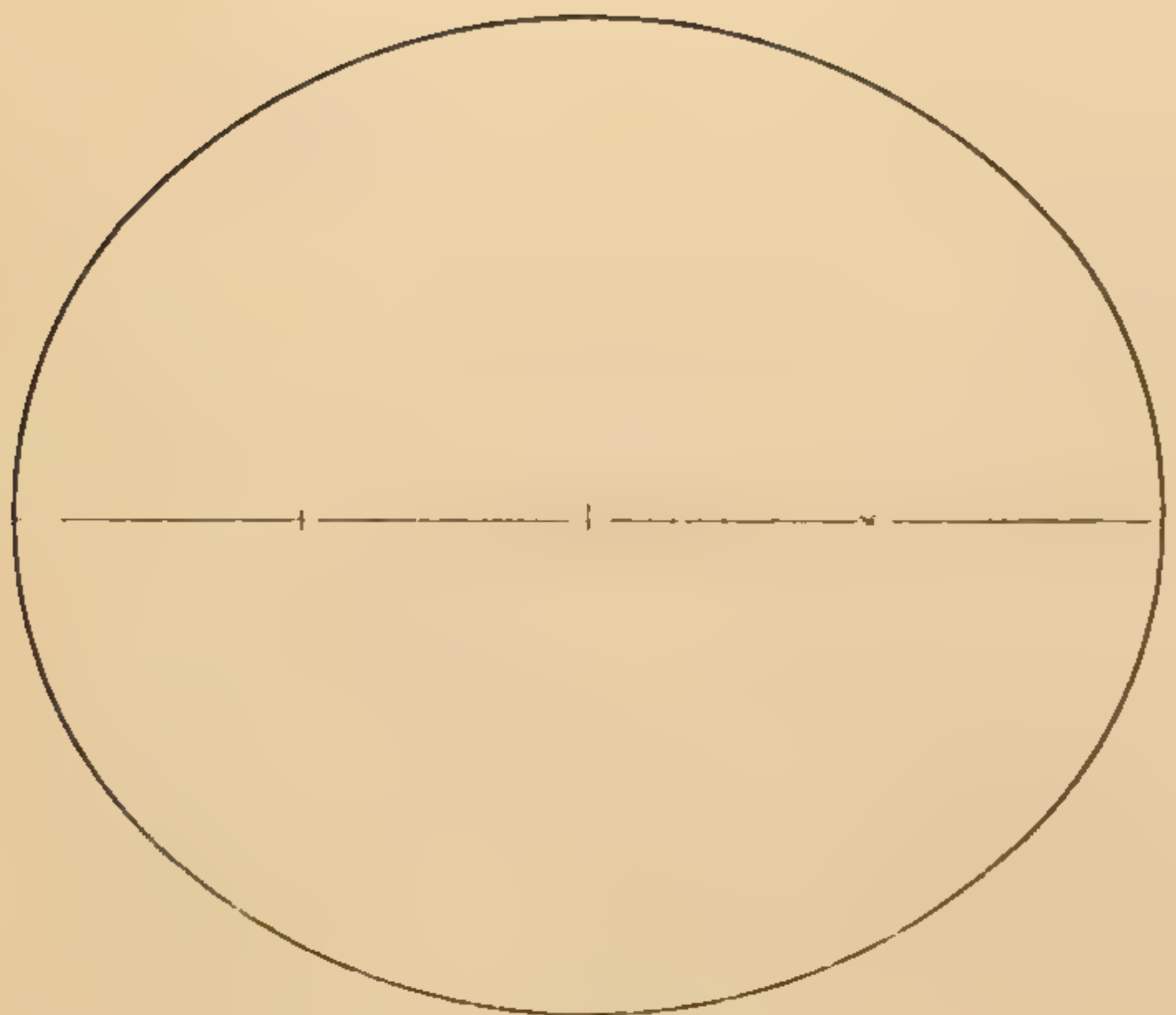
1 Coronae Borealis.



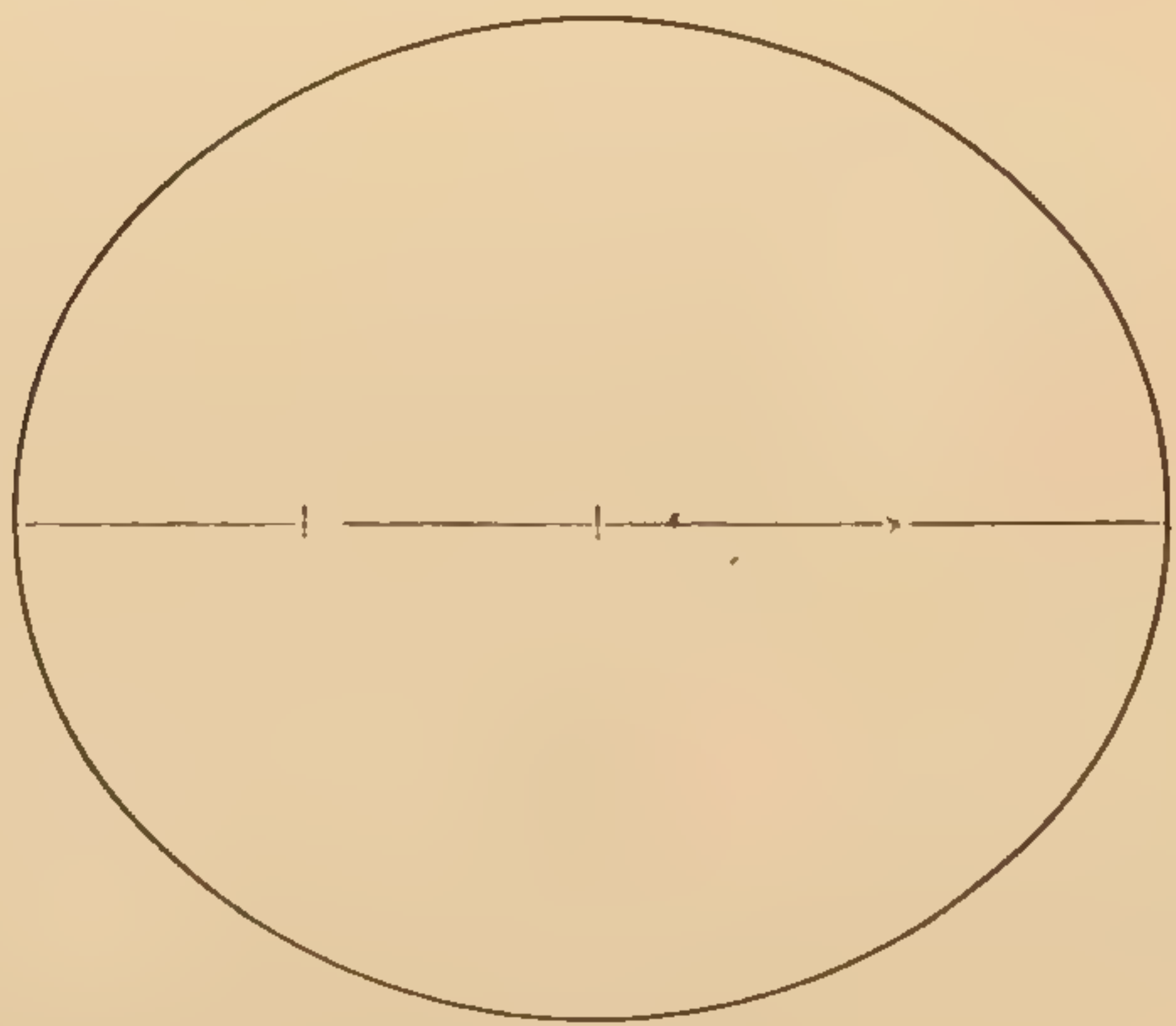
κ Pegasi.



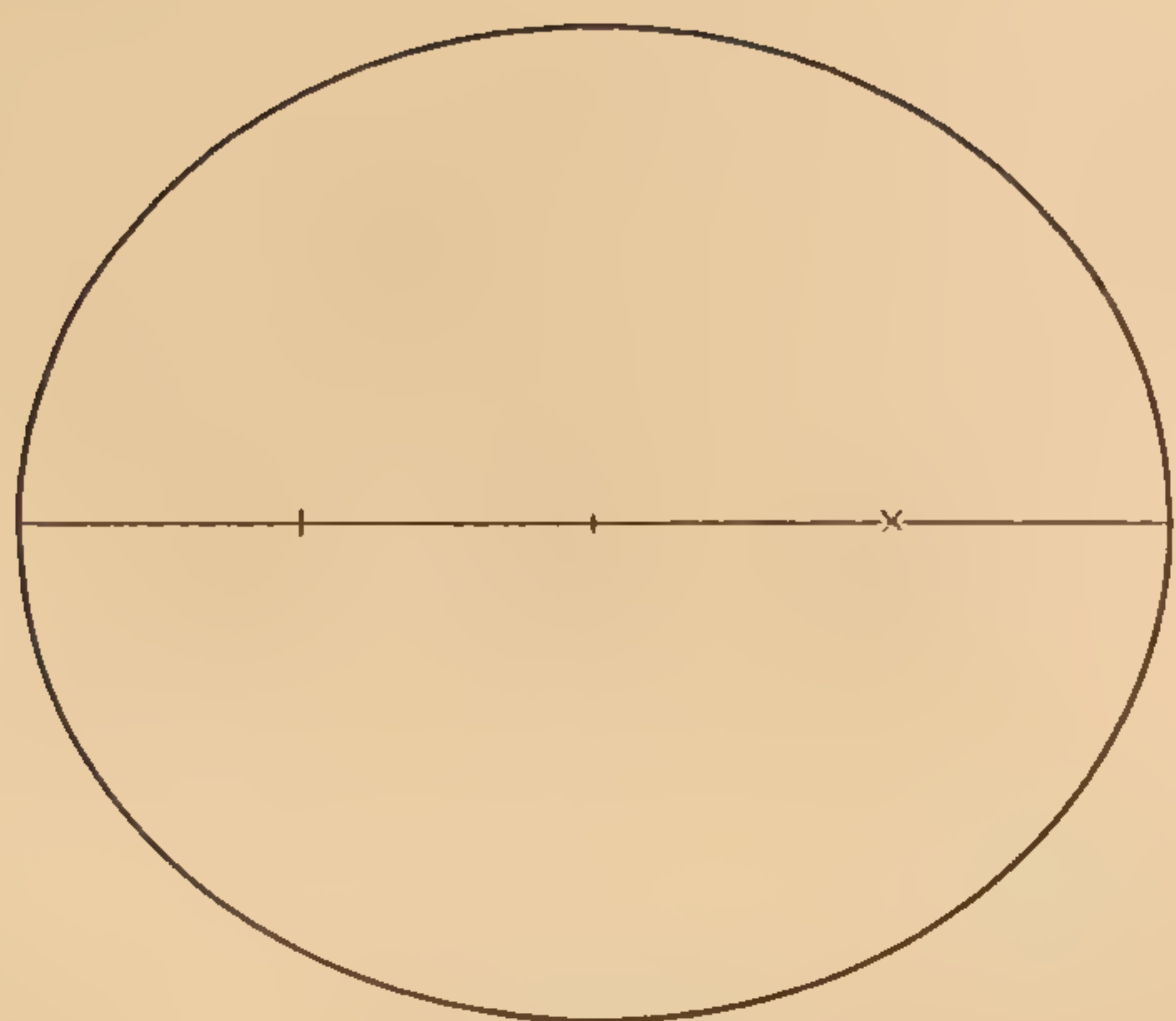
ζ Herculis.



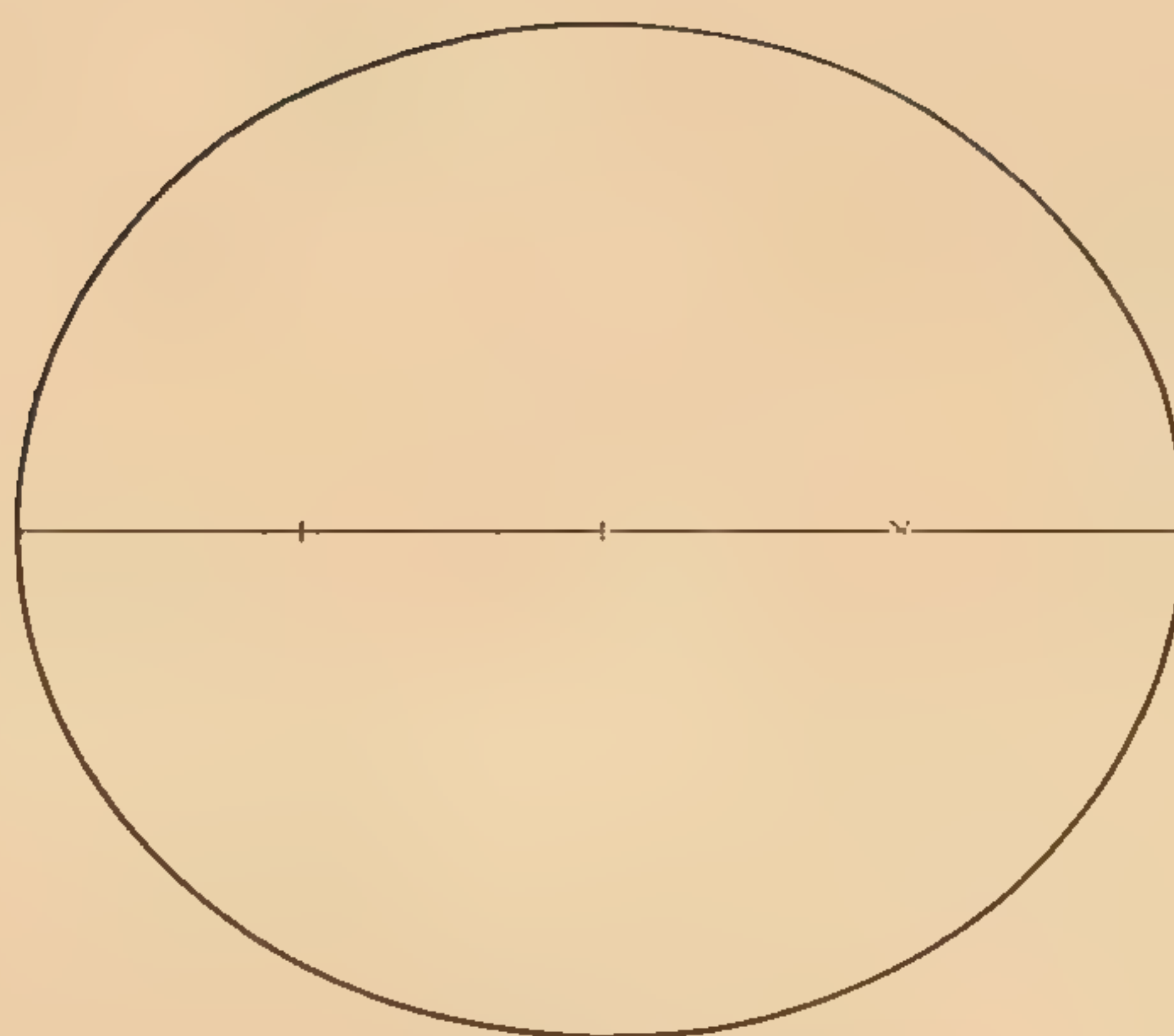
70 Ophiuchi.



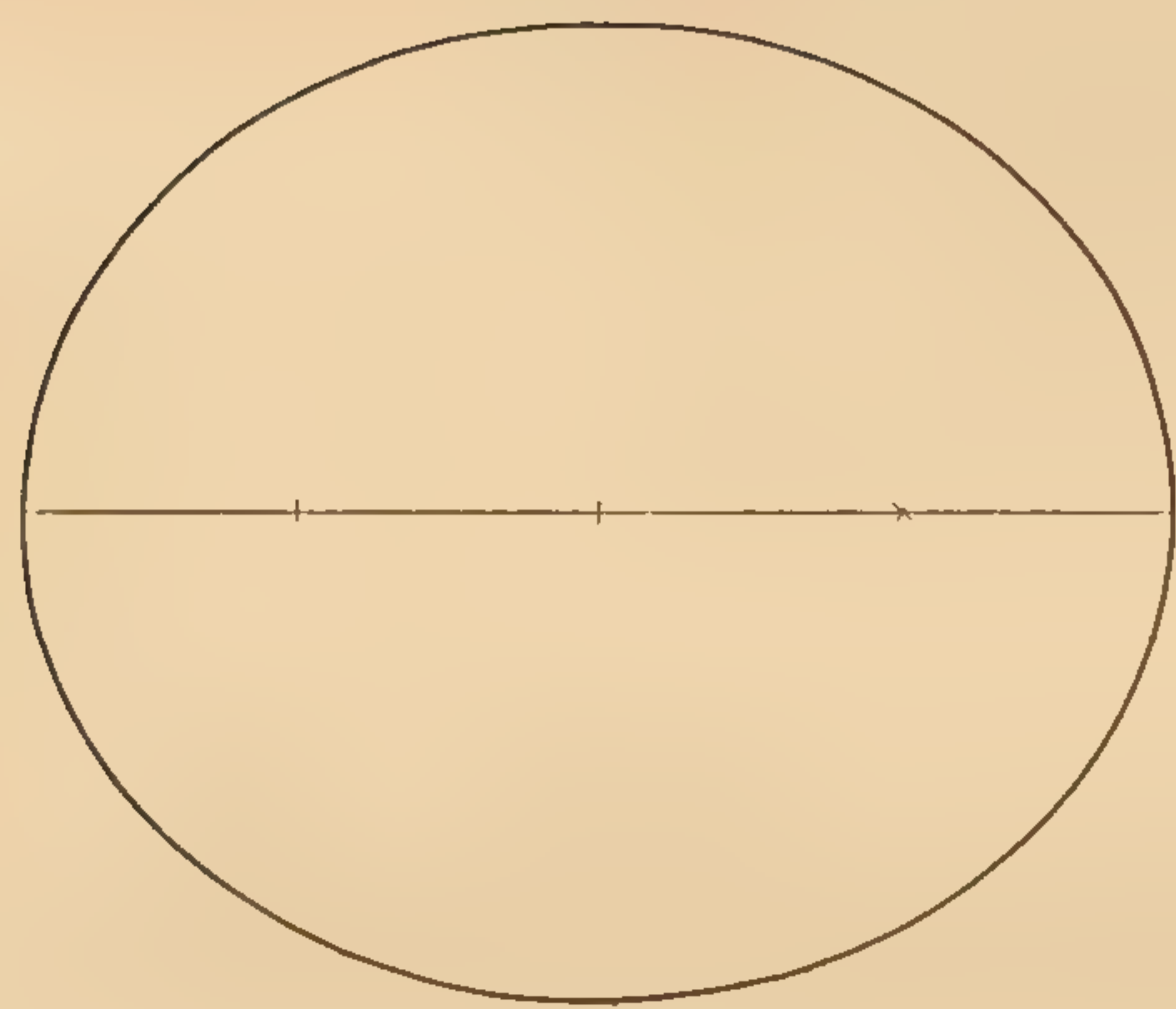
β 416.



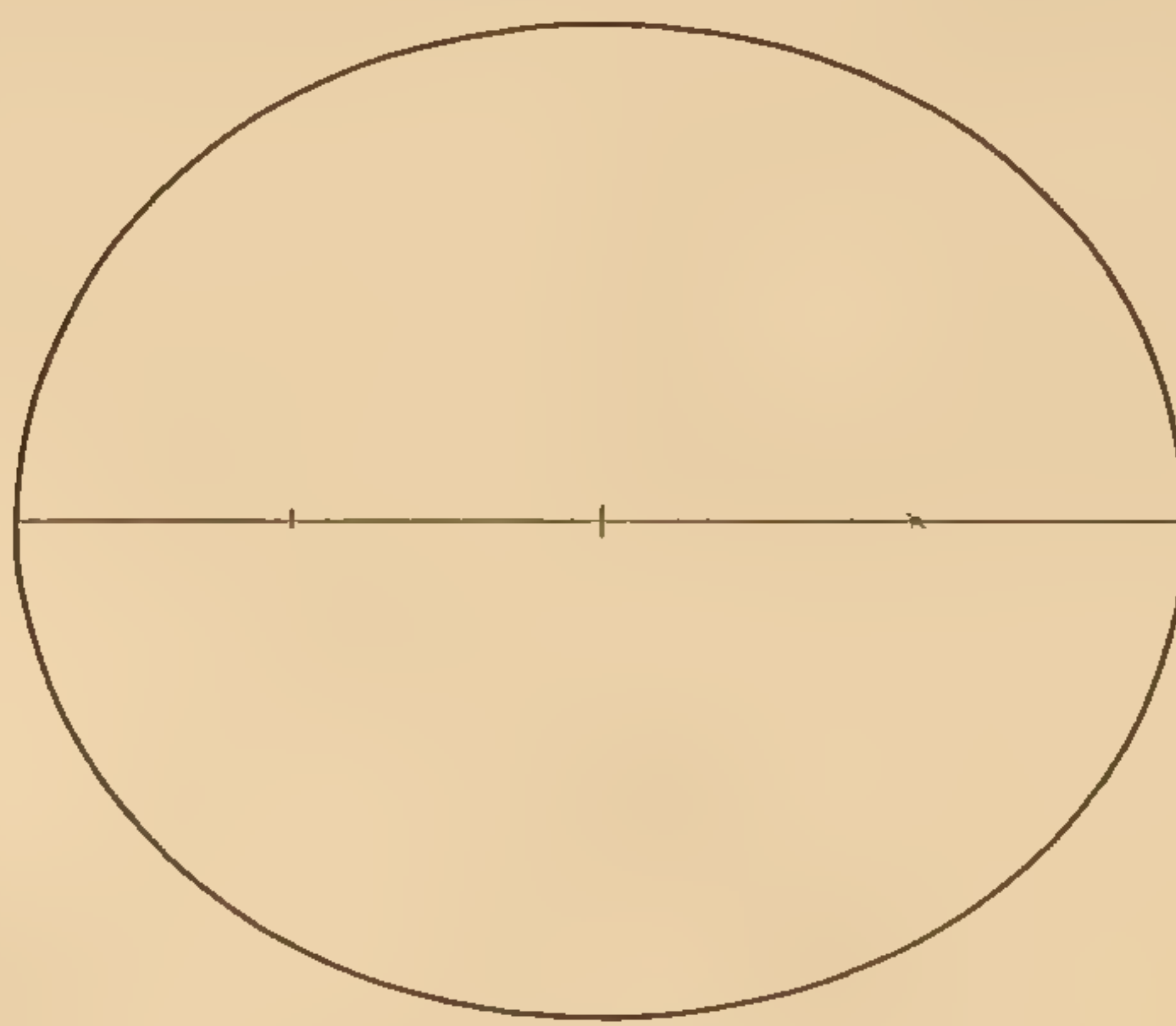
η Cassiopeae.



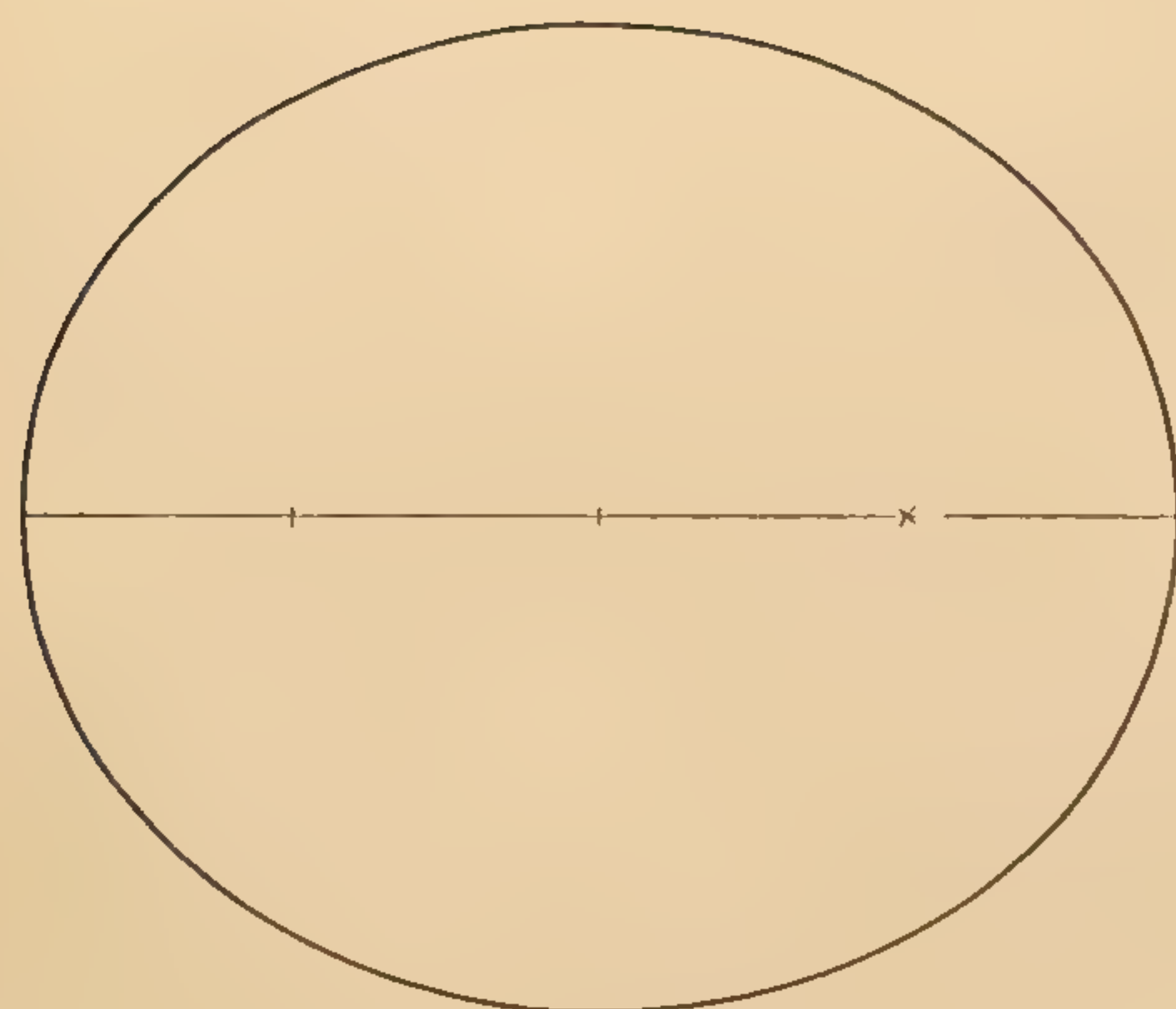
4 Aquarii.



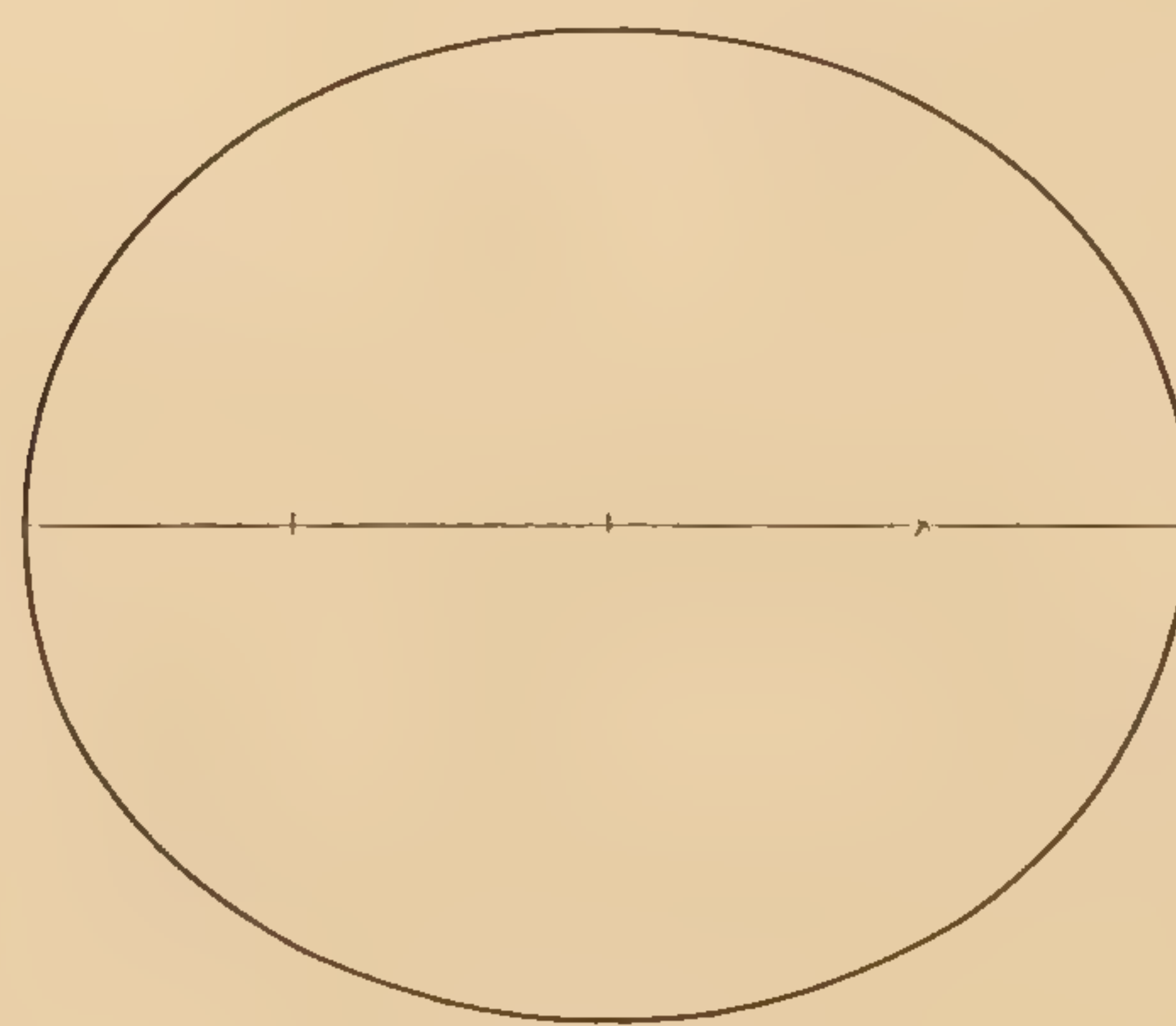
α Centauri.



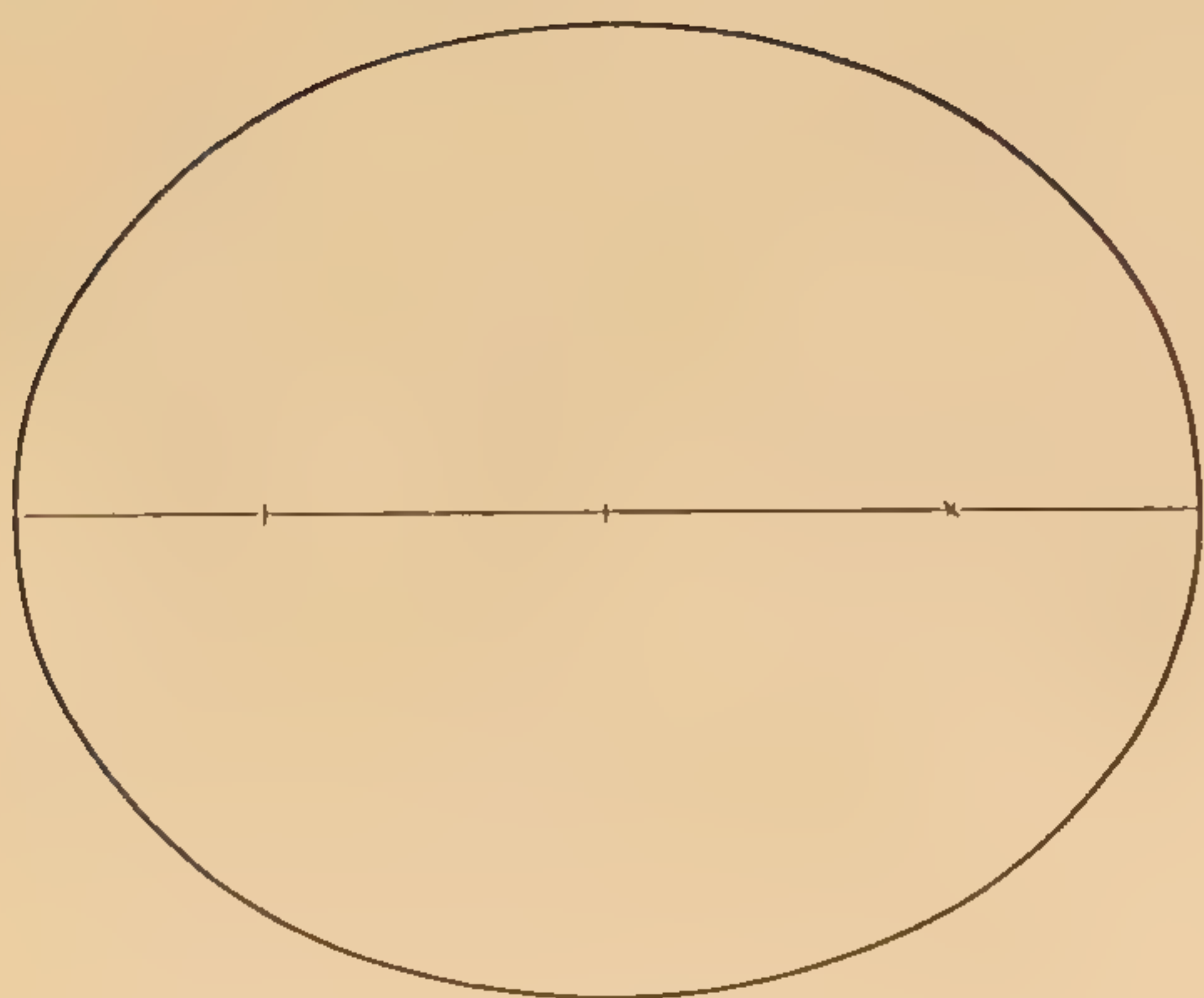
ω Leonis.



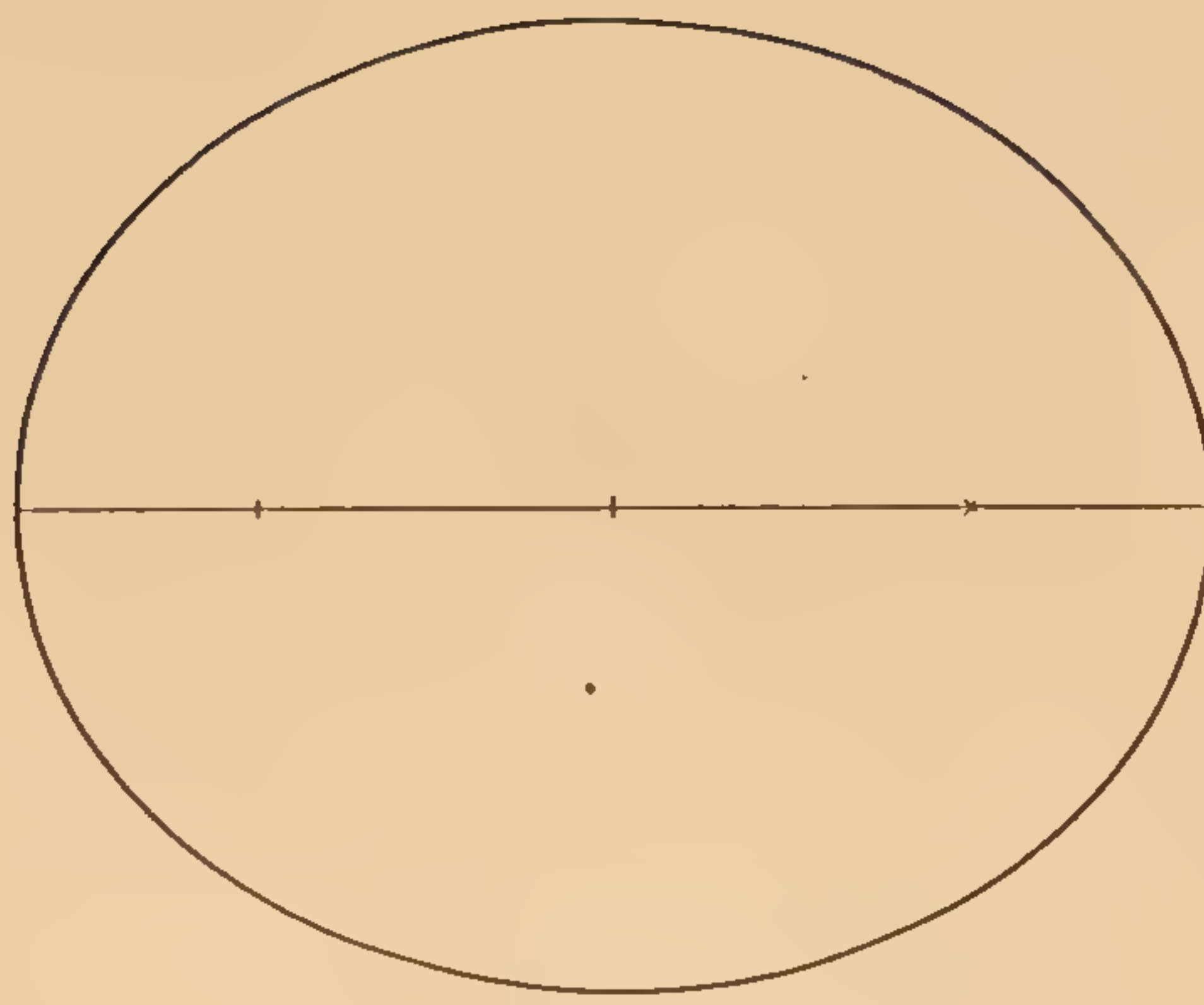
μ Bootis.



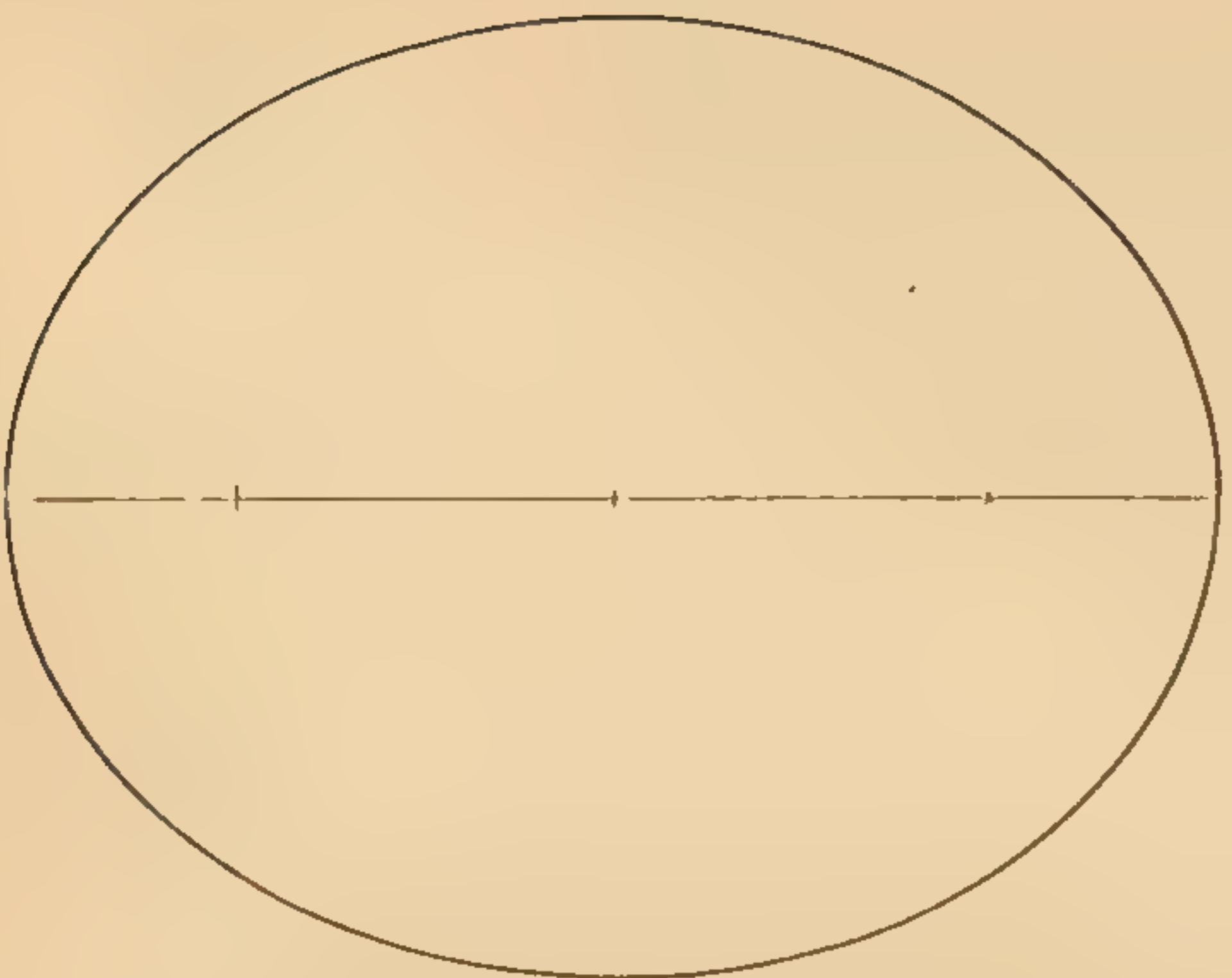
σ Coronae Borealis.



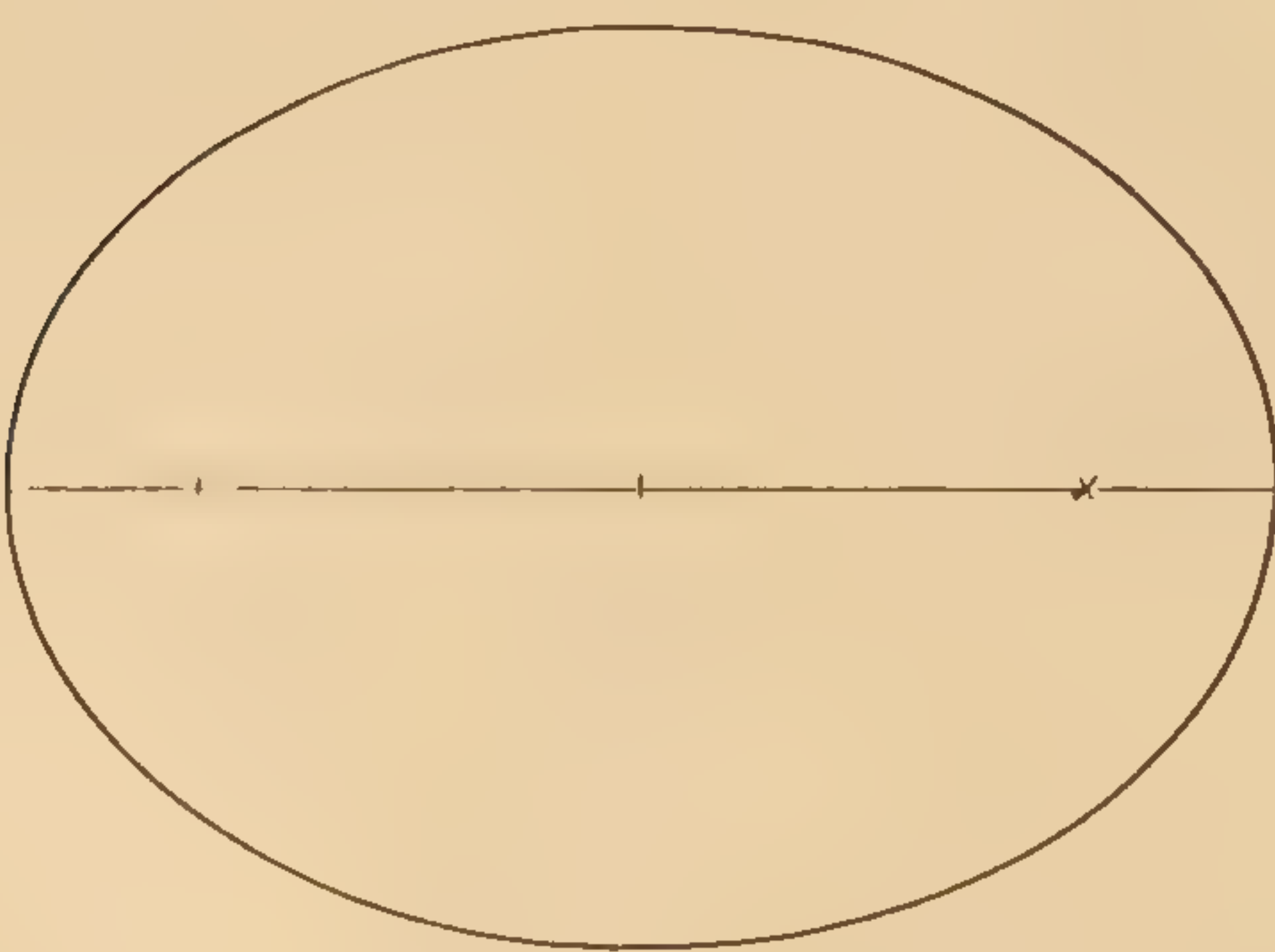
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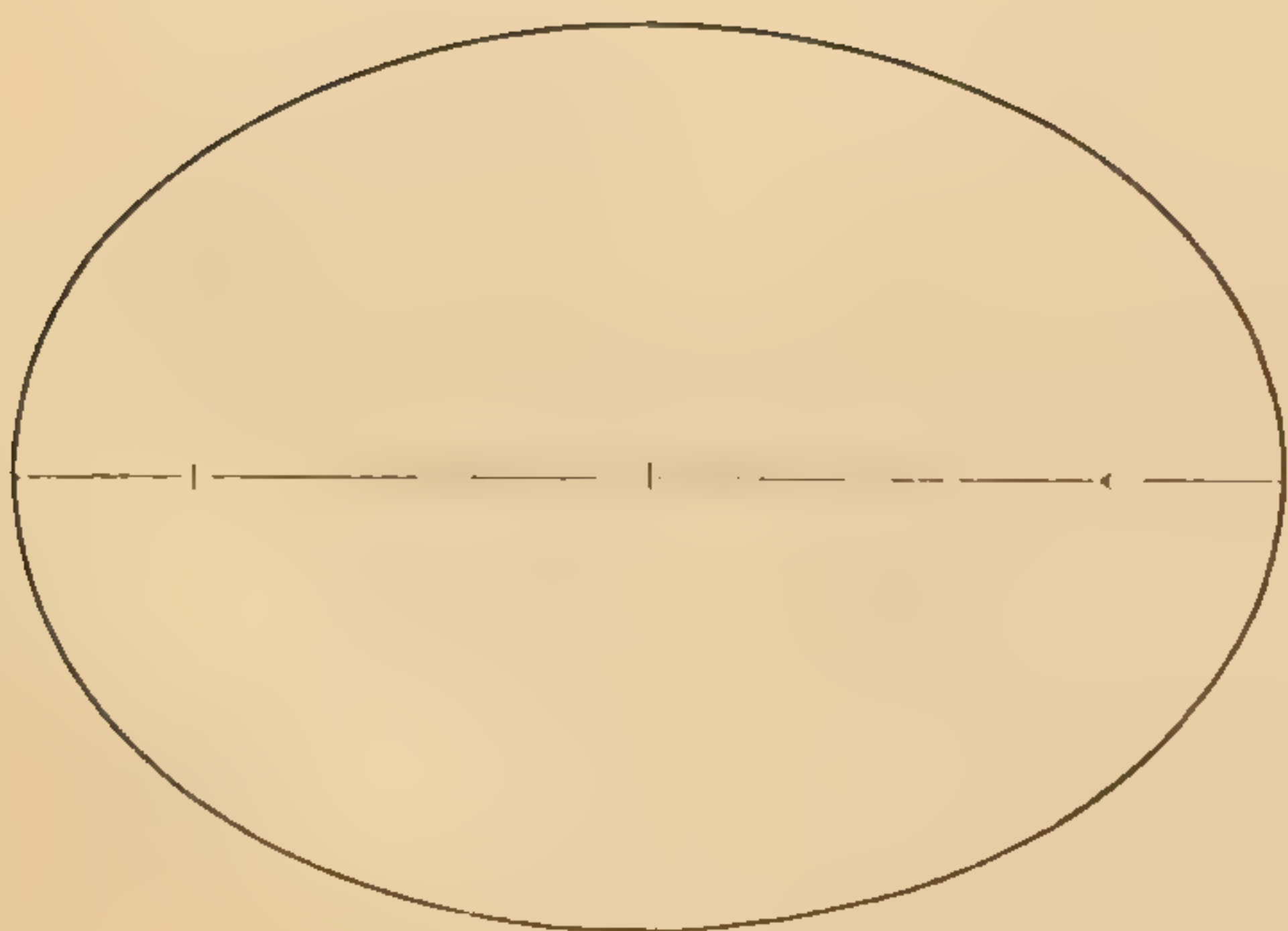
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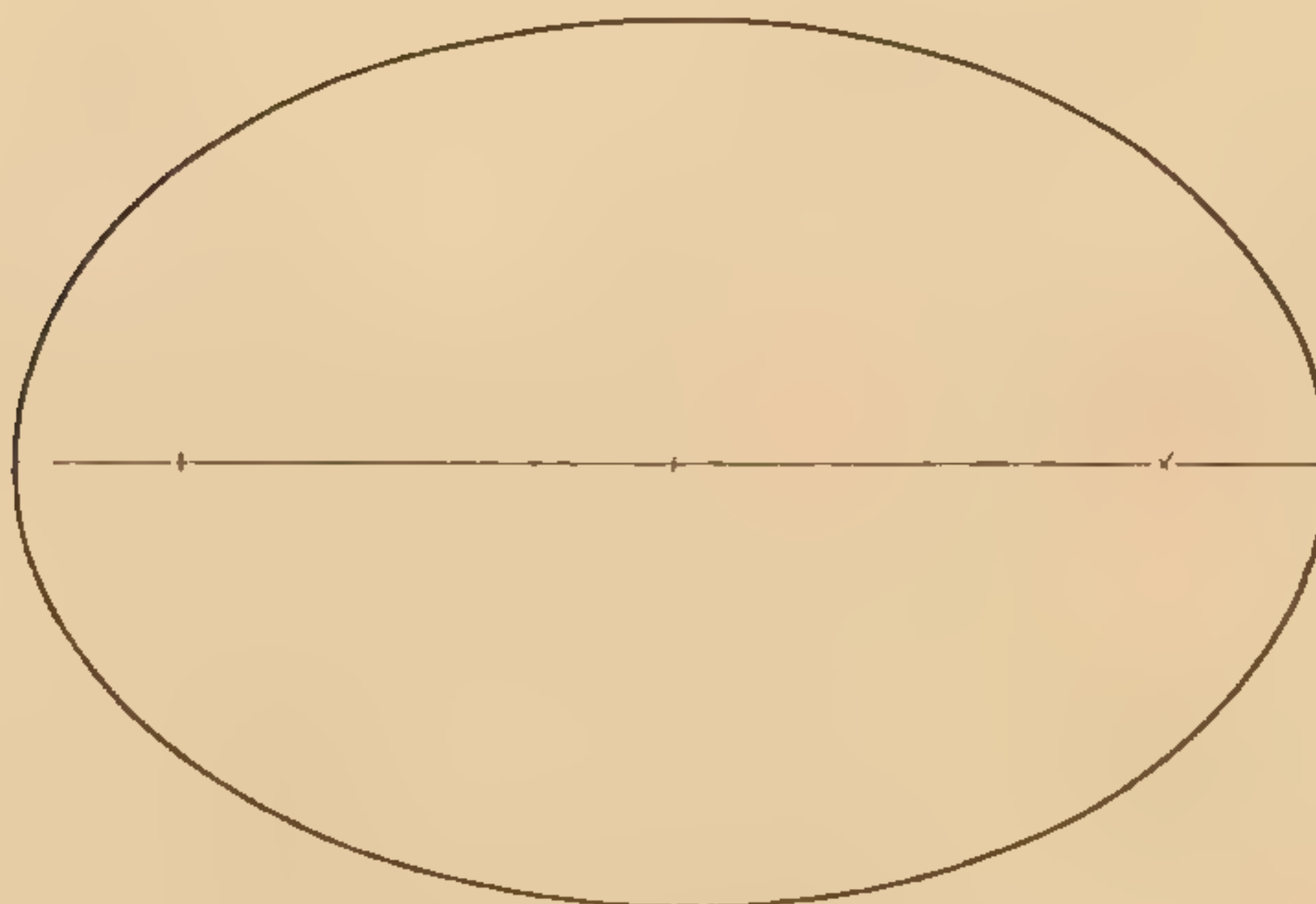
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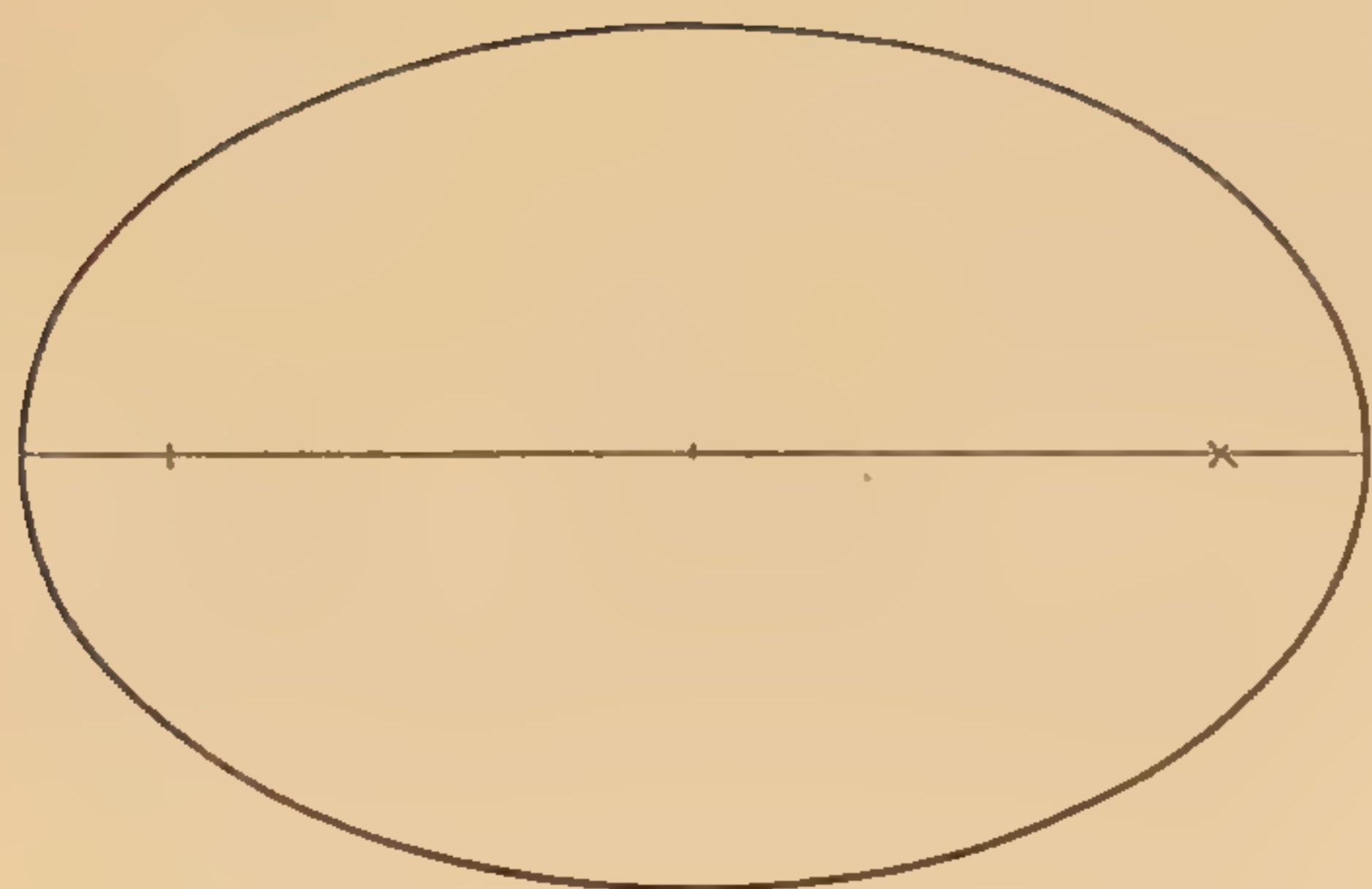
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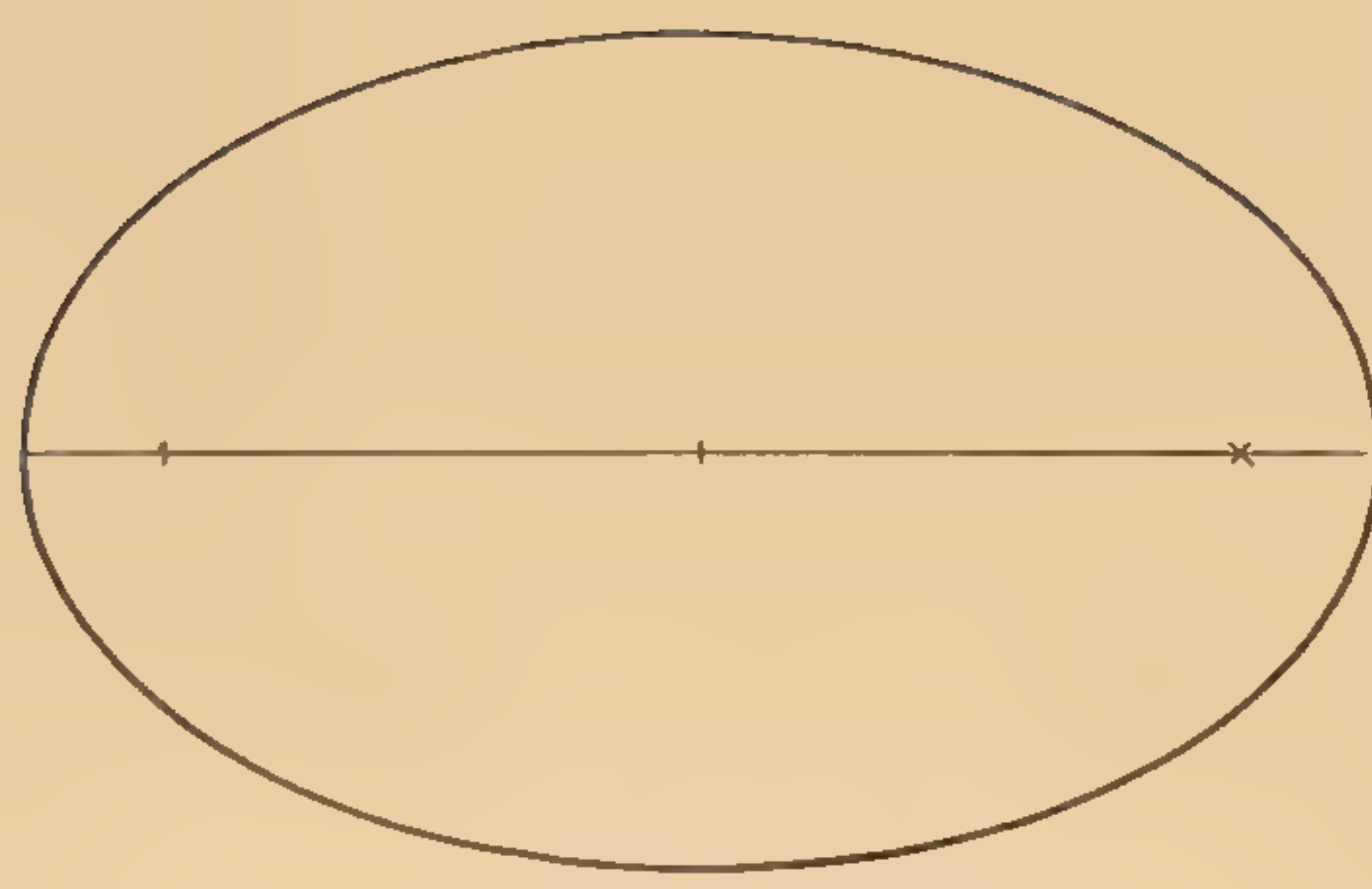
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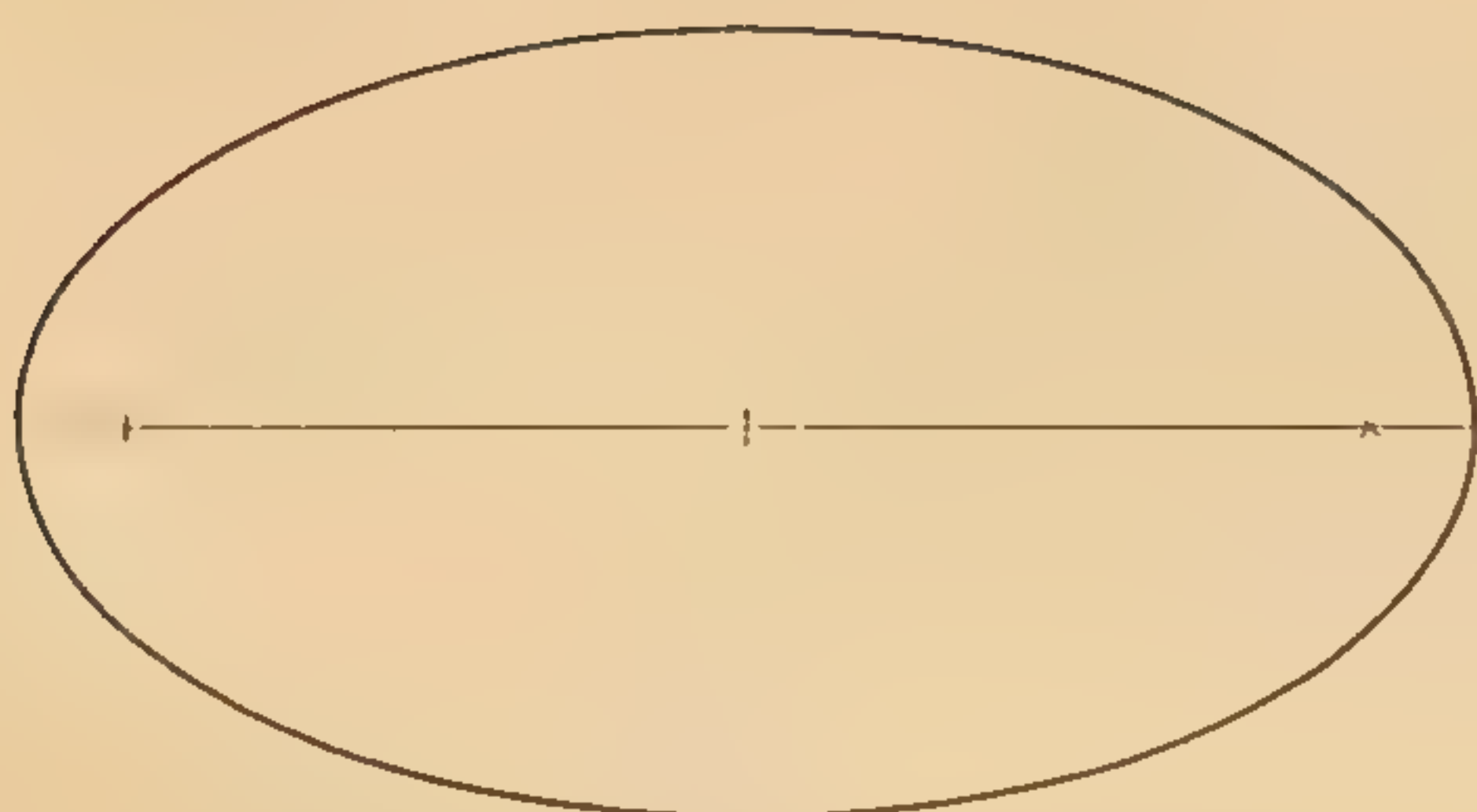
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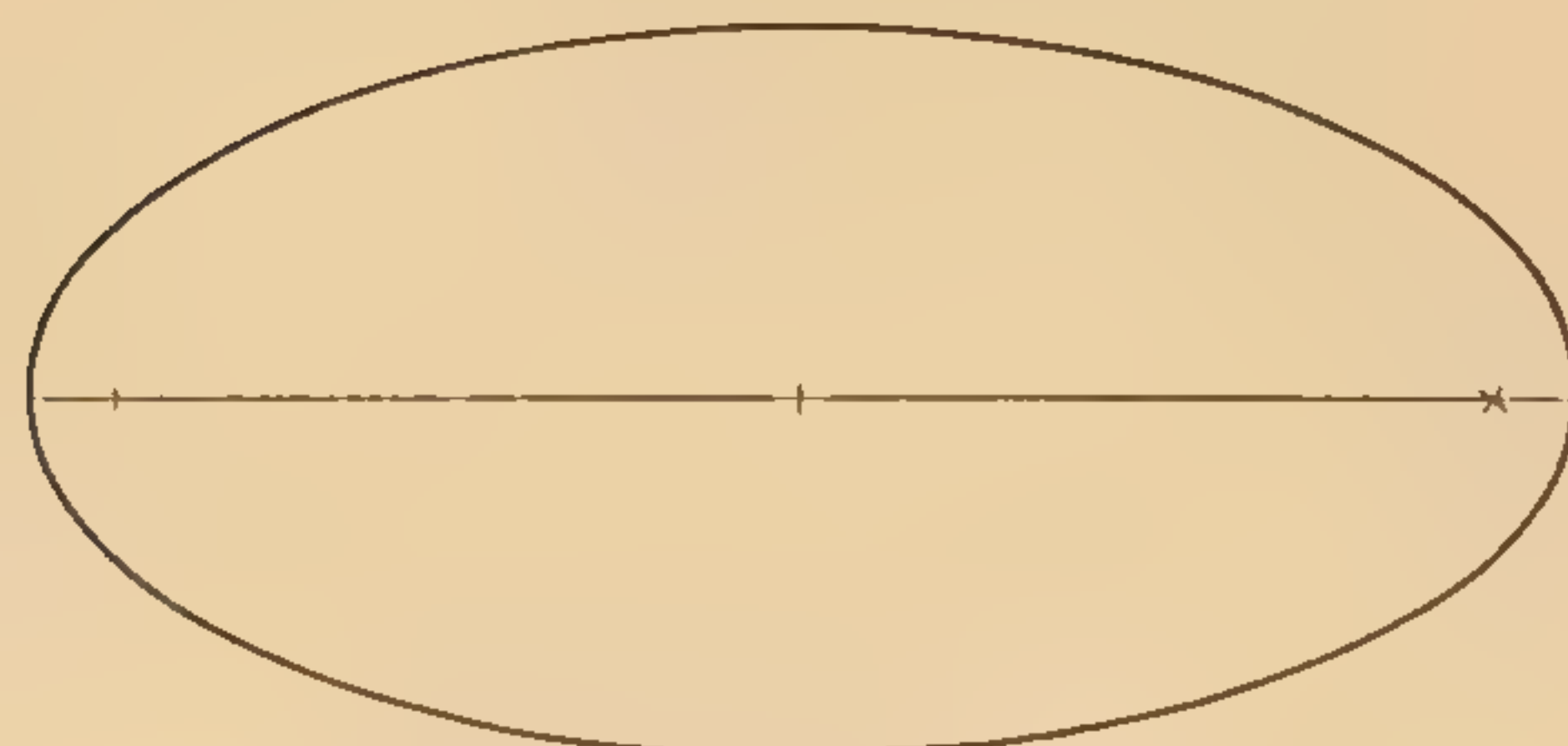
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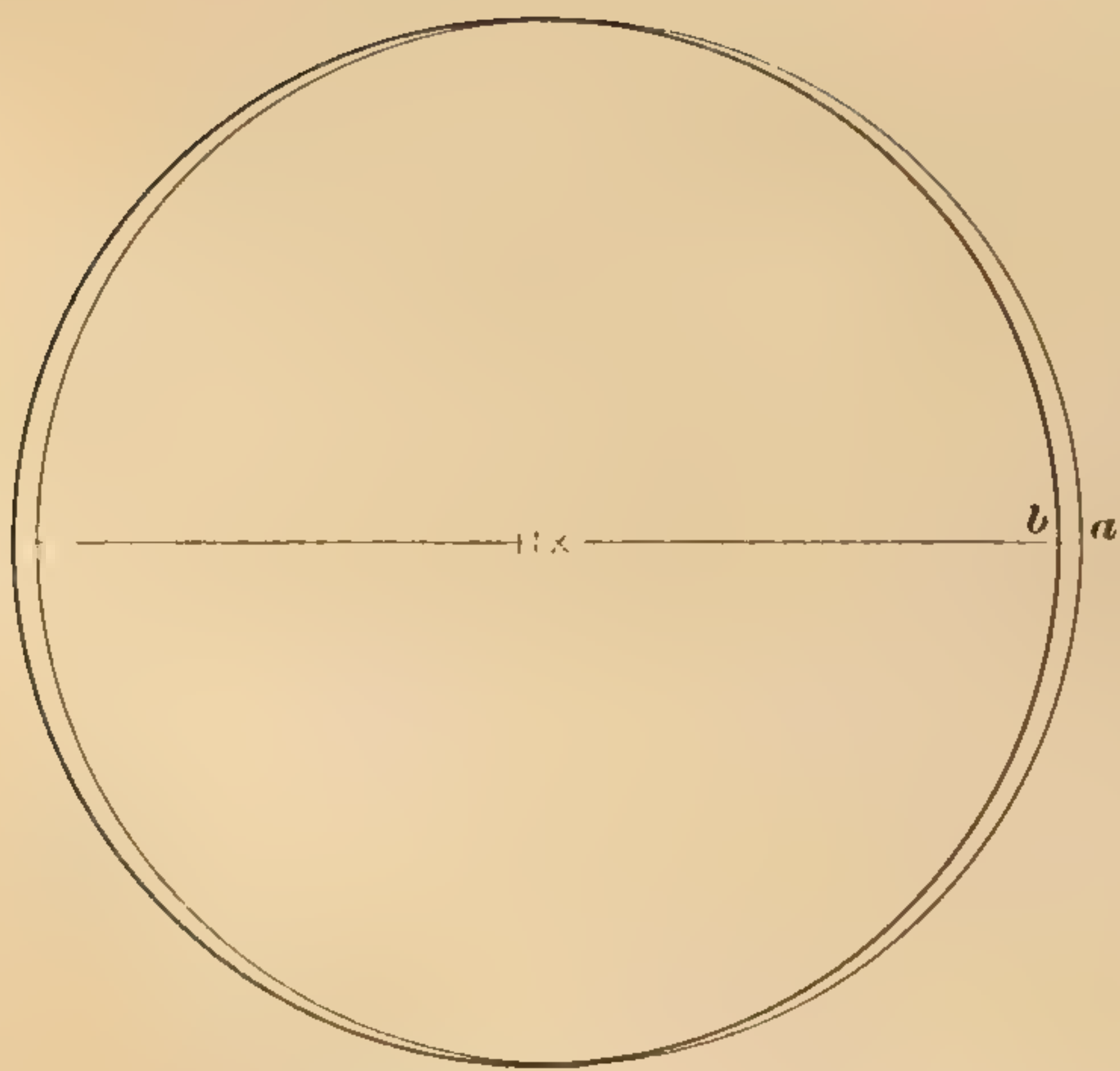
γ Centauri.



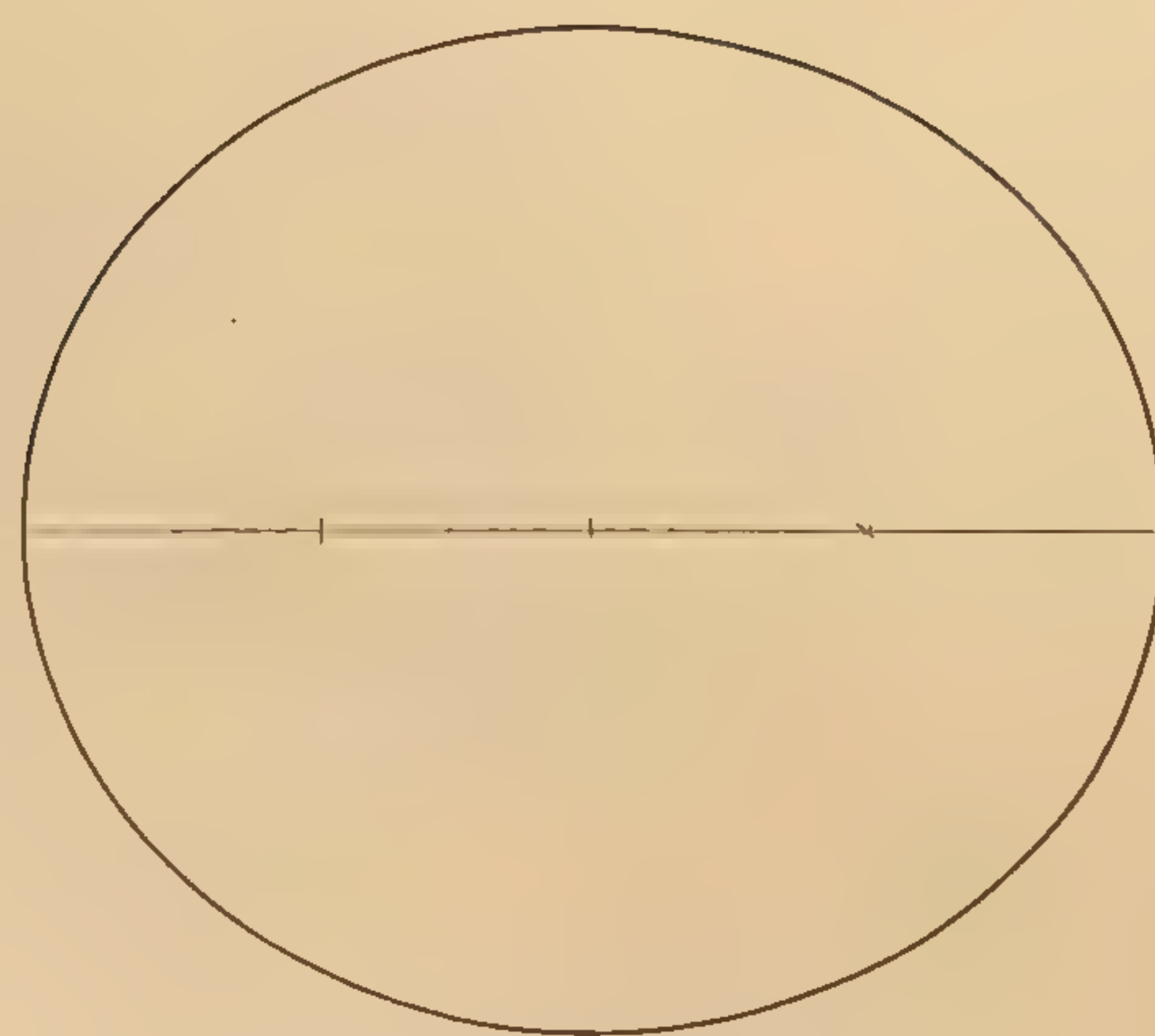
γ Andromedae



γ Virginis.



a =Circle
 b =Mean Planetary Orbit, $e=0.0389$.



Mean Double-Star Orbit, $e=0.482$.

The observer who is aware of the high eccentricities and different inclinations of the orbits will know that in many cases the length of the apparent radius-vector is subject to great variations, and as a shortening of the radius-vector corresponds to accelerated angular motion of the companion, he will never find it safe to assume that the motion is uniform. The forty stars treated in this work present several instances where the angular motion at certain epochs has been extremely rapid, and it is much to be regretted that more observations were not secured at such critical points of the orbits. These general results may prove of value to the observer of the future, and stimulate an increased interest in the systematic measurement of revolving binaries.

§5. *Relative Masses of the Components in Stellar Systems.*

A problem of fundamental importance in the study of the stars is the determination of the relative masses of the components of a system. Such determinations have been made heretofore in very few cases, and even when undertaken have been seriously embarrassed by the errors of observation. It has been customary to base the investigations upon absolute positions determined with the Meridian Circle. The errors of our absolute positions deduced in this way are so large in comparison with the delicate quantities depending on the irregularity of the proper motions of the individual components of a system whose centre of gravity moves uniformly on the arc of a great circle, that the results obtained are affected by large probable errors.

The systems in which such researches have been attempted are:

(1) *α Canis Majoris*, where AUWERS finds the masses to be in the ratio of 1:2.119.

(2) *α Centauri*, in which STONE found the masses approximately equal; ELKIN made them as 1:1.124; and ROBERTS finally concludes from a more elaborate investigation that they are in the ratio of 1:1.041.

(3) *η Cassiopeae*, investigated in 1881 by LUDWIG STRUVE, who found the masses to be in the ratio of 1:3.731.

So far as we are aware these three wide systems are the only ones whose relative masses have been investigated, and we may remark that the condition of each star is favorable to a determination from the circumstance that the pairs are wide and tolerably rapid in their orbital motion, and therefore the

irregularity of the proper motions of the components is conspicuous in comparison with the errors of observation.

There are other systems such as 70 *Ophiuchi*, ξ *Boötis*, and γ *Virginis*, which are favorable for similar investigations, but none have yet been attempted. It would be all the more interesting to investigate the relative masses of 70 *Ophiuchi* from the circumstance that the system contains a dark body which sensibly perturbs the visible components.

In the case of γ *Virginis* we might infer that the masses are nearly equal, as in the system of α *Centauri*.

But even if the bright and widely-separated pairs were all investigated, it would still be difficult to reach any of the small, close stars whose distances are less than two seconds of arc. The investigation of the relative masses of the components of such systems by means of absolute positions determined with the Meridian Circle seems forever impossible, since the stars under such power would seldom be separated, and when separated the errors of observation would be larger than the quantities involved in the determination of the relative masses. The old method is therefore very limited in its application, and a new method must be invented if we are ever to have precise knowledge of the relative masses of the components of binary systems.

We suggest the following method as much more general and also much more exact than the one depending on absolute positions. The distance and position-angle of each component with respect to a neighboring star should be determined at different epochs, the measures being taken with the Heliometer if the distance is large, with the Micrometer if the neighboring star is close or very faint. A series of such relative positions would disclose the location of the centre of gravity by its uniform motion and the resulting conservation of areas with respect to the neighboring star. And since the measures are differential only, it ought to be possible to attain the desired degree of accuracy; the only difficulty likely to arise in practice would be one depending on the personal equations and the constant errors affecting the work of individual observers. Experience alone could determine how serious this difficulty would be, but it seems probable from the results obtained in the measurement of double stars that it would become considerable only in the case of pairs which have no near companion.

Indeed, this method for finding the relative masses of stars is exactly the same as that employed in parallax measurement, except that the observations must extend over the period of a revolution (or a large part of such a period) instead of over the period of one year.

The proposed method therefore is as follows: Let the differences in right ascension and declination with respect to either of the components at the epochs t, t', t'' be

$$\begin{aligned}\Delta\alpha_0 &= \rho_0 \sin\theta_0 \sec\delta_0 & ; & \quad \Delta\delta_0 = \rho_0 \cos\theta_0 \\ \Delta\alpha'_0 &= \rho'_0 \sin\theta'_0 \sec\delta'_0 & ; & \quad \Delta\delta'_0 = \rho'_0 \cos\theta'_0 \\ \Delta\alpha''_0 &= \rho''_0 \sin\theta''_0 \sec\delta''_0 & ; & \quad \Delta\delta''_0 = \rho''_0 \cos\theta''_0\end{aligned}$$

Let the differences in right ascension and declination of the components of the system in like manner be

$$\begin{aligned}\Delta\alpha &= \rho \sin\theta \sec\delta & ; & \quad \Delta\delta = \rho \cos\theta \\ \Delta\alpha' &= \rho' \sin\theta' \sec\delta' & ; & \quad \Delta\delta' = \rho' \cos\theta' \\ \Delta\alpha'' &= \rho'' \sin\theta'' \sec\delta'' & ; & \quad \Delta\delta'' = \rho'' \cos\theta''\end{aligned}$$

Then the coördinates of the centre of gravity of the system referred to the neighboring star will be given by the expressions,

$$\begin{aligned}\Delta\alpha_0 + \frac{M_1}{M_1+M_2} \Delta\alpha & \quad ; \quad \Delta\delta_0 + \frac{M_1}{M_1+M_2} \Delta\delta \\ \Delta\alpha'_0 + \frac{M_1}{M_1+M_2} \Delta\alpha' & \quad ; \quad \Delta\delta'_0 + \frac{M_1}{M_1+M_2} \Delta\delta' \\ \Delta\alpha''_0 + \frac{M_1}{M_1+M_2} \Delta\alpha'' & \quad ; \quad \Delta\delta''_0 + \frac{M_1}{M_1+M_2} \Delta\delta'',\end{aligned}$$

where the formulæ are arranged for the case of the smaller star, which is generally to be preferred, as the magnitude of the absolute orbital motion about the centre of gravity is in the inverse ratio of the masses of the components.

In these expressions the only unknown quantity is the ratio $\frac{M_1}{M_1+M_2}$. The most natural condition for the determination of this unknown is furnished by the principle of the conservation of the motion of the centre of gravity of a system of bodies. When the arc described by the centre of gravity is small, the motion in right ascension and declination is uniform like that in the arc of a great circle. Thus we have

$$\frac{\Delta\alpha'_0 - \Delta\alpha_0 + \frac{M_1}{M_1+M_2} (\Delta\alpha' - \Delta\alpha)}{\Delta\alpha''_0 - \Delta\alpha_0 + \frac{M_1}{M_1+M_2} (\Delta\alpha'' - \Delta\alpha)} = \frac{\Delta\delta'_0 - \Delta\delta_0 + \frac{M_1}{M_1+M_2} (\Delta\delta' - \Delta\delta)}{\Delta\delta''_0 - \Delta\delta_0 + \frac{M_1}{M_1+M_2} (\Delta\delta'' - \Delta\delta)} = \frac{t' - t}{t'' - t}$$

When n sets of independent observations have been secured, the number of equations for the determination of the most probable value of the ratio $\frac{M_1}{M_1+M_2}$ is $2(n-2)$.

If the precession is sensible, the observations of $\theta_o, \theta'_o, \theta''_o$, and $\theta, \theta', \theta''$, etc., must be referred to a common epoch. An independent formula for the determination of the ratio $\frac{M_1}{M_1 + M_2}$ may be deduced from the criterion that the motion of the centre of gravity is confined to the arc of a great circle.

While the method may not prove to be entirely general, owing to the occasional absence of suitable comparison stars, there is reason to think that the Heliometer and Micrometer together ought to prove very effective. Such measurements, if extended to groups of perspective involving two or more objects, will furnish the means also of detecting the existence of any possible irregularities in the proper motions of single stars. In the early days of star cataloguing it was difficult to believe that the proper motions were uniform and rectilinear, but as this has been found to be the general rule, it is now difficult for some to credit the existence of irregularities in the proper motions, or the presence of dark bodies perturbing the motions of the stars. The errors of observation are relatively so large that sound method of procedure requires caution in attributing anomalies to foreign causes, lest by undue credulity we be led to introduce all manner of vain fictions; yet it is certainly unphilosophical to doubt the existence of numerous dark companions which disturb the motions of the fixed stars. It will ultimately be a matter of great interest to determine the extent and the character of such perturbations. These considerations suggest fields of inquiry of the widest scope, and assure us that while exact Astronomy shall be cultivated, the Heliometer and the Micrometer are not likely to lose their present importance, through the introduction of any sort of mechanical methods.

It will be some years before the above method can be applied, and hence it is interesting to reach some general result as to the relative masses of binary stars. The determinations above spoken of, except in the case of *Sirius*, show that the masses are roughly in proportion to the brightness of the stars. This rule would doubtless lead to erroneous conclusions in a good many individual cases, yet in taking double stars as a class, it will give results which are not far from the truth; and hence the light-ratios of the forty stars given in the Table show that on the average the components of binaries are comparable, and frequently almost equal, in mass. This we may infer to be a general law for all binaries, and the corresponding relative masses accord perfectly with those of the double nebulae drawn by SIR JOHN HERSCHEL, and with the mass-ratios resulting from the rupture of the figures of equilibrium of rotating mass of fluid investigated by POINCARÉ and DARWIN.

§ 6. *Exceptional Character of the Planetary System.*

The fundamental result indicated in the foregoing section is in striking contrast with the phenomena presented in the solar system. The masses of the planets are very small compared to that of the Sun, and the masses of the satellites are very small compared to those of the planets around which they revolve. The mass-ratio in the case of the Earth and Moon amounts to $\frac{1}{80}$, and is by far the largest in the solar system. The mass of *Jupiter*, 1047.36 , is much larger than that of any other planet, and yet such a body is wholly insignificant compared to the Sun. If such inconsiderable companions attend the fixed stars, they would neither be visible, nor could they be discovered by any perturbations which they might produce. It is therefore impossible to determine whether the stellar systems include such bodies as the planets, and we are thus unaware of the existence of any other systems like our own. On the other hand the heavens present to our consideration an indefinite number of *double systems*, each of which is divided into comparable masses. These *double systems* stand in direct contrast to the planetary system, where the central body has 746 times the mass of all the other bodies combined. In binary stars the mass distribution is evidently *double*, while in the solar system it is essentially *single*. By a process extending throughout the universe it seems that the nebulae frequently divide into approximately equal or comparable masses, and develop into double stars, while in the case of our own nebula substantially all the matter has gone into the Sun.

Therefore while observation gives us no ground for denying the existence of other systems like our own, it does not enable us on the other hand to affirm or even to render it probable that such systems do exist. And in this state of insufficient evidence we are confronted by the undoubted existence of a great number of systems of an entirely different type. Whatever theories of Cosmogony are proposed, it is evident that in order to have any claim to acceptance, they must be based upon what is really known, not upon what may or may not exist. Those who have proceeded to deduce Cosmogonic processes from our own isolated and abnormal system, have therefore pursued an illogical course, and it is not remarkable that they have failed to throw much light upon the laws of Cosmogony.

The solar system is rendered abnormal by the great number and small masses of its attendant bodies and by the circularity of their orbits about the large central bodies which govern their motion. The system is throughout so

regular, and adjusted to such admirable conditions of stability, that among known systems it stands absolutely unique. Whether observation will ever disclose any other system of such complexity, regularity and harmony, is an interesting question for the future of Astronomy. It is certain that the number of double stars will be augmented in proportion to the diligence of observers and the improvement of our telescopes; and we may reasonably expect a sensible increase in the number of triple and quadruple stars and of stars attended by dark bodies.

Such systems as *Sirius*, *Procyon*, ζ *Canceri* and 70 *Ophiuchi* are not likely to be isolated cases; but caution is required where the observations are not decisive, lest the number be unduly increased by imaginary bodies resulting from errors of observation. It seems probable that a number of double stars are likely to disclose perturbations which can be investigated, and we have already some indications that the motions of ζ *Herculis*, ξ *Ursae Majoris*, μ^1 *Herculis* and η *Coronae Borealis* are not perfectly regular. But in the present state of the measures it seemed best to attribute the apparent irregularities to errors of observation. ζ *Herculis* especially merits the most careful attention of observers; after its periastron passage a refined investigation will show whether the motion is really perturbed.

The question naturally arises whether the stars of these double systems are attended by small dark bodies of a planetary character. We have seen that most of the binaries have highly eccentric orbits, and hence if planetary bodies revolved around either component, they would experience great perturbations, besides the most violent changes of light and heat. It seems probable that planets could not be formed without developing very eccentric orbits, and if once in existence, it is questionable whether such bodies could endure under the violent perturbations to which they would be subjected at periastron passage. Even if a planet were very close to its central star, its motion would be affected by an inequality of enormous magnitude analogous to the annual equation in the moon's motion; and if not destroyed by collision with one of the stars or by disintegration under the tidal forces within ROCHE'S limit, in all probability it would sooner or later be driven from the system on a curve analogous to a parabola or an hyperbola. Thus, while the motion of a planet around one of the components could hardly be so stable as the corresponding phenomena of the solar system, it might yet continue for long ages if the orbit of the binary be not too eccentric; the final state of the system would depend upon the densities, relative masses and distances of the components, the mutual inclinations, and above all, the eccentricities, of their orbits.

See, Thomas Jeff. J.

Researches on the evolution of the stellar systems

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